

CHARACTERIZATIONS OF SUBMACHINE OF INTERVAL NEUTROSOPHIC AUTOMATA

V. KARTHIKEYAN¹ AND R. KARUPPAIYA

ABSTRACT. The main aim of this paper is to study various characterizations of interval neutrosophic automaton through submachines. We introduce immediate successor, successor, submachine, submachine generated by states set, using interval neutrosophic automaton and discuss their properties.

1. INTRODUCTION

The theory of fuzzy sets was introduced by Zadeh in 1965 [5] as a generalizations of crisp sets. After these fuzzy sets, Atanasiu introduced the concept of intuitionistic fuzzy sets in 1986 [1] which is an extension of fuzzy set. In intuitionistic fuzzy set, each element of the set representing by a membership grade and non-membership grade.

The neutrosophic set(NS) was introduced by Florentin Smarandache [3]. A neutrosophic set N is classified by a T_N , I_N , and F_N where it means Truth, Indeterminacy and Falsity membership function. T_N , I_N , and F_N are real standard and non-standard subsets of $]0^-, 1^+[$.

Wang et al. [4] introduced the notion of interval-valued neutrosophic sets. The concept of interval neutrosophic finite state machine was introduced by Tahir Mahmood [2].

¹corresponding author

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2. PRELIMINARIES

Definition 2.1. [3] Let U be the universe of discourse. A neutrosophic set (NS) N in U is characterized by a truth membership function T_N , an indeterminacy membership function I_N and a falsity membership function F_N , where T_N , I_N , and F_N are real standard or non-standard subsets of $]0^-, 1^+[$. That is

$$N = \{ \langle x, (T_N(x), I_N(x), F_N(x)) \rangle, x \in U, T_N, I_N, F_N \in]0^-, 1^+[\}$$

and with the condition $0^- \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3^+$. We need to take the interval $[0, 1]$ for technical applications instead of $]0^-, 1^+[$.

Definition 2.2. [4] Let U be a universal set. An interval neutrosophic set (INS for short) is of the form

$$N = \{ \langle \alpha_N(x), \beta_N(x), \gamma_N(x) \rangle | x \in U \} = \{ \langle x, [\inf \alpha_N(x), \sup \alpha_N(x)], [\inf \beta_N(x), \sup \beta_N(x)], [\inf \gamma_N(x), \sup \gamma_N(x)] \rangle | x \in U \},$$

where $\alpha_N(x)$, $\beta_N(x)$, $\gamma_N(x) \subseteq [0, 1]$ and the condition that

$$0 \leq \sup \alpha_N(x) + \sup \beta_N(x) + \sup \gamma_N(x) \leq 3.$$

3. INTERVAL NEUTROSOPHIC AUTOMATA

Definition 3.1. [2] $M = (Q, \Sigma, N)$ is called interval neutrosophic automaton (INA for short), where Q and Σ are non-empty finite sets called the set of states and input symbols respectively, and $N = \{ \langle \alpha_N(x), \beta_N(x), \gamma_N(x) \rangle \}$ is an INS in $Q \times \Sigma \times Q$. The set of all words of finite length of Σ is denoted by Σ^* . The empty word is denoted by ϵ , and the length of each $x \in \Sigma^*$ is denoted by $|x|$.

Definition 3.2. [2] $M = (Q, \Sigma, N)$ be an INA. Define an INS,

$$N^* = \{ \langle \alpha_{N^*}(x), \beta_{N^*}(x), \gamma_{N^*}(x) \rangle \}$$

in $Q \times \Sigma^* \times Q$ by

$$\alpha_{N^*}(q_i, \epsilon, q_j) = \begin{cases} [1, 1] & \text{if } q_i = q_j \\ [0, 0] & \text{if } q_i \neq q_j \end{cases}, \quad \beta_{N^*}(q_i, \epsilon, q_j) = \begin{cases} [0, 0] & \text{if } q_i = q_j \\ [1, 1] & \text{if } q_i \neq q_j \end{cases},$$

$$\gamma_{N^*}(q_i, \epsilon, q_j) = \begin{cases} [0, 0] & \text{if } q_i = q_j \\ [1, 1] & \text{if } q_i \neq q_j \end{cases},$$

$$\begin{aligned}
\alpha_{N^*}(q_i, w, q_j) &= \alpha_{N^*}(q_i, xy, q_j) = \bigvee_{q_r \in Q} [\alpha_{N^*}(q_i, x, q_r) \cup \alpha_{N^*}(q_r, y, q_j)], \\
\beta_{N^*}(q_i, w, q_j) &= \beta_{N^*}(q_i, xy, q_j) = \bigvee_{q_r \in Q} [\beta_{N^*}(q_i, x, q_r) \cup \beta_{N^*}(q_r, y, q_j)], \\
\gamma_{N^*}(q_i, w, q_j) &= \gamma_{N^*}(q_i, xy, q_j) = \bigvee_{q_r \in Q} [\gamma_{N^*}(q_i, x, q_r) \cup \gamma_{N^*}(q_r, y, q_j)], \\
\forall q_i, q_j \in Q, w = xy, x \in \Sigma^* \text{ and } y \in \Sigma.
\end{aligned}$$

Definition 3.3. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton and let $q_i, q_j \in Q$. Then q_i is called a immediate successor of q_j if the following condition holds. $\exists x \in \Sigma$ such that $\alpha_{N^*}(q_j, x, q_i) > [0, 0]$, $\beta_{N^*}(q_j, x, q_i) < [1, 1]$, and $\gamma_{N^*}(q_j, x, q_i) < [1, 1]$.

We say that q_i is a successor of q_j if the following condition holds. $\exists w \in \Sigma^*$ such that $\alpha_{N^*}(q_j, w, q_i) > [0, 0]$, $\beta_{N^*}(q_j, w, q_i) < [1, 1]$, and $\gamma_{N^*}(q_j, w, q_i) < [1, 1]$.

We denote by $S(q_j)$ the set of all successors of q_j . For any subset Q' of Q , the set of all successors of Q' denoted by $S(Q')$ is defined to be the set $S(Q') = \bigcup \{S(q_j) \mid q_j \in Q'\}$.

Definition 3.4. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Let $Q' \subseteq Q$. Let $N_{Q'}$ be an interval neutrosophic subset of $Q' \times \Sigma \times Q$ and let $M' = (Q', \Sigma, N')$. The interval neutrosophic automaton M' is called submachine of M if

- (i) $\alpha_N|_{(Q' \times \Sigma \times Q')} = \alpha'_{N'}$.
- (ii) $\beta_N|_{(Q' \times \Sigma \times Q')} = \beta'_{N'}$.
- (iii) $\gamma_N|_{(Q' \times \Sigma \times Q')} = \gamma'_{N'}$, and
- (iv) $S(Q') \subseteq Q'$.

Definition 3.5. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Let $M' = (Q', \Sigma, N') \neq \phi$ be a submachine of M . Then M' is separated if $S(Q \setminus Q') = (Q \setminus Q')$.

Definition 3.6. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Then M is called strongly connected if $\forall q_i, q_j \in Q, q_i \in S(q_j)$.

Definition 3.7. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton and let $M' = (Q', \Sigma, N')$ be a submachine of M . M' is called proper if $Q' \neq Q$ and $Q' \neq \phi$. If M is strongly connected then M has no proper submachine.

Definition 3.8. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton and M having the exchange property if $q_i, q_j \in Q$ and let $Q' \subseteq Q$. If $q_i \in S(Q' \cup \{q_j\})$, $q_i \notin S(Q')$, then $q_j \in S(Q' \cup \{q_i\})$.

Definition 3.9. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton. Let $Q' \subseteq Q$. If $M = \langle Q' \rangle$ then M is generated by Q' .

4. CHARACTERIZATIONS OF SUBMACHINE OF INTERVAL NEUTROSOPHIC AUTOMATA

Theorem 4.1. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automata. Let $q_i, q_j, q_k \in Q$. Then subsequent statements hold.

- (i) q_i is a successor of q_i .
- (ii) if q_j is a successor of q_j and q_k is a successor of q_i , then q_k is a successor of q_i .

Proof.

(i) Since $\alpha_{N^*}(q_i, \epsilon, q_i) = [1, 1] > [0, 0]$, $\beta_{N^*}(q_i, \epsilon, q_i) = [0, 0] < [1, 1]$, and $\gamma_{N^*}(q_i, \epsilon, q_i) = [0, 0] < [1, 1]$, q_i is a successor of q_i .

(ii) Now $\exists x, y \in \Sigma^*$ such that $\alpha_{N^*}(q_i, x, q_j) > [0, 0]$, $\beta_{N^*}(q_i, x, q_j) < [1, 1]$ and $\gamma_{N^*}(q_i, x, q_j) < [1, 1]$. Then $\alpha_{N^*}(q_i, xy, q_k) \geq \alpha_{N^*}(q_i, x, q_j) \wedge \alpha_{N^*}(q_j, y, q_k) > [0, 0]$, $\beta_{N^*}(q_i, xy, q_k) \leq \beta_{N^*}(q_i, x, q_j) \vee \beta_{N^*}(q_j, y, q_k) < [1, 1]$, and $\gamma_{N^*}(q_i, xy, q_k) \leq \gamma_{N^*}(q_i, x, q_j) \vee \gamma_{N^*}(q_j, y, q_k) < [1, 1]$. Hence, q_k is a successor of q_i by Lemma 3.4 from [2]. $\square$

Theorem 4.2. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automata. Then subsequent statements are equivalent.

- (i) M assure the exchange property.
- (ii) $q_i, q_j \in Q$, $q_j \in S(q_i)$ iff $q_i \in S(q_j)$.

Proof.

(1) $\Rightarrow$ (2) : Let $q_i, q_j \in Q$ and $q_i \in S(q_j)$. Now $q_i \notin S(\phi)$. Hence $q_j \in S(q_i)$. Similarly if $q_j \in S(q_i)$ then $q_i \in S(q_j)$.

(2) $\Rightarrow$ (1) : Let $Q' \subseteq Q$, $q_i, q_j \in Q$. Suppose $q_i \in S(Q' \cup q_j)$, $q_i \notin S(Q')$. Then $q_i \in S(q_j)$. Hence $q_j \in S(q_i) \subseteq S(Q' \cup q_i)$. $\square$

Theorem 4.3. Let $M = (Q, \Sigma, N)$ be an interval neutrosophic automaton and $Q' \subseteq Q$. Then $M' = (S(Q'), \Sigma, N_{Q'})$ is a submachine of M where

$$N_{Q'} = \{\langle \alpha_{Q'}, \beta_{Q'}, \gamma_{Q'} \rangle\},$$

$$\alpha_{Q'} = \alpha|_{S(Q') \times \Sigma \times S(Q')}, \beta_{Q'} = \beta|_{S(Q') \times \Sigma \times S(Q')}, \text{ and } \gamma_{Q'} = \gamma|_{S(Q') \times \Sigma \times S(Q')}.$$

Proof. It is obvious that $\alpha_{Q'} = \alpha|_{S(Q') \times \Sigma \times S(Q')}$, $\beta_{Q'} = \beta|_{S(Q') \times \Sigma \times S(Q')}$, and $\gamma_{Q'} = \gamma|_{S(Q') \times \Sigma \times S(Q')}$. Clearly, $S(Q') \subseteq S(S(Q'))$. Let $q_j \in S(S(Q'))$. Then $q_j \in S(Q')$ for some $q_i \in S(Q')$. Thus $q_i \in S(q_k)$ for some $q_k \in Q'$. Now, q_j is a successor of q_i and q_i is a successor of q_k . Hence, q_j is a successor of q_k . Thus, $q_j \in S(q_k) \subseteq S(Q')$. Hence $S(S(Q')) = S(Q')$. Thus $M' = (S(Q'), \Sigma, N_{Q'})$ is a submachine of M . $\square$

5. CONCLUSION

The main aim of the paper is to study the various characterizations of an interval neutrosophic automaton. For Instance, we discuss immediate successor, successor, submachine, submachine generated by states set, using interval neutrosophic automaton.

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DEPARTMENT OF MATHEMATICS
GOVERNMENT COLLEGE OF ENGINEERING
DHARMAPURI-636704, TAMIL NADU, INDIA
E-mail address: vkarthikau@gmail.com

DEPARTMENT OF MATHEMATICS
ANNAMALAI UNIVERSITY
CHIDAMBARAM-608002, TAMIL NADU, INDIA
E-mail address: rajaanju.40@gmail.com