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A MODERN APPROACH FOR SOLVING INTERVAL BASED ASSIGNMENT PROBLEM

M. RADHA [1], S. ANANTHALAKSHMI, AND R. USHA PARAMESWARI

ABSTRACT. The assignment problem (AP) is a special case of the transportation problem, in which the objective is to assign a number of resources to the equal number of activities at a minimum cost (or maximum profit). We endeavour in this paper to introduce a modern approach to assignment problem based on intervals for solving wide range of problem. Considering the recent complexity, it is not enough to make what should be a perfect assignment plan only based on. In this paper, the aim is to evaluate an assignment problem under uncertainty (in interval form) and also convert interval assignment problem into its mathematical form. At the end, a numerical example is given to show the optimal solution of the model.

1. INTRODUCTION

"The best person for the job" is an apt description of the assignment model. An important topic, put forward immediately after the transportation problem, is the assignment problem.

The assignment model is a special state of linear programming models and it is similar to the transportation model. Assignment models deal with the topic how to assign n workers to n jobs such that the cost incurred is minimized. It was developed and published in 1955 by H. Kuhn, who gave the name "Hungarian method" because the method is in general based on the earlier works

[1] *corresponding author*

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of two Hungarian mathematicians: D. Konig and J. Egervary and is therefore known as Hungarian method of assignment models [2]. In most cases that parameters of the problem are not available in precise values, they are expressed in an interval. While dealing with assignment problem, the most important tool namely range convert the interval into their equivalent cost entries. The situation can be illustrated by the assignment of workers with varying degrees of skill to jobs [4]. Saragam Majumdar [6] has introduced an interval linear assignment problems. G. Ramesh and Ganesan and Deepa [3, 5] have proposed a new computational technique to solve assignment problem with generalized interval Hungarian method. Amutha et al. [1] has studied a method of solved extension of the interval in assignment problem.

2. Interval arithmetic

The interval formed with the parameters may be written as where is the left value $[\underline{x}]$ and is the right value $[\bar{x}]$ of the interval respectively.

Let $[\underline{x}, \bar{x}]$ and $[\underline{y}, \bar{y}]$ be two elements. Then the following equalities are well known:

(i) $[\underline{x}, \bar{x}] + [\underline{y}, \bar{y}] = [\underline{x} + \underline{y}, \bar{x} + \bar{y}]$
(ii) $[\underline{x}, \bar{x}] - [\underline{y}, \bar{y}] = [\underline{x} - \underline{y}, \bar{x} - \bar{y}]$
(iii) $[\underline{x}, \bar{x}] \times [\underline{y}, \bar{y}] = [min\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}, max\{\underline{x}\underline{y}, \underline{x}\bar{y}, \bar{x}\underline{y}, \bar{x}\bar{y}\}]$

3. Interval assignment problem

Consider a problem of assignment of n resources to m activities so as to minimize the overall cost or time in such a way that each resource can associate with one and only one job based on interval. The cost matrix $[\underline{(C_{ij})}, (\bar{C_{ij}})]$ is given as below:

	$Activity$	A_1	A_2	$\cdots$	$\cdots$	A_n	$Available$
	R_1	$[\underline{C_{11}}, \bar{C_{11}}]$	$[\underline{C_{12}}, \bar{C_{12}}]$	$\cdots$	$\cdots$	$[\underline{C_{1n}}, \bar{C_{1n}}]$	1
	R_2	$[\underline{C_{21}}, \bar{C_{21}}]$	$[\underline{C_{22}}, \bar{C_{22}}]$	$\cdots$	$\cdots$	$[\underline{C_{2n}}, \bar{C_{2n}}]$	1
$Resource$	$\vdots$	$\vdots$	$\vdots$			$\vdots$	$\vdots$
	$\vdots$	$\vdots$	$\vdots$			$\vdots$	$\vdots$
	R_n	$[\underline{C_{n1}}, \bar{C_{n1}}]$	$[\underline{C_{n2}}, \bar{C_{n2}}]$	$\cdots$	$\cdots$	$[\underline{C_{nm}}, \bar{C_{nm}}]$	1
	$Required$	1	1	$\cdots$	$\cdots$	1	

4. INTERVAL HUNGARIAN ALGORITHM

In this section, it is proposed interval Hungarian method to solve the interval linear assignment problems.

Step (1): Subtract the lowest value from the highest value of the interval and then convert the entire interval cost matrix to single value cost matrix.

Step (2): Select the smallest number from each row and subtract it from the other numbers in the same row as well as for the columns.

Step (3): Draw the minimum number of vertical and horizontal lines to cover all zeros in all rows and columns in the matrix. Let the minimum number of lines is L and the number of columns or rows is n.

If $L = n$ in the matrix, then an optimal assignment can be find, and we proceed to step (6).

If $L < n$ then proceed to step (4).

Step (4): Determine the smallest number in the matrix, not covered by L lines. Subtract this minimum number from all uncovered numbers and add the same number at the intersection of horizontal and vertical lines.

Step (5): Repeat step (3) and step (4) until $L = n$ become.

Step (6): To find assignment by test assigning all the zeros in the rows and columns. The solution is optimal when assigning one and only one zero per row and column in given matrix.

Step (7): Repeat the step (6) until to exactly find one zero to be assignment in each row (column), then process ends.

Step (8): Write the numbers that correspond to the zeros assigned in the previous step in the main matrix and calculate the objective function.

Example 1. *Consider the following interval assignment problem. Assign the four jobs to the four machines so as to minimize the total cost.*

	I	II	III	IV
1	$[10, 20]$	$[10, 15]$	$[2, 15]$	$[0, 15]$
2	$[1, 4]$	$[3, 12]$	$[5, 23]$	$[4, 7]$
3	$[5, 15]$	$[2, 9]$	$[3, 6]$	$[6, 8]$
4	$[2, 7]$	$[4, 15]$	$[4, 13]$	$[2, 9]$

Convert the interval assignment problem to usual cost matrix by using range.

Step (1):

	I	II	III	IV
1	10	5	13	15
2	3	9	18	3
3	10	7	3	2
4	5	11	9	7

Applying Hungarian method:

Step (2): Row reduction

	I	II	III	IV
1	5	0	8	10
2	0	6	15	0
3	8	5	1	0
4	0	6	4	2

Step (3): Column reduction

	I	II	III	IV
1	5	0	7	10
2	0	6	14	0
3	8	5	0	0
4	0	6	3	2

Step (4):

	I	II	III	IV
1	5	0	7	10
2	0	6	14	0
3	8	5	0	0
4	0	6	3	2

So the optimal assignment for all jobs to machines is the following:

$1 \to II,\ 2 \to IV,\ 3 \to III,\ 4 \to I.$

The minimum total interval assignment cost is attained by replacing the above optimal assignment solution in the interval cost objective function as [10,15] + [4,7] + [3,6] + [2,7] = [19,35] = 16.

5. Conclusion

This paper is mainly focused on fixed cost entries of assignment problem when parameters are vague in their nature. In particular, when parameters are delivered in intervals, different approaches which are considered while dealing with interval parameters have been investigated. But in this paper proposed intervals are converted into crisp values by using range method. A numerical example has been presented for demonstrating the solution procedure of the proposed method. This proposed method can also be applicable for the maximization and unbalanced assignment problem.

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Research Scholar
Govindammal Aditanar College for Women
Affiliated to Manonmaniam Sundaranar University
Tirunelveli, India
E-mail address: `palani@apcmcollege.ac.in`

Department of Mathematics
Annai Hajira Women's College
Melapalayam,Tirunelveli-627 012

Department of Mathematics
Govindammal Aditanar College for Women
Tiruchendur