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JOHNSONS ALGORITHM IN FUZZY ENVIRONMENT

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ABSTRACT. Classical Johnson algorithm is extended to find fuzzy number job sequencing problem. The processing time is represented by trapezoidal fuzzy number. The trapezoidal fuzzy numbers are defuzzified by using linear ranking function proposed by Maleki [14]. The modified algorithm is illustrated with suitable examples.

1. INTRODUCTION

Scheduling consists of planning and arranging jobs in an orderly sequence of operations in order to 'satisfy the customer's requirements'. Scheduling involves sequencing of activities under time and resource constrains to meet a specific objective. In 1954, Johnson developed an algorithm [5] for solving to machine problems. This approach also extended to solve m-machine problems. In recent years several technique are adopted for solving fuzzy job sequencing problems [7–13].

Here, we apply classical Johnson's algorithm to solve fuzzy sequencing model. The processing time is represented by trapezoidal fuzzy number. The linear ranking function is used to compare fuzzy numbers. This paper is organised as follows: In Section 2 some fundamental concept on fuzzy number and ranking function are given. In Section 3 and 4, the problem statement, terminology are discussed. In Section 5, 6, 7, 8, formulation of fuzzy Johnson's algorithm and examples are discussed. The paper is concluded in Section 9.

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2. Fundamental Of Fuzzy Set Theory

The term "fuzzy" was proposed by Zadeh in 1962. In 1965, he published the paper "Fuzzy Sets". Other references that we can see are [1–6].

Definition 2.1. *(Fuzzy Sets) Let X is a collections of objects denoted generically by X, then a fuzzy set $\underline{A}$ in X is a set of ordered pairs*

$$\underline{A} = \{(x, \mu_{\underline{A}}(x)) \,|x \in X, \mu_{\underline{A}}(x) \in [0,1]\},$$

where $\mu_{\underline{A}}(x)$ is called the membership function.

Definition 2.2. *(Support) The support of a fuzzy set $\underline{A}$ is the crisps set defined by*

$$\underline{A} = \{x \in X | \mu_{\underline{A}}(x) > 0\}.$$

Definition 2.3. *(core) The core of a fuzzy set $\underline{A}$ is the crisp set of points $x \in X$ with $\mu_{\underline{A}}(x) = 1$.*

Definition 2.4. *(Boundary) The boundary of a fuzzy set $\underline{A}$ are defined set of points $x \in X$ such that $0 < \mu_{\underline{A}}(x) < 1$. It is evident that the boundary is defined as the region of the universal set containing elements that have non-zero membership but not complete membership. Fig.1 illustrates the region.*

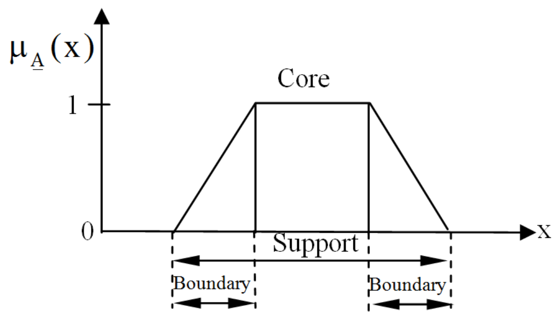

Fig.1

Definition 2.5. *(Normality) A fuzzy set $\underline{A}$ is normal if and only there exists $x_i \in X$ such that $\mu_{\underline{A}}(x_i) = 1$.*

Definition 2.6. *(Sub-normality) A fuzzy set $\underline{A}$ is sub normal if $\mu_{\underline{A}}(x) < 1$.*

Definition 2.7. *(α-cut and strong α-cut) The α-cut of a fuzzy set $\underline{A}$ denoted by $[\underline{A}]_\alpha$ and is defined by $[\underline{A}]_\alpha = \{x \in X | \mu_{\underline{A}}(x) \geq \alpha\}$. If $\mu_{\underline{A}}(x) > \alpha$, then $[\underline{A}]_\alpha$ is called strong α-cut. It is clear that α-cut (strong α-cut) is a crisp set.*

Definition 2.8. *(Convexity) A fuzzy set $\underline{A}$ on X is convex if, for any $x_1, x_2 \in X$ and $\lambda \in [0,1]$,*

$$\mu_{\underline{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_{\underline{A}}(x_1), \mu_{\underline{A}}(x_2)\}.$$

It is to be noted that a fuzzy set is convex if and only if its α-cut is convex.

Definition 2.9. *(Fuzzy number) A fuzzy number is a fuzzy subset in universal set X which is both convex and normal.*

Definition 2.10. *A fuzzy number $\underline{A} = \{a,b,c,d\}$ is said to be a trapezoidal fuzzy number if its membership function is given by*

$$\mu_{\underline{A}}(x) \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x < b \\ 1 & b \leq x \leq c \\ \frac{(x-d)}{(c-d)}, & c < x \leq d \\ 0 & \textit{otherwise} \end{cases}.$$

Definition 2.11. *(Trapezoidal Fuzzy number) Let $\underline{A} = (a^L, a^U, \alpha, \beta)$ be the TrFN, where $(a^L - \alpha, a^U + \beta)$ is the support $\underline{A}$ and $[a^L, a^U]$ is the core of $\underline{A}$.*

Arithmetic on Trapezoidal Fuzzy Numbers

Let $F(R)$ be set of all trapezoidal fuzzy numbers over the real line R. The arithmetic operations on trapezoidal fuzzy numbers are defined as follows:
Let $\underline{a} = (a^L, a^U, \alpha, \beta)$ and $\underline{b} = (b^L, b^U, \gamma, \delta)\left(\frac{\pi}{2} - \theta\right)$ be two trapezoidal fuzzy numbers and $x \in R$. We define

$$\begin{aligned} x > 0, x \in R; x\underline{a} =& (xa^L, xa^U, x\alpha, x\beta) \\ x < 0, x \in R; x\underline{a} =& (xa^U, xa^L, -x\beta, -x\alpha) \\ \underline{a} + \underline{b} =& (a^L + b^L, a^U + b^U, \alpha+\gamma, \beta+\delta) \\ \underline{a} - \underline{b} =& (a^L - b^L, a^U - b^U, \alpha+\delta, \beta+\gamma). \end{aligned}$$

Ranking Function

Ranking is one of the effective method for ordering fuzzy numbers. Various types of ranking function have been introduced and some have been used for solving linear programming problems with fuzzy parameters. An effective approach for ordering the element of $F(R)$ is to define a ranking function. Let $\mathfrak{R} : F(R) \to (R)$. We define order on $F(R)$ as follows:

$$(1) \quad \underset{\mathfrak{R}}{a \geq b} \text{ iff } \mathfrak{R}(\underline{a}) \geq \mathfrak{R}(\underline{b}) \quad (2) \quad \underset{\mathfrak{R}}{a \geq b} \text{ iff } \mathfrak{R}(\underline{a}) > \mathfrak{R}(\underline{b})$$

$$(3) \quad \underset{\mathfrak{R}}{\underline{a = b}} \text{ iff } \mathfrak{R}(\underline{a}) = \mathfrak{R}(\underline{b}) \qquad (4) \quad \underset{\mathfrak{R}}{\underline{a \leq b}} \text{ iff } \mathfrak{R}(\underline{b}) \leq \mathfrak{R}(\underline{a})$$

Here $\mathfrak{R}$ is the ranking functions, such that $\mathfrak{R}(k\underline{a} + \underline{b}) = k\mathfrak{R}(\underline{a}) + \mathfrak{R}(\underline{b})$.

Now we introduce a linear ranking function that is similar to the ranking function adopted by Maleki [14]. For a trapezoidal fuzzy number $\underline{a} = (a^L, a^U, \alpha, \beta)$, we use ranking function as follows:

$$\mathcal{R}(\underline{a}) = \int_0^1 (\inf \underline{a}_\alpha + \sup \underline{a}_\alpha) d\alpha, \qquad \mathcal{R}(\underline{a}) = (a^L + a^U) + \frac{1}{2}(\beta - \alpha).$$

For any trapezoidal fuzzy numbers $\underline{a} = (a^L, a^U, \alpha, \beta)$ and $\underline{b} = (b^L, b^U, \gamma, \delta)$.

3. Problem Statement

Consider n different jobs that has to be processed through m-machines. Here we have to find the sequence for processing the jobs that minimizes total time.

4. Terminology and Notations

Processing order: Refers to sequence of machines to be used.

Fuzzy Processing time: Refers to time for a job on each machine.

Fuzzy idle time on a machine: Time for which a machine does not process a job.

Fuzzy makespan: Refers to total time for completing the first and last job including the idle time.

Notation:

$\underline{t_{ij}}$= Fuzzy processing time for job i on machine j

$\underline{\mathrm{T}}$ = Fuzzy makespan

$\underline{1_{ij}}$= Fuzzy idle time on machine j from the end of job $(i-1)$ to the start of i.

5. Fuzzy Johnson's Algorithm For Processing N Jobs Through Two Machines

Step 1: Express the given data in a tabular form:

Step 2: Find Minimum of $\{\mathfrak{R}(\underline{t}_{1j}), \mathfrak{R}(\underline{t}_{2j})\} \forall j$.

Job	1	2	3	$\cdots$	n
Fuzzy processing time Machine 1	$\underline{t}_{11}$	$\underline{t}_{12}$	$\underline{t}_{13}$	$\cdots$	$\underline{t}_{1n}$
Fuzzy processing time Machine 2	$\underline{t}_{21}$	$\underline{t}_{22}$	$\underline{t}_{23}$	$\cdots$	$\underline{t}_{2n}$

Step 3: (a) If the minimum belongs to $M|C-1$, process 1^{st} & if belongs to $M|C-2$ process last.

(b) In case of tie, we have 3 cases:

(i) If $\min(\underline{t}_{1j}, \underline{t}_{2j}) = \Re(\underline{t}_{1k}) = \Re\underline{t}_{2r}$ then process k^{th} job $1st$ and rth job last.

(ii) If tie belongs to $M|C-1$ only, then select the job with smallest job subscript.

(iii) If tie belongs to $M|C-2$, then select the job corresponding to the largest job subscript last.

Step 4: Remove the assigned jobs. If the table is empty, stop and go to **Step 5**, otherwise, go to **Step 2**.

Step 5: Find fuzzy idle time for $M|C-1$ & $M|C-2$.

(a) Fuzzy idle time for $M|C-1$ = (Total fuzzy elapsed time) - (time when the last job in a sequence finishes in $M|C-1$)

(b) Fuzzy idle time for $M|C-2$ = Time at which the first job in a sequence finishes on $M|C-1+\sum_{j=1}^{n}$(time when the j^{th} job in a sequence starts on $M|C-2$)-(Time when the $(j-1)^{th}$ j on in a sequence finishes on $M|C-2$)

Step 6: Total fuzzy elapsed time = Time when n^{th} job in a sequence finishes on

$$M|C-2 = \sum_{j=1}^{n} \underline{B}_{2j} + \sum_{j=1}^{n} \underline{1}_{2j}$$

where $\underline{B}_{2j}$= Time for processing j^{th} job on $M|C-2$, $\underline{1}_{2j}$=Fuzzy idle time.

6. Numerical Example

Example 1. *Find optimum sequence and fuzzy makespan:*

Job	1	2	3	4	5	6
Machine 1	(3,4,5,6)	(3,4,7,10)	(1,2,3,5)	(8,2,3,5)	(10,11,14,15)	(7,9,10,13)
Machine 2	(8,9,10,12)	(6,7,7.5,8)	(6,8,10,14)	(6,7,8,9)	(5,9,10,12)	(10,12,14,15)

Step 1: Expressing the data as:

Job	1	2	3	4	5	6
Machine 1	(3,4,5,6) $\Re(t_{11} = 7.5)$	(3,4,7,10) $\Re(t_{12} = 8.5)$	(1,2,3,5) $\Re(t_{13} = 4)$	(8,2,3,5) $\Re(t_{14} = 21)$	(10,11,14,15) $\Re(t_{15} = 21.5)$	(7,9,10,13) $\Re(t_{16} = 17.5)$
Machine 2	(8,9,10,12) $\Re(t_{21} = 18)$	(6,7,7.5,8) $\Re(t_{22} = 13.25)$	(6,8,10,14) $\Re(t_{23} = 16)$	(6,7,8,9) $\Re(t_{24} = 13.5)$	(5,9,10,12) $\Re(t_{25} = 15)$	(10,12,14,15) $\Re(t_{26} = 22.5)$

Step 2: Since the minimum fuzzy processing time is (1, 2, 3, 5) corresponding to job 3 occurring under machine 1. Hence, job 3 is to be processed first. Next minimum fuzzy processing time is (3, 4, 5, 6) against the job 1 under machine 1. Hence, job 1 is to be processed next to job 3.
The next minimum fuzzy processing time is (3, 4, 7, 10) against the job 2 under machine 1. Hence job 2 is to be processed next to job 1. The next minimum fuzzy processing time is (6, 7, 8, 9) against the job 4 under machine 2. Hence job 4 is to be processed last.

Continuing in the same manner the optimal processing jobs is given by

3	1	2	6	5	4

Hence, the flow of jobs through machine 1 and 2 using the optimal sequence:

$$3 \to 1 \to 2 \to 6 \to 5 \to 4$$

Table 1: Computation of total fuzzy elapsed time (fuzzy makespan) of the sequence (in Section 10. Tables).

Therefore the minimum fuzzy elapsed time = (42, 54, 62.5, 75)
Fuzzy idle time for M|C-1 = (0, 22,126.5, 127)
Fuzzy idle time for M|C-2 = (1, 2, 3, 5).

7. N Jobs Through 3 MCs

Consider n jobs $(1, 2, 3, \ldots, n)$ processing through 3 machine $M1$, $M2$, $M3$ in the order $M1M2M3$. Fuzzy Johnson's method can be extended when either one or both of the following conditions hold.

(1) Min time on $M|C \underset{\Re}{M_1 \geq}$ Max time on $M|CM2$.

(2) Mini time on $M|C \underset{\Re}{M_3 \geq}$ Max time on $M|CM2$.

Optimal sequence of n jobs 3 machines can be obtained by converting into a problem involving n jobs and 2 imaginary $M|Cs$ X and Y.

The following steps are used to convert the given problem into a 2 machine problem.

Step 1: Find the minimum fuzzy processing time for the jobs on the first and last machines and the maximum fuzzy processing time for the second machine.

Step 2: Check the following inequality

$$\underset{i}{\textit{Min}\ \underline{M}_{1i}} \ \underset{\Re}{\geq} \ \underset{i}{\textit{Max}\ \underline{M}_{2i}} \quad \textit{or} \quad \underset{i}{\textit{Min}\ \underline{M}_{3i}} \ \underset{\Re}{\geq} \ \underset{i}{\textit{Max}\ \underline{M}_{2i}} .$$

Step 3: If none of the inequalities in **Step 2** are satisfied, then method cannot be applied.

Step 4: If at least one of the inequalities in **Step 2** is satisfied, we define two hypothetical or imaginary machine X and Y and corresponds fuzzy processing time. $\underline{X_i}$ and $\underline{Y_i}$ are defined by

$$\begin{aligned} \underline{X_i} &= \underline{M_{1i}} + \underline{M_{2i}}, \qquad & i = 1, 2, 3, \ldots, n \\ \underline{Y_i} &= \underline{M_{2i}} + \underline{M_{3i}}, \qquad & i = 1, 2, 3, \ldots, n \end{aligned}$$

Step 5: For the converted machines X and Y we obtain the optimum sequence using n-job-2 machine algorithm.

Example 2. *Find optimum sequence and fuzzy makespan:*

Table 2 (in Section 10. Tables).

Step 1: Expressing the data

Table 3 (in Section 10. Tables).

Step 2: Since minimum of fuzzy processing time in machine 3 = Max of fuzzy processing time in machine 2. Hence Fuzzy Johnson's algorithm can be applied for finding the optimal sequence.

Step 3: Let us define two imaginary machine X and Y and fuzzy processing time expressed in tabular form.
Table 4 (in Section 10. Tables).

Step 4: Since minimum fuzzy processing time is (8, 13, 16, 22) and (10, 12, 14, 18). But (8,13, 16, 22) corresponds to job 1 occurring under imaginary machine X (First Machine). Here job 1 is to be processed first. Again (10, 12, 14, 18) corresponds to job 5 occurring under imaginary machine Y (second machine). Hence job 5 is to be processed last.

Next minimum fuzzy processing time is (10, 12, 14, 19)corresponds to job 3 occurring under imaginary machine Y (second machine). Hence job 3 is to be processed last but one position i.e. before job 5 is processed.

Continuing in the same manner the optimal sequence of processing the job is given by

1	4	7	2	6		5

Table 5 (in Section 10. Tables).

Therefore the minimum fuzzy elapsed time = (55, 78, 100, 116)
Fuzzy idle time for Machine 1 = (-5, 36,191, 193)
Fuzzy idle time for Machine 2 = (-44, 92, 616, 613)
Fuzzy idle time for Machine 3 = (8, 13, 16, 22)

8. N Jobs Through M machines

Suppose there are n jobs to be processed through m $M|Cs$, $M1$, $M2$, ..., Mm in the order $M1M2\ldots Mm$. Let T_{ij} be fuzzy processing time taken by i_{th} $M|C$ to complete the j_{th} job. There is so general method available by which we can obtain optimal sequence(s) in problems of this type.

This can be transformed to a problem of processing n jobs through $2M|Cs$ if no passing of jobs is permissible and if either or both of the conditions given in Step 2 of the procedure that follows is satisfied. Procedure for obtaining optimal sequence.

Step 1: Find

(i) $\min_j \Re(\underline{T}_{1j})$

(ii) $\min_j \Re(\underline{T}_{mj})$ and

(iii) $\min_j \left[\Re(\underline{T}_{2j}), \Re(\underline{T}_{3j}), \ldots, \Re(\underline{T}_{(m-1)j})\right]$, for $j = 1, 2, \ldots, n$.

Step 2: Check whether

(i) $\min_j \Re(\underline{T}_{ij}) \geq \max_j \Re(\underline{T}_{ij})$, for $i = 1, 2, 3, \ldots, m-1$ (or)

(i) *Min*$_j$ $\Re(\underline{T}_{ij}) \geq$ *Max*$_j$ $\Re(\underline{T}_{ij})$, for $i = 1, 2, 3, \ldots, m-1$

Step 3: If in equations of **Step 2** are not satisfied, this method fails. Otherwise, go to next step.

Step 4: Convert the m machine problem into a 2-machine problem considering two imaginary $M|Cs$ G and H, so that

$$\underline{T}_{Gj} = \underline{T}_{1j} + \underline{T}_{2j} + \ldots + \underline{T}_{(m-1)j}$$
$$\underline{T}_{Hj} = \underline{T}_{2j} + \underline{T}_{3j} + \ldots + \underline{T}_{Mj}$$

Now, determine the optimal sequence on n jobs through 2 machines by using the optimal sequence algorithm.

Step 5: Along with conditions in **Step 4**, If $\Re(T_{2j}) + \Re(T_{3j}) + \ldots + \Re(T_{(m-1)j}) = C, \forall j = 1, 2, \ldots, n$, then determine the optimal sequence for n jobs and two $M|Cs$ $M1$ and Mm in the order $M1Mm$ by using the fuzzy Jhonson's algorithm.

Example 3. *Find optimum sequence and fuzzy makespan:*

Job-j	M_1	M_2	M_3	M_4	M_5	M_6
1	(9,10,12,13)	(3,5,7,9)	(3,4,5,7)	(1,2,4,5)	(4,5,6,9)	(12,13,15,17)
2	(8,10,12,13)	(3,4,5,6)	(4,5,6,8)	(3,4,5,6)	(3,5,7,9)	(9,10,12,14)
3	(5,7,10,11)	(2,3,5,7)	(3,5,7,9)	(2,3,5,7)	(3,4,5,7)	(6,9,11,13)
4	(10,11,14,15)	(2,3,5,6)	(1,2,3,5)	(2,3,5,6)	(3,5,7,9)	(5,7,10,12)

Step 1: Expressing the data.

Table 6 (in Section 10. Tables).

Step 2: Here $\min_j \Re(\underline{T}_{ij}) = 1.5$, $\min_j \Re(\underline{T}_{6j}) = 1.3$, $\max_j \Re(\underline{T}_{2j}) = 9$, $\max_j \Re(\underline{T}_{3j}) = 10$, $\max_j \Re(\underline{T}_{4j}) = 7.5$, $\max_j \Re(\underline{T}_{5j}) = 10.5$, Hence the condition of **Step 2** satisfied. So the problem can be converted into 4-job are 2 machines problem. Thus we define $\underline{G}$ and $\underline{H}$ two imaginary machine such that

$$\underline{T}_{Gj} = \sum_{i=1}^{5} \underline{T}_{ij}, \qquad \underline{T}_{Hj} = \sum_{i=2}^{6} \underline{T}_{ij},$$

Then the problem can be reformulated as:

Job-j	1	2	3	4
Machine $\underline{G}$	(20,26,34,43)	(21,28,35,42)	(15,22,32,41)	(18,24,34,41)
Machine $\underline{H}$	(23,29,37,47)	(22,28,35,43)	(16,24,43,33)	(13,20,30,38)
$\Re(\underline{G})$	52.5	52.5	41.5	45.5
$\Re(\underline{H})$	57	54	45	37

Step 3: Applying fuzzy Johnson algorithm for processing n-jobs through two machines the optimal job sequence is given by

3	1	2	4

Step 4: Computation of total fuzzy elapsed time (fuzzy makespan) of the sequence.

Table 7 (in Section 10. Tables).

Table 8 (in Section 10. Tables).

Therefore the minimum fuzzy elapsed time = (51, 63, 81, 97).

Fuzzy idle time for Machine 1 = (13, 31,133, 145).

Fuzzy idle time for Machine 2 = (19, 56, 156, 274).

Fuzzy idle time for Machine 3 = (14, 58, 289, 300).

Fuzzy idle time for Machine 4 = (18, 70, 330, 341).

Fuzzy idle time for Machine 5 = (10, 69, 369, 377).

Fuzzy idle time for Machine 6 = (41, 74, 180, 175).

9. Conclusion

The proposed fuzzy job sequencing algorithm can be applied to solve the problems occurring in real life situation.

10. TABLES

TABLE 1. Table 1: Computation of total fuzzy elapsed time (fuzzy makespan) of the sequence

		$M\|C-1$			$M\|C-2$			
Job	Time in	Fuzzy Proce -ssing Time	Time out	Time in	Fuzzy Proce -ssing Time	Time out Time out	Fuzzy idle time Machine 1	Fuzzy idle time Machine 2
3	(0,0,0,0) 0	(1,2,3,5) 5	(1,2,3,5) 5	(1,2,3,5) 5	(6,8,10,14) 18	(7,10,13,19) 23	(0,0,0,0)	(1,2,3,5)
1	(1,2,3,5) 5	(3,4,5,6) 8	(4,6,8,11) 13	(7,10,13,19) 23	(8,9,10,12) 11	(15,19,23,31) 42	-	-
2	(4,6,8,11) 13	(3,4,7,10)	(7,10,15,21)	(15,19,23,31)	(6,7,7.5,8)	(21,26,30.5,39)	-	-
6	(7,10,15,21)	(7,9,10,13)	(14,19,25,34)	(21,26,30.5,39)	(10,12,14,15)	(31,38,44.5,54)	-	-
5	(14,19,25,34)	(10,11,14,15)	(24,30,39,49)	(31,38,44.5,54)	(5,9,10,12)	(36,47,54.5,66)	-	-
4	(24,30,39,49)	(8,12,13,15)	(32,42,52,64)	(36,47,54.5,66)	(6,7,8,9)	(42,54,62.5,75)	-	-

TABLE 2. Table 2

Job	1	2	3	4	5	6	7
Machine 1	(3,5,7,12)	(7,10,13,14)	(6,10,13,14)	(5,7,9,11)	(8,9,11,13)	(7,10,13,14)	(6,9,11,13)
Machine 2	(5,8,9,10)	(3,5,7,12)	(4, 5,6,9)	(6,7,8,10)	(4,5,6,8)	(5,8,9,10)	(3,5,7,12)
Machine 3	(5,10,15,16)	(6,9,11,13)	(6,7,8,10)	(9,11,13,14)	(6,7,8,10)	(5,10,15,16)	(10,11,14,15)

TABLE 3. Table 3

Job	1	2	3	4	5	6	7
Machine 1	(3,5,7,12) $\mathfrak{R}(\underline{t}_{11}) = 10.5$	(7,10,13,14) $\mathfrak{R}(\underline{t}_{12}) = 17.5$	(6,10,13,14) $\mathfrak{R}(\underline{t}_{13}) = 16.5$	(5,7,9,11) $\mathfrak{R}(\underline{t}_{14}) = 13$	(8,9,11,13) $\mathfrak{R}(\underline{t}_{15}) = 18$	(7,10,13,14) $\mathfrak{R}(\underline{t}_{16}) = 17.5$	(6,9,11,13) $\mathfrak{R}(\underline{t}_{17} = 16)$
Machine 2	(5,8,9,10) $\mathfrak{R}(\underline{t}_{21}) = 13.5$	(3,5,7,12) $\mathfrak{R}(\underline{t}_{22}) = 10.5$	(4, 5,6,9) $\mathfrak{R}(\underline{t}_{23}) = 10.5$	(6,7,8,10) $\mathfrak{R}(\underline{t}_{24}) = 14$	(4,5,6,8) $\mathfrak{R}(\underline{t}_{25}) = 10$	(5,8,9,10) $\mathfrak{R}(\underline{t}_{26}) = 13.5$	(3,5,7,14) $\mathfrak{R}(\underline{t}_{27} = 11.5)$
Machine 3	(5,10,15,16) $\mathfrak{R}(\underline{t}_{31}) = 15.5$	(6,9,11,13) $\mathfrak{R}(\underline{t}_{32}) = 16$	(6,7,8,10) $\mathfrak{R}(\underline{t}_{33}) = 14$	(9,11,13,14) $\mathfrak{R}(\underline{t}_{34}) = 20.5$	(6,7,8,10) $\mathfrak{R}(\underline{t}_{35}) = 14$	(5,10,15,16) $\mathfrak{R}(\underline{t}_{36}) = 15.5$	(10,11,14,15) $\mathfrak{R}(\underline{t}_{37} = 21.5)$

TABLE 4. Table 4

Job	1	2	3	4	5	6	7
Machine $\underline{X}$	(8,13,16,22)	(10,15,20,26)	(10,15,19,23)	(11,14,17,21)	(12,14,17,21)	(12,18,22,24)	(9,14,18,27)
Machine $\underline{Y}$	(10,18,24,26)	(9,14,18,25)	(10,12,14,19)	(15,18,21,24)	(10,12,14,18)	(10,18,24,26)	(13,16,21,29)
$\mathfrak{R}(\underline{X})$	24	28	27	27	28	31	27.5
$\mathfrak{R}(\underline{Y})$	29	26.5	24.5	34.5	24	29	32

TABLE 5. Table 5: Computation of total fuzzy elapsed time (fuzzy makespan) of the sequence

		$M\|C-1$			$M\|C-2$			$M\|C-3$				
Job	Time in	F.P.T	Time out	Time in	F.P.T	Time out	Time in	F.P.T	Time out	F.I.T on $M\|C-1$	F.I.T on $M\|C-2$	F.I.T on $M\|C-3$
1	(0,0, 0,0)	(3,5, 7,12)	(3,5, 7,12)	(3,5, 7,12)	(5,8, 9,10)	(8,13, 16,22)	(8,13, 16,22)	(5,10, 15,16)	(13,23, 31,38)	-	(3,5, 7,12)	(8,13, 16,22)
4	(3,5, 7,12)	(5,7, 9,11)	(8,12, 16,23)	(8,12, 16,23)	(6,7, 8,10)	(14,19, 24,33)	(13,23, 31,38)	(9,11, 13,14)	(22,34, 44,52)	-	-	-
7	(8,13, 16,22)	(6,9, 11,13)	(14,21, 27,36)	(14,21, 27,36)	(3,5, 7,12)	(17,26, 34,48)	(22,34, 44,52)	(10,11, 14,15)	(32,45, 58,67)	-	(-5,7, 60,60)	-
2	(14,21, 27,36)	(7,10, 13,14)	(21,31, 40,50)	(21,31, 40,50)	(3,5, 7,12)	(24,36, 47,62)	(32,45, 58,67)	(6,9, 11,13)	(38,54, 69,80)	-	(-5,14, 90,84)	
6	(21,31, 40,50)	(7,10, 13,14)	(28,41, 53,64)	(28,41, 53,64)	(5,8, 9,10)	(33,49, 62,74)	(38,54, 69,80)	(15,10, 15,16)	(43,64, 84,96)	-	(-8,17, 115,111)	
3	(28,41, 53,64)	(6,10, 13,14)	(34,51, 66,78)	(34,51, 66,78)	(4,5, 6,9)	(38,56, 72,87)	(43,64, 84,96)	(6,7, 8,10)	(49,71, 92,106)	-	(-15,18, 140, 140)	
5	(34,51, 66,78)	(8,9,) 11,13)	(42,60, 77,91)	(43,64, 84,96)	(4,5, 6,8)	(47,69, 90,104)	(49,71, 92,106)	(6,7, 8,10)	(55,78, 100,116)	(-5,36, 191,193)	(-14,31, 204,106)	

TABLE 6. Table 6

Job-j	M_1	$\Re(\underline{T}_{1j})$	M_2	$\Re(\underline{T}_{2j})$	M_3	$\Re(\underline{T}_{3j})$	M_4	$\Re(\underline{T}_{4j})$	M_5	$\Re(\underline{T}_{5j})$	M_6	$\Re(\underline{T}_{6j})$
1	(9,10,12,13)	19.5	(3,5,7,9)	9	(3,4,5,7)	8	(1,2,4,5)	3.5	(4,5,6,9)	10.5	(12,13,15,17)	26
2	(8,10,12,13)	18.5	(3,4,5,6)	7.5	(4,5,6,8)	10	(3,4,5,6)	7.5	(3,5,7,9)	9	(9,10,12,14)	20
3	(5,7,10,11)	12.5	(2,3,5,7)	6	(3,5,7,9)	9	(2,3,5,7)	6	(3,4,5,7)	8	(6,9,11,13)	16
4	(10,11,14,15)	21.5	(2,3,5,6)	5.5	(1,2,3,5)	4	(2,3,5,6)	5.5	(3,5,7,9)	9	(5,7,10,12)	13

TABLE 7. Table 7

		$M\|C-1$			$M\|C-2$			$M\|C-3$			$M\|C-4$	
Job	Time in	F.P.T	Time out	Time in	F.P.T	Time out	Time in	F.P.T	Time out	Time in	F.P.T	Time out
3	(0,0, 0,0)	(5,7, 10,11)	(5,7, 10,11)	(5,7, 10,11)	(2,3, 5,7)	(7,10, 15,18)	(7,10, 15,18)	(3,5, 7,9)	(10,15, 22,27)	(10,15, 22,27)	(2,3, 5,7)	(12,18, 27,34)
1	(5,7, 10,11)	(9,10, 12,13)	(14,17, 22,24)	(14,17, 22,24)	(3,5, 7,9)	(17,22, 29,33)	(17,22, 29,33)	(3,4, 5,7)	(20,26, 34,40)	(20,26, 34,40)	(1,2, 4,5)	(21,28, 38,45)
2	(14,17, 22,24)	(8,10, 12,13)	(22,27, 34,37)	(22,27, 34,37)	(3,4, 5,6)	(25,31, 39,43)	(25,31, 39,43)	(4,5, 6,8)	(29,36, 45,51)	(29,36, 45,51)	(3, 4, 5, 6)	(32,40, 50,37)
4	(22,27, 34,37)	(10,11, 14,15)	(32,38, 48,52)	(32,38, 48,52)	(2, 3, 5, 6)	(34,41, 53,80)	(34,41, 53,80)	(1,2, 3,5)	(35,43, 56,63)	(35,43, 56,63)	(2,3, 5,6)	(37,46, 61,69)

TABLE 8. Table 8

		$M\|C-5$			$M\|C-6$			Fuzzy	Idle	Time		
Job	Time in	F.P.T	Time out	Time in	F.P.T	Time out	$M\|C1$	$M\|C2$	$M\|C3$	$M\|C4$	$M\|C5$	$M\|C6$
3	(12,18, 27,34)	(3,4, 5,7)	(15,22, 32,41)	(15,22, 32,41)	(6,9, 11,13)	(21,31, 43,54)		(5,7, 10,11)	(5,7, 10,15)	(10,15, 22,27)	(12,18, 27,34)	(15,22, 32,41)
1	(21,28, 38,45)	(4,5, 6,9)	(25,33, 44,54)	(25,33, 44,54)	(12,13, 15,17)	(37,46, 59,71)		(4,10, 40,39)	(2,12, 56,55)	(2,14, 68,67)	(-1,13, 79,77)	(-6,12, 98,97)
2	(32,40, 50,37)	(3,5, 7,9)	(35,45, 57,66)	(37,46, 59,71)	(9,10, 12,14)	(46,56, 71,85)		(0,10, 67,66)	(-1,11, 79,77)	(1,15, 90,89)	(-1,15, 104,101)	(32,40, 50,37)
4	(37,46, 61,69)	(3,5, 7,9)	(40,51, 68,78)	(46,56, 71,85)	(5,7, 10,12)	(51,63, 81,97)						

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