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A NOTE ON HOMEOMORPHISM USING GRILLS

J. R. SUJITHRA[1] AND N. CHANDRAMATHI

ABSTRACT. In this research article we give emphasize to a make out the characterizations and properties of the new type of homeomorphisms called $\zeta - \Re\omega g-$homeomorphism and $\zeta - \Re\omega g^{\circ}-$ homeomorphism described in the grill topological spaces.

1. INTRODUCTION

It is obvious that closed sets are the basic objects in a topological space. Benchalli and Wali introduced Regular weakly closed sets [1]. Levine introduced generalized closed set in topology [4]. N. Chandramathi introduced $\zeta\omega-$closed set and $\zeta\omega$-homeomorphism [2]. We introduce $\zeta - \Re\omega g-$Homeomorphism and $\zeta - \Re\omega g^{\circ}-$Homeomorphism in grill topological space without assuming any separation axioms.

2. PRELIMINARIES

Definition 2.1. *[3] A bijective function $f : (X, \tau) \to (Y, \sigma)$ is called $R-$ homeomorphism if f is both $R-$ continuous and R-open.*

Definition 2.2. *A bijective function $f : (X, \tau) \to (Y, \sigma)$ is called $R^{*}-$ homeomorphism if both f and f^{-1} are continuous.*

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[1]*corresponding author*

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3. $\zeta - \mathfrak{R}\omega g-$ HOMEOMORPHISM

Definition 3.1. *A bijection $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$ is $\zeta - \mathfrak{R}\omega g-$ continuous and $\zeta - \mathfrak{R}\omega g-$open then it is called as a $\zeta - \mathfrak{R}\omega g-$ homeomorphism.*

Case 3.1: Let $X = \{1, 2, 3\}, Y = \{1, 2, 3\}, \tau = \{\varphi, X, \{1\}\}, \upsilon = \{\varphi, X, \{3\}, \{1, 3\}, \{2, 3\}\}$, $\zeta = \{\{1\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}\}$. Define an identity function $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$ as $f\{1\} = \{1\}$, $f\{2\} = \{2\}$, $f\{3\} = \{3\}$. Here f is bijective, $\zeta - \mathfrak{R}\omega g-$ continuous, $\zeta - \mathfrak{R}\omega g-$open. Thus f is satisfying all the conditions of homeomorphisms. So f is a $\zeta - \mathfrak{R}\omega g-$homeomorphism.

Remark 3.1. *All homeomorphisms are $\zeta - \mathfrak{R}\omega g-$homeomorphism.*

Case 3.2: Let $X = Y = \{1, 2, 3\}$, $\tau = \{\varphi, X, \{1\}, \{1, 2\}, \{1, 3\}\}$, $\upsilon = \{\varphi, X, \{1\}, \{3\}, \{1, 3\}, \{2, 3\}\}$ and an identity function $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$ as $f\{1\} = \{1\}$, $f\{2\} = \{2\}$, $f\{3\} = \{3\}$. Here f is not a homeomorphism but it is a $\zeta - \mathfrak{R}\omega g-$homeomorphism.

Theorem 3.1. *All $\zeta-$homeomorphisms are $\zeta - \mathfrak{R}\omega g-$homeomorphism.*

Proof. Consider a ζ-homeomorphic function $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$. Since f is ζ- homeomorphism we know that the function f and its inverse are $\zeta-$continuous functions, one to one and onto functions. But all $\zeta-$ continuous functions are $\zeta - \mathfrak{R}\omega g-$ continuous function, f and its inverse functions are $\zeta - \mathfrak{R}\omega g-$ continuous functions. Hence f is a $\zeta - \mathfrak{R}\omega g-$homeomorphism. □

Theorem 3.2. *All $rwg-$homeomorphism are $\zeta - \mathfrak{R}\omega g-$ homeomorphism.*

Proof. Consider a $rwg-$ homeomorphic function $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$. Since f is $rwg-$homeomorphism we know that the function f and its inverse are $rwg-$ continuous functions, one to one and onto functions. But all $rwg-$ continuous functions are $\zeta - \mathfrak{R}\omega g-$continuous function, f and its inverse functions are $\zeta - \mathfrak{R}\omega g-$continuous functions.

Hence f is a $\zeta - \mathfrak{R}\omega g-$homeomorphism. □

Theorem 3.3. *Define a bijective function $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$. Then the following are equivalent:*

(a) *Its inverse function $f^{-1} : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$ is $\zeta - \mathfrak{R}\omega g-$irresolute.*
(b) *The function is a $\zeta - \mathfrak{R}\omega g^{\circ}-$open function.*
(c) *The function is a $\zeta - \mathfrak{R}\omega g^{\circ}-$closed function.*

Proof.

$(a) \Rightarrow (b)$: Consider a $\zeta - \Re\omega g-$open set S in (X, τ, ζ). According to (a) $(f^{-1})^{-1}(S) = f(S)$ is $\zeta - \Re\omega g-$open in (Y, υ, ζ). Thus the given function is a $\zeta - \Re\omega g^{\circ}-$open function.

$(b) \Rightarrow (c)$: Consider a $\zeta - \Re\omega g-$closed set P. Then $X - P$ is $\zeta - \Re\omega g-$open and by (b) $f(X - P) = Y - f(V)$ is $\zeta - \Re\omega g-$open function in (Y, υ, ζ). So $f(P)$ is $\zeta - \Re\omega g-$closed function in Y. So f is a $\zeta - \Re\omega g^{\circ}-$closed function.

$(c) \Rightarrow (a)$: Let us take a $\zeta - \Re\omega g-$closed set P. By (c) $f(P)$ is $\zeta - \Re\omega g-$closed in (Y, υ, ζ). But $f(P) = (f^{-1})^{-1}(P)$. Hence $(c) \Rightarrow (a)$ holds. □

Remark 3.2. *Let f, k be any two $\zeta - \Re\omega g-$homeomorphisms. Then $k \circ f$ need not to be a $\zeta - \Re\omega g-$homeomorphism.*

Case 3.3: Let $X = \{1, 2, 3\}$, $Y = \{1, 2, 3\}$, $\tau = \{\varphi, X, \{2\}, \{1, 2\}, \{2, 3\}\}$, $\alpha = \{\varphi, X, \{1\}\}$, $\beta = \{\varphi, X, \{1\}, \{3\}, \{1, 3\}\}$. Let us take the corresponding grill as $\zeta = \{\{1\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}\}$. Define an identity function $f : (X, \tau, \zeta) \to (Y, \alpha, \vartheta)$ and $k : (Y, \alpha, \vartheta) \to (Z, \beta, \mu)$ as $f(1) = 1$, $f(2) = 2$, $f(3) = 3$. We can see that the function f and k are $\zeta - \Re\omega g-$homeomorphisms. But we can see that their composition $k \circ f : (X, \tau, \zeta) \to (Z, \beta, \mu)$ is not a $\zeta - \Re\omega g-$homeomorphism. $k \circ f$ is a $\zeta - \Re\omega g-$continuous function but $(k \circ f)(\{1, 2\}) = \{1, 2\}$ is not a $\zeta - \Re\omega g-$open function in (Z, β, μ). Hence $k \circ f$ is not a $\zeta - \Re\omega g-$homeomorphism.

4. $\zeta - \Re\omega g^{\circ}-$ HOMEOMORPHISM

Here we introduce $\zeta - \Re\omega g^{\circ}-$homeomorphism in grill topological space.

Definition 4.1. *A bijection $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$ is called $\zeta - \Re\omega g^{\circ}-$ homeomorphism if both f and f^{-1} are $\zeta - \Re\omega g-$irresolute.*

Theorem 4.1. *All $\zeta - \Re\omega g^{\circ}-$homeomorphisms are $\zeta - \Re\omega g-$homeomorphism. That is $\zeta - \Re\omega g^{\circ} - H(X, \tau, \zeta)$ is a subset of $\zeta - \Re\omega g - H(X, \tau, \zeta)$.*

Proof. We know that all $\zeta - \Re\omega g-$irresolute functions are $\zeta - \Re\omega g-$continuous function and all $\zeta - \Re\omega g^{\circ}-$open functions are $\zeta - \Re\omega g-$open. Hence proved. □

Theorem 4.2. *Let $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$ is a $\zeta - \Re\omega g^{\circ}-$homeomorphism then $f(\zeta - \Re\omega g - int(B)) = \zeta - \Re\omega g - int f(B)$ for all $B \subseteq X$.*

Proof. We have

$$(\zeta - \mathfrak{R}\omega g - \textit{int}(A))^c = (\zeta - \mathfrak{R}\omega g - \textit{cl}(A^c)),$$
$$(\zeta - \mathfrak{R}\omega g - \textit{int}(B)) = (\zeta - \mathfrak{R}\omega g - \textit{cl}(B^c))^c.$$

Then

$$f(\zeta - \mathfrak{R}\omega g - \textit{int}(B)) = f(\zeta - \mathfrak{R}\omega g - \textit{cl}(B^c))^c = (f(\zeta - \mathfrak{R}\omega g - \textit{cl}(B^c)))^c$$
$$= (\zeta - \mathfrak{R}\omega g - \textit{cl}(f(B^c)))^c = (\zeta - \mathfrak{R}\omega g - \textit{int}(f(B))).$$

□

Corollary 4.1. *$f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$ is a $\zeta - \mathfrak{R}\omega g^\circ -$homeomorphism then $f^{-1}(\zeta - \mathfrak{R}\omega g - \textit{int}(B)) = \zeta - \mathfrak{R}\omega g - \textit{int} f^{-1}(B)$ for all $B \subseteq Y$*

Theorem 4.3. *Consider a $\zeta - \mathfrak{R}\omega g^\circ -$homeomorphism $f : (X, \tau, \zeta) \to (Y, \upsilon, \zeta)$. Then f induces an isomorphism from the group $\zeta - \mathfrak{R}\omega g^\circ - H(X, \tau, \zeta)$ onto $\zeta - \mathfrak{R}\omega g^\circ - H(Y, \upsilon, \zeta)$.*

Proof. Let us define a function $\varpi_f : \zeta - \mathfrak{R}\omega g^\circ - H(X, \tau, \zeta) \to \zeta - \mathfrak{R}\omega g^\circ - H(Y, \upsilon, \zeta)$ through the function f by $\varpi_f(g) = f \circ k \circ f^{-1}$ for all $g \in \zeta - \mathfrak{R}\omega g^\circ - H(X, \tau, \zeta)$. Then ϖ_f is a bijective function. Also for every $k_1, k_2 \in \zeta - \mathfrak{R}\omega g^\circ - H(X, \tau, \zeta)$,

$$\varpi_f(k_1 \circ k_2) = f \circ (k_1 \circ k_2) \circ f^{-1} = (f \circ k_1 \circ f^{-1}) \circ (f \circ k_2 \circ f^{-1})$$
$$= \varpi_f(k_1) \circ \varpi_f(k_2).$$

Hence, ϖ_f is a homeomorphism. Therefore it is an isomorphism induced by f. □

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Department of Mathematics
Government Arts College
Ooty-643002, Tamil Nadu, India
E-mail address: sujithra.pkr@gmail.com

Department of Mathematics
Government Arts College
Udumalpet-641126, Tamil Nadu, India
E-mail address: drmathi@gmail.com