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FUZZY SOFT CS-CLOSED SPACES IN FUZZY SOFT TOPOLOGICAL SPACES

V. CHANDRASEKAR[1] AND G. ANANDAJOTHI

ABSTRACT. In this paper, we present a fuzzy soft Cs-closed spaces in fuzzy soft Topological Spaces. A few properties and portrayals of this space are discussed. Fuzzy soft regular semi open set is introduced with example.

1. INTRODUCTION AND PRELIMINARIES

Zadeh [12], presented the fuzzy set in 1965 to solve the many real life problems and in 1999, Molodtsov [5] established the concept of soft sets as a sufficient mathematical tool for dealing the problems with uncertainties. Many researchers have applied the concept of fuzzy sets and soft sets separately. Later Maji [2] have initiated the generalized concept of fuzzy soft sets which combines the fuzzy and soft sets. The fuzzy soft topological structure was introduced by Tanay [10] in 2011. Based on this work, some authors studied the concept of fuzzy soft topological spaces [5, 8].

In 2012, Zahran [13] introduced the concept of fuzzy Cs-closed and some of the characterizations in fuzzy topological spaces were studied. Let U be an initial Universe $\&$ E be a set of parameters, $P(U)$ denote the power set of U and A be a nonempty subset of E. In 2001 fuzzy soft set (briefly, fSs) [2], fuzzy soft topology (briefly, fSt) [9], fuzzy soft neighborhood [10], fuzzy soft closure (resp. interior) F_A [6], fuzzy soft semi open (briefly, $fSso$), fuzzy soft semi closed (briefly, $fSsc$) set [8], fuzzy soft semi closure (resp. interior) of

[1]*corresponding author*
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F_A and will be denoted by $Scl(F_A)$ (resp. $Sint(F_A)$), fuzzy soft regular open (resp. closed) set [4], fSo cover [7], fuzzy soft compact [7], fuzzy soft nearly C-compact [3], fuzzy soft filter [1] were given.

Definition 1.1. *[11] Let F_A & G_B be two fSs's over (U, E). The following operations are defined as:*

Subset: $F_A \subset G_B$, if $F_A({}_1e) \subseteq G_B({}_1e)$, $\forall\ {}_1e \in E$.
Equal: $F_A = G_B$, if $F_A({}_1e) \subseteq G_B({}_1e)$ & $G_B({}_1e) \subseteq F_A({}_1e)$.
Union : $H_{A\cup B} = F_A \cup G_B$ where $H_{A\cup B}({}_1e) = F_A({}_1e) \cup G_B({}_1e)\ \forall\ {}_1e \in E$.
Intersection: $H_{A\cap B} = F_A \cap G_B$ where $H_{A\cap B}({}_1e) = F_A({}_1e) \cap G_B({}_1e)\ \forall\ {}_1e \in E$.

Definition 1.2. *[11] The fSs F_A over (U, E) is called a fuzzy soft point in (U, E) denoted by ${}_1e(F_A)$, if for the element ${}_1e \in F_A$, $F({}_1e) \neq 0$ and $F({}_1e) = 0\ \forall\ {}_1e \notin F_A$.*

Definition 1.3. *[8] Let (U_1, E_1, τ_1) and (U_2, E_2, τ_2) be two $fSts$'s. A fuzzy soft function $f_{up} : (U_1, E_1, \tau_1) \to (U_2, E_2, \tau_2)$ is said to be fuzzy*

(i) *soft semi-continuous (resp. fSsCts) if $f_{up}^{-1}(g_B)$ is a $fSsc$ set in (U_1, E_1, τ_1) $\forall$ fuzzy soft closed set g_B in (U_2, E_2, τ_2).*
(ii) *fSsCts if $\forall$ fSs g_B in (U_2, E_2, τ_2), $Scl(f_{up}^{-1}(g_B)) \subseteq f_{up}^{-1}(cl(g_B))$.*
(iii) *soft semi-irresolute (resp. fSsIrr) if the inverse image of each $fSsc$ set is $fSsc$ (U_1, E_1, τ_1).*
(iv) *fSsIrr if $\forall$ fSs g_B in (U_2, E_2, τ_2), $Scl(f_{up}^{-1}(g_B)) \subseteq f_{up}^{-1}(Scl(g_B))$.*

2. Fuzzy soft Cs-closed space

Definition 2.1. *Let (U, E, τ) be $fSts$. Then (U, E, τ) is said to be a fuzzy soft Cs-closed (resp. $fSCsc$) if given a $fSsc$ set F_A on (U, E) and $\forall$ $fSso$ cover ψ of F_A $\exists$ a finite subfamily $\{F_{A_i} : i = 1, 2, 3, \cdots, n\}$ of $\psi \ni F_A \subseteq \bigcup_{i=1}^{n} Scl(F_{A_i})$.*

Remark 2.1. *It is clear that $fSCsc$ implies fuzzy soft nearly C-compactness.*

Definition 2.2. *A fSs F_A in a $fSts$ (U, E, τ) is called a fuzzy soft regular semi open (briefly, $fSrso$) set iff $\exists$ a fuzzy regular open set $G_B \ni G_B \subseteq F_A \subseteq cl(G_B)$.*

Example 1. *Let $X = \{a_1, b_1, c_1\}$, $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2\} \subseteq E$. Let us consider the following fSs's over (X, E):*

$F_{A_1} = \{F(e_1) = \{a_1/0.5, b_1/0.3, c_1/0.3\}, F(e_2) = \{a_1/0.3, b_1/0.3, c_1/0.3\}\}$,
$F_{A_2} = \{F(e_1) = \{a_1/1, b_1/0, c_1/0.5\}, F(e_2) = \{a_1/0.5, b_1/0.3, c_1/1\}\}$,

$F_{A_3} = \{F(e_1) = \{a_1/0.5, b_1/0, c_1/0.3\}, F(e_2) = \{a_1/0.3, b_1/0.3, c_1/0.3\}\}$,
$F_{A_4} = \{F(e_1) = \{a_1/1, b_1/0.3, c_1/0.5\}, F(e_2) = \{a_1/0.5, b_1/0.5, c_1/1\}\}$.
Let us consider the fSt $\tau = \{0_E, 1_E, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}\}$ *over* (X, E). *Now,*
$F^c_{A_1} = \{F^c(e_1) = \{a_1/0.5, b_1/0.7, c_1/0.7\}, F^c(e_2) = \{a_1/0.7, b_1/0.7, c_1/0.7\}\}$,
$F^c_{A_2} = \{F^c(e_1) = \{a_1/0, b_1/1, c_1/0.5\}, F^c(e_2) = \{a_1/0.5, b_1/0.7, c_1/0\}\}$,
$F^c_{A_3} = \{F^c(e_1) = \{a_1/0.5, b_1/1, c_1/0.7\}, F^c(e_2) = \{a_1/0.7, b_1/0.7, c_1/0.7\}\}$,
$F^c_{A_4} = \{F^c(e_1) = \{a_1/0, b_1/0.7, c_1/0.5\}, F^c(e_2) = \{a_1/0.5, b_1/0.5, c_1/0\}\}$.
Clearly, $F^c_{A_1}, F^c_{A_2}, F^c_{A_3}, F^c_{A_4}$ *are fuzzy soft closed sets.*

Obviously $F^c_{A_1}$ *is fuzzy soft regular open set. Consider the* fSs $G_A = \{G(e_1) = \{a_1/0.5, b_1/0.4, c_1/0.3\}, G(e_2) = \{a_1/0.3, b_1/0.5, c_1/0.5\}\}$ *in* τ. *Since* $F_{A_1} \subseteq G_A \subseteq cl(F_{A_1})$. *Hence* G_A *is* $fSrso$ *set in* τ.

Lemma 2.1. *For a* fSs F_A *in a* $fSts$ (U, E, τ). *The following*

(i) *Every* $fSrso$ *set is* $fSso$ *and* $fSsc$.
(ii) $Scl(F_A)$ *is* $fSrso$ $\forall$ $fSso$ *set* F_A *in* (U, E) *are hold.*

Theorem 2.1. *In a* $fSts$ (U, E, τ) *the following conditions are equivalent:*

(i) U *is* $fSCsc$.
(ii) $\forall$ $fSsc$ *set* F_A *on* (U, E) *and* $\forall$ $fSrso$ *cover* ψ *of* F_A $\exists$ *a finite subfamily* $\{F_{A_i} : i = 1, 2, 3, \cdots, n\}$ *of* $\psi \ni F_A \subseteq \bigcup_{i=1}^n F_{A_i}$.
(iii) $\forall$ $fSsc$ *set* F_A *on* (U, E) *and* $\forall$ *family* $\xi = \{G_{A_\alpha}\}_{\alpha\in\Delta}$ *of non-empty* $fSsc$ *sets* $\ni \cap\xi \cap F_A = \phi$ $\exists$ *a finite subfamily* $\{G_{A_i} : i = 1, 2, 3, \cdots, n\}$ *of* $\xi \ni \bigcap_{i=1}^n Sint(G_{A_i}) \cap F_A = \phi$.
(iv) $\forall$ $fSsc$ *set* F_A *on* (U, E) *and* $\forall$ *family* $\xi = \{G_{A_\alpha}\}_{\alpha\in\Delta}$ *of* $fSsc$ *sets, if* $\forall$ *finite subfamily* $\{G_{A_i} : i = 1, 2, 3, \cdots, n\}$ *of* ξ *we have* $\bigcap_{i=1}^n Sint(G_{A_i}) \cap F_A = \phi$ *then* $\cap\xi \cap F_A \neq \phi$.

Proof.

$(i) \Rightarrow (ii)$: Suppose (i) holds. Let F_A be $fSsc$ set and ψ of be a $fSrso$ cover. Then by Lemma 2.1 (i), ψ is a $fSso$ cover of F_A. There exist a finite subfamily $\{F_{A_i} : i = 1, 2, 3, \cdots, n\}$ of $\psi \ni F_A \subseteq \bigcup_{i=1}^n Scl(F_{A_i})$. By Lemma 2.1 (ii), $Scl(F_{A_i})$ is regular semi open and $Scl(F_{A_i})$ is $fSsc$, since F_{A_i} $fSsc$. Hence $F_A \subseteq \bigcup_{i=1}^n Scl(F_{A_i}) \subseteq \bigcup_{i=1}^n F_{A_i}$.

$(ii) \Rightarrow (i)$: Suppose (ii) holds. Let F_A be $fSsc$ set and $\psi = \{F_{A_i} : i = 1, 2, 3, \cdots, n\}$ be a $fSso$ cover of F_A . Then $\xi = \{Scl(F_{A_i})\}$ is a $fSrso$ cover of F_A. By (ii) there exist a finite subfamily $\{Scl(F_{A_i}) : i = 1, 2, 3, \cdots, n\} \ni F_A \subseteq \bigcup_{i=1}^{n} Scl(F_{A_i})$.

$(ii) \Rightarrow (iii)$: Let $\xi = \{G_{A_\alpha}\}_{\alpha\in\Delta}$ be a family of $fSsc$ sets of $fSts$ $(U, E, \tau) \ni \cap\xi \cap F_A = \phi\ \forall\ fSsc$ sets of (U, E, τ). Then $\zeta = \{Scl(G_{A_\alpha})\}_{\alpha\in\Delta}$ is a $fSrso$ cover of F_A. Thus there exist a finite subfamily $Scl(F_A) = \{Scl(G_{A_\alpha}), i = 1, 2, 3, \cdots, n\}$ of $\zeta \ni F_A \subseteq \bigcup_{i=1}^{n} Scl(F_{A_i})$.

Now for each i, we have $int(G_{A_i}) = Sint(F^c_{A_i}) = Sint(E - F_{A_i}) = E - Scl(E - (E - F_{A_i})) = E - Scl(F_{A_i})$.

So $\bigcap_{i=1}^{n} Sint(G_{A_i}) = E - \bigcup_{i=1}^{n} Scl(F_{A_i}) \subseteq E - F_A$, By (i). i.e., $\bigcap_{i=1}^{n} Sint(G_{A_i}) \cap F_A = \phi$.

$(iii) \Rightarrow (ii)$: Let $\psi = \{F_{A_i} : i = 1, 2, 3, \cdots, n\}$ be a $fSso$ cover of the $fSsc$ set F_A of $fSts$ (U, E, τ). Since $F_A h \subseteq \bigcup_{\alpha\in\Delta} F_{A_\alpha}$. We will show that $\bigcap_{i=1}^{n} F^c_{A_i} \cap F_A = \phi$. Since, F^c_A is a family of soft semi closed sets, then by (iii), $\bigcup_{\alpha\in\Delta} F_{A_\alpha} \cap F_A = \phi$, there exist a finite subfamily $F^c_{A_i}$, such that $\bigcap_{i=1}^{n} Sint(F^c_{A_i}) \cap F_A = \phi$. Thus $F_A \subseteq \bigcup_{i=1}^{n}(E - Sint(E - F_{A_i}))$. Now for each i, $Sint(E - F_{A_i}) = E - Scl(E - (E - F_{A_i})) = E - Scl(F_{A_i})$. So $F_A \subseteq \bigcup_{i=1}^{n} Scl(F_{A_i})$. Since F_{A_i} are $fSsc$ sets, Hence $F_A \subseteq \bigcup_{i=1}^{n} F_{A_i}$.

$(iii) \Rightarrow (iv)$: Let F_A be $fSsc$ set and $\xi = \{G_{A_\alpha}\}_{\alpha\in\Delta}$ be a family of $fSsc$ sets, if for each finite subfamily $\{G_{A_i} : i = 1, 2, 3, \cdots, n\}$ of ξ, $\bigcap_{i=1}^{n} Sint(G_{A_i}) \cap F_A = \phi$.

Suppose that $\cap G_{A_i} \cap F_A = \phi$. Then by (ii) $\exists$ a finite subfamily $\{G_{A_i} : i = 1, 2, 3, \cdots, n\}$ of $\xi \ni \bigcap_{i=1}^{n} Sint(G_{A_i}) \cap F_A = \phi$, which is a contradiction. Hence $\cap G_{A_i} \cap F_A = \phi$.

$(iv) \Rightarrow (iii)$: Obvious. □

Theorem 2.2. *Every $fSsc$ subset of a $fSCsc$ space (U, E, τ) is Cs-closed.*

Theorem 2.3. *In a $fSts$ (U, E, τ) the following statements are equivalent:*

(i) *U is $fSCsc$.*
(ii) *If F_A is a proper $fSsc$ set & ϕ is a family of $fSsc$ sets of $(U, E, \tau) \ni F_A \subseteq (E - \bigcap_{i=1}^{n} F_{A_i})$ then there exist a finite subfamily of ϕ say $F_{A_1}, F_{A_2}, \cdots, F_{A_n}$ $\ni F_A \subseteq (E - \bigcap_{i=1}^{n} Sint(F_{A_i}))$.*

Definition 2.3. *Let (U, E, τ) be $fSts$. A fuzzy soft filter in U is said to be semi adherence convergent if every $fSso$ neighborhood of the adherence set of ζ contains an element of ζ where the adherence set is defined by $\bigcap\{Scl(F_A) : F_A \in \zeta\}$.*

Theorem 2.4. *If (U, E, τ) is $fSCsc$ then every $fSso$ filter is semi adherence convergent.*

Theorem 2.5. *Let $f_{up} : (U_1, E_1, \tau_1) \to (U_2, E_2, \tau_2)$ be a fuzzy soft function from a $fSts$ (U_1, E_1, τ_1) to a $fSts$ (U_2, E_2, τ_2). Then the image of a $fSCsc$ space under a fuzzy soft irresolute function is $fSCsc$ space.*

Proof. Let $f_{up} : (U_1, E_1, \tau_1) \to (U_2, E_2, \tau_2)$ be a fuzzy soft irresolute function from $fSCsc$ space U_1 onto U_2 and let F_A be a proper $fSsc$ set in U_2. Let $\psi = \{F_{A_\alpha}\}_{\alpha \in \Delta}$ be a $fSso$ cover of F_A in U_2. Since f_{up} is fuzzy soft irresolute, then $f_{up}^{-1}(F_A)$ is a $fSsc$ set in U_1 & $\{f_{up}^{-1}(F_{A_\alpha})\}_{\alpha \in \Delta}$ is a $fSso$ cover of $f_{up}^{-1}(F_{A_\alpha})$ in U_1. Since U_1 is $fSCsc$, then there is a finite subfamily $\{f_{up}^{-1}(F_{A_\alpha}), i = 1, 2, \cdots, n\}$ such that $f_{up}^{-1}(F_{A_\alpha}) \subseteq \bigcup_{i=1}^{n} Scl(f_{up}^{-1}(F_{A_\alpha})) \subseteq \bigcup_{i=1}^{n} f_{up}^{-1}(Scl(F_{A_\alpha}))$ by Definition 1.3 (iv) and hence $F_A \subseteq \bigcup_{i=1}^{n} Scl(F_{A_i})$. Thus U_2 is $fSCsc$ Space. □

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DEPARTMENT OF MATHEMATICS
KANDASWAMI KANDAR'S COLLEGE
VELUR (NAMAKKAL) - 638 182
TAMIL NADU, INDIA
E-mail address: vckkc3895@gmail.com

DEPARTMENT OF MATHEMATICS
KANDASWAMI KANDAR'S COLLEGE
VELUR (NAMAKKAL) - 638 182
TAMIL NADU, INDIA
E-mail address: jothi.anandhi@gmail.com