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SOME TYPES OF IDEALS IN SYMMETRIC RINGS

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ABSTRACT. In Ring theory, a branch of abstract algebra, an ideal is a special subset of a ring. Ring theory is an extension of Group theory. Ideals generalize certain subsets of the integers, such as the even number or the multiple of 3. The concept of an order ideal in order theory is derived from the notion of ideal in ring theory. Ideals were introduced by Marshall H. Stone, who derived their name from the ring ideals of Abstract algebra. Ideals were proposed by Richard Dedekind in 1876 in the third edition of his book Vorlesungen Uber Zahlentheore (English: Lecturers on Number Theory). They were a generalization of the concept of ideal numbers developed by Ernt Kummer. Later the concept was expanded by David Hilbert and especially Emmy Noether. In this paper we would like to introduce a new type of ideals in symmetric ring that is in two cases of S_2^* ring, S_3^* ring and we define two type of ideals in S_2^* ring, S_3^* ring. We give some properties of symmetric ideals and symmetric group and we introduce a new concept of reverse composition and plus circle compo.

1. INTRODUCTION

In algebra, which is a broad division of mathematics, Abstract algebra is a study of algebraic structures. Algebraic structures include groups, rings, fields, modules, vector spaces, lattices and algebras. The term abstract algebra was coined in the early 20th century to distinguish this area of study from the other

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parts of algebra. Permutations were studied by Joseph-Louis Lagrange in his 1770 paper "Refexions sur la resolutions algebriquedes equations" devoted to solutions of algebraic equations in which he introduced Lagrange reselvents. Paolo Ruffini was the first person who developed the theory of permutation groups in the context of solving algebraic equations, like his predecessors. His goal was to establish the impossibility of an algebraic solution to a general algebraic equation of degree greater than four. The next step was taken by Evarsite Galois in 1832, although his work remained unpublished until 1846, when he considered for the first time the closure property of a group of permutations. The theory of permutation groups received further far reaching development in the hands of Augustin Cauchy and Camile Jordan, both through introduction of new concepts and primarily, a great wealth of results about special classes of permutation groups and even some general theorems. Permutation groups are central to the study of geometric symmetries and to Galois Theory, the study of finding solutions of polynomial equations.

Symmetric groups on infinite sets behave quite differently from symmetric groups on finite sets, and are discussed in Scott 1987, Dixon & Mortimer 1996 and Cameron 1999. In Ring theory, a branch of abstract algebra, an ideal is a special subset of a ring. Ring theory is an extension of Group theory. Ideals generalize certain subsets of the integers, such as the even number or the multiple of 3. The concept of an order ideal in order theory is derived from the notion of ideal in ring theory. Ideals were introduced by Marshall H. Stone, who derived their name from the ring ideals of Abstract algebra. Ideals were proposed by Richard Dedekind in 1876 in the third edition of his book Vorlesungen Uber Zahlentheore (English: Lecturers on Number Theory). They were a generalization of the concept of ideal numbers developed by Ernt Kummer. Later the concept was expanded by David Hilbert and especially Emmy Noether. The notion of ideal comes from a generalization of modular arithmetic. It is a refinement (by Dedekind) of Kummer's notion of ideal number, which arose from attempts to prove Fermat's Last Theorem (or some special cases). In Mathematical order theory, an ideal is a special subset of a partially ordered set (poset). Although this term historically was derived from the notion of a ring ideal of Abstract algebra, it has subsequently been generalized to a different notion, [5, 6]. Ideals are of great importance for many constructions in order and Lattice theory. Cryptography is an area of study with significant application of ring theory, [4]. Ideals play an important role in the development of ring theory similar to the role played by normal subgroups in group theory, [2, 3, 7].

2. Preliminaries

The following definitions are given in [1].

Definition 2.1. *Let A be non empty set. A binary operation $*$ on A is a function $*: A \times A \to A$. The image of an ordered pair $(a, b) \in A \times A$ under $*$ is denoted by $a * b$. A set A with a binary operation $*$ defined on it is denoted by $(A, *)$. In simple, a binary operation is a "way of putting two things together".*

Definition 2.2. *A non empty set G together with a binary operation $*$:G×G→ G is called a group if the following conditions are satisfied,*

(i) *$*$ is associative, that is $a * (b * c) = (a * b) * c$ for all $a, b, c \in G$;*
(ii) *there exists an element $e \in G$ such that $a * e = e * a = a$ for all $a \in G$;*
(iii) *For any element a in G, there exists an element $a' \in G$ such that $a * a' = a' * a = e$, then a' is called the inverse of a.*

Definition 2.3. *Let G be a group, a subset H of G is called a subgroup of G if H itself is a group under the operation induced by G.*

Definition 2.4. *A group G is said to be abelian if $ab = ba$ for all $a, b \in G$. A group which is not abelian is called a non abelian.*

Definition 2.5. *Let A be finite set. A bijection from A to itself is called a permutation of A.*

Definition 2.6. *Let A be a finite set containing n elements. The set of all permutations of A is clearly a group under the composition of functions. This group is called the symmetric group of degree n and is denoted by S_n.*

Definition 2.7. *A non empty set R together with two binary operations, addition denoted by $'+'$ and multiplication denoted by $'.'$ is called a ring if for all $a, b, cinR$. The following conditions are satisfied,*

(i) *additive commutative, that is $a + b = b + a$;*
(ii) *additive associative, that is $(a + b) + c = a + (b + c)$*
(iii) *additive identity, that is there exists 0 in R such that $a + 0 = 0$;*

(iv) *additive inverse, that is there exists* $-a$ *in* R *such that* $a + (-a) = 0$*;*
(v) *multiplicative associative, that is* $(a \cdot b) \cdot c = a \cdot (b \cdot c)$*;*
(vi) *distributive that is* $a \cdot (b + c) = a \cdot b + a \cdot c$ *and* $(b + c).a = b \cdot a + c \cdot a$

If the multiplication is commutative then the ring is called commutative ring.

Definition 2.8. *Let* R *be a ring. A non-empty subset of* R *is called a left ideal of* R *if*

(i) $a, b \in I \Rightarrow a - b \in I$
(ii) $a \in I$ *and* $r \in R \Rightarrow ra \in I$.

I *is called a right ideal of* R *if*

(i) $a, b \in I \Rightarrow a - b \in I$
(ii) $a \in I$ *and* $r \in R \Rightarrow ar \in I$.

I *is called an ideal of* R *if* I *is both a left ideal and a right ideal.*

Definition 2.9. S_2 *is a symmetric group. The elements of* S_2 *are* $\left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\} = \{e, p_1\}.$

The Inverse Composition is defined as in S_2, $e^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$

(i.e) $e(1) = 1 \Rightarrow e^{-1}(1) = 1$
$e(2) = 2 \Rightarrow e^{-1}(2) = 2.$
Similarly, $p_1(1) = 2 \Rightarrow p_1^{-1}(2) = 1$
$p_1(2) = 1 \Rightarrow p_1^{-1}(1) = 2.$
$\Rightarrow (p_1)^{-1} = p_1.$

3. Main Results

Definition 3.1. *Let us consider a symmetric group* S_2*.*

The elements of S_2 *are* $\left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\} = \{e, p_1\}.$

The Reverse Composition is defined as in S_2,

$$e \; O_R \; p_1 = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \; O_R \; \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$

The composition mapping is $1 \to 1 \to 2$ *here we define the reverse composition mapping as* $1 \to 1 \to 2$ *(i.e.)* $2 \to 1$.

Similarly, $2 \to 2 \to 1$ *(i e)* $1 \to 2$

$$e\ O_R\ p_1 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = p_1$$

and also $p_1\ O_R\ e = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}\ O_R \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.

(i e) $1 \to 1 \to 2 \Rightarrow 2 \to 1$

$2 \to 2 \to 1 \Rightarrow 1 \to 2$.

$$p_1\ O_R\ e = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = p_1$$

It's clearly O_R *is also a binary operation.*

Definition 3.2. *A binary operation* O_R *on* S_2 *is said to be commutative if* $e\ O_R\ p_1 = p_1\ O_R\ e$ *for all* $e, p_1 \in S_2$.

Definition 3.3. *A symmetric group* (S_2, O_R) *is said to be Abelian if* $e\ O_R\ p_1 = p_1\ O_R\ e$ *for all* $e, p_1 \in S_2$.

Definition 3.4. *The symmetric group* S_2 *together with two binary operation* O *- composition, and* O_R *- reverse composition, is called* S_2^*- *ring (symmetric ring) if staisfies the following conditions:*

(i) (S_2, o) *is a symmetric group*
(ii) (S_2, O_R) *is an Abelian group.*

Definition 3.5. *Let* S_2^* *be a symmetric ring. A non empty subset of* S_2^* *is called a left ideal of* S_2^* *if*

(i) $e, p_1 \in S \Rightarrow e\ o\ p_1 \in S$
(ii) $e \in S$ *and* $p_1 \in S_2^* \Rightarrow e\ O_R\ p_1 \in S$.

and S *is called a right ideal of* S_2^* *if*

(i) $e, p_1 \in S \Rightarrow e\ o\ p_1 \in S$
(ii) $e \in S$ *and* $p_1 \in S_2^* \Rightarrow p_1\ O_R\ e \in S$.

S *is called an* S_2^**- ideal of* S_2^* *if* S *is both a left ideal and right ideal.*

Verification:

Let $S = \{e, p_1\} = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\}$

$S_2^* = e, p_1 = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\}$

(i) $e \; o \; p_1 = p_1 \in S$

(ii) $e \; O_R \; p_1 = p_1 \; O_R \; e = p_1 \in S.$

Theorem 3.1. *Let* (S_2, o) *be a symmetric group with identity* e. *If* S *is an ideal of* S_2^* *(symmetric) ring and* $e \in S$ *then* $S = S_2^*$.

Proof. Obviously $S \subseteq S_2^*$.

Let $p_1 \in S_2^*$, since $e \in S$.

$\Rightarrow e \; o \; p_1 = p_1 \in S$

$\Rightarrow S_2^* \subseteq S$

$\Rightarrow S = S_2^*$ □

Corollary 3.1. *Let* (S_2, O_R) *be a symmetric group with identity* e. *If* S *is an ideal of* S_2^* *(symmetric) ring and* $e \in S$ *then* $S = S_2^*$.

Proof. Obviously $S \subseteq S_2^*$.

Let $p_1 \in S_2^*$. Since $e \in S$.

$\Rightarrow e \; O_R \; p_1 = p_1 \in S$

$\Rightarrow S_2^* \subseteq S$

$\Rightarrow S = S_2^*$ □

Theorem 3.2. *Let* (S_2, O_R) *be a symmetric group. Let* $e, p_1 \in S_2$ *then* $(e \; O_R \; p_1)^{-1} = p_1^{-1} \; O_R \; e^{-1}$ *and* $(e^{-1})^{-1} = e$*,* $(p_1^{-1})^{-1} = p_1$.

Proof. Let consider the elements of $S_2 = \{e, p_1\}$,

i.e. $S_2 = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \right\}$

Consider, $e \; O_R \; p_1 = p_1 = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

$$(e \; O_R \; p_1)^{-1} = (p_1)^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = p_1$$

$$(p_1)^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = p_1$$

$$(e)^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = e$$

$(p_1)^{-1}\, O_R\, (e)^{-1} = p_1\, O_R\, e = p_1$
$\Rightarrow (e\, O_R\, p_1)^{-1} = (p_1)^{-1}\, O_R\, (e)^{-1}$
Hence the theorem. □

Remark 3.1.

(i) *A symmetric group S_n is said to be symmetric Abelian if $ab = ba$ for all $a, b \in S_n$ $(n = 2)$.*
A symmetric group which is not Abelian is called a non Abelian group.

(ii) *An element $a \in S_n$ is called symmetric idempotent if $a^2 = a$. Thus we have shown that in a group S_n, The identity element is the only idempotent element in S_n.*

(iii) *Let composition be a binary operation defined on S_n. An element $e \in S_n$ is called a left identity if $e\ o\ a = a$ for all $a \in S_n$, e is called a right identity if $a\ o\ e = a$ for all $a \in S_n$.*

(iv) *Let composition be a binary operation defined on S_n. Let $e \in S_n$ be an identity element.*
Let $a \in S_n$. An element $a' \in S_n$ is called a left inverse of a if $a'\ o\ a = e$, a' is called a right inverse of a if $a\ o\ a' = e$.

Definition 3.6. *We define a new operation on S_3, called as plus circle compo, which is satisfying the following conditions:*

First consider the elements of S_3

$$\left\{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}\right\} = \{e, p_1, p_2, p_3, p_4, p_5\}$$

Consider the identity mapping,

$$e +^o e = e$$

i.e. we are adding both the mapping are in the same, then,

$$\begin{aligned} e(1) +^o e(1) &= 1 \\ e(2) +^o e(2) &= 2, \\ e(3) +^o e(3) &= 3. \end{aligned}$$

$[1 +^o 1 = 1$ and $2 +^o 2 = 2, 3 +^o 3 = 3]$.

Similarly,

$$e +^o p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1. \end{pmatrix}$$

In this case the summing elements are greater than the base element 3. Therefore we subtract the base elements 1, 2, 3 in all mappings, i.e.

$$\begin{aligned} e(1) +^o p_1(1) &= 1 + 2 = 3 - 1 = 2 \\ e(2) +^o p_1(2) &= 2 + 3 = 5 - 2 = 3 \\ e(3) +^o p_1(3) &= 3 + 1 = 4 - 3 = 1 \end{aligned}$$

Therefore $e +^o p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = p_1.$

Similarly, $p_1 +^o e = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

$$\begin{aligned} p_1(1) + e(1) &= 2 + 1 = 3 - 1 = 2 \\ p_1(2) + e(2) &= 3 + 2 = 5 - 2 = 3 \\ p_1(3) + e(3) &= 1 + 3 = 4 - 3 = 1 \\ p_1 +^o e &= p_1. \end{aligned}$$

i.e. first we add the mapping element and eliminate the base values of the identity element.

$$p_2 +^o e = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\begin{aligned} p_2(1) +^o e(1) &= 1 + 3 = 4 - 1 = 3 \\ p_2(2) +^o e(2) &= 2 + 1 = 3 - 2 = 1 \\ p_2(3) +^o e(3) &= 3 + 2 = 5 - 3 = 2 \\ p_2 +^o e &= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = p_2. \end{aligned}$$

Similarly, $e +^o p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

$$\begin{aligned} e(1) +^o p_2(1) &= 1+3 = 4-1 = 3 \\ e(2) +^o p_2(2) &= 2+1 = 3-2 = 1 \\ e(3) +^o p_2(3) &= 3+2 = 5-3 = 2 \\ p_2 +^o e &= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = p_2. \\ &\Rightarrow e +^o p_2 = p_2. \end{aligned}$$

And also we get,

$$\begin{aligned} p_3 +^o e &= p_3. \\ p_4 +^o e &= p_4. \\ p_5 +^o e &= p_5. \end{aligned}$$

And $e +^o p_3 = p_3,\ p_4 +^o e = p_4,\ p_5 +^o e = p_5$.

We do the operation in all cases, then we get a following table:

$+^o$	e	p_1	p_2	p_3	p_4	p_5
e	e	p_1	p_2	p_3	p_4	p_5
p_1	p_1	p_1	e	e	e	e
p_2	p_2	e	p_2	e	e	e
p_3	p_3	e	e	p_3	e	e
p_4	p_4	e	e	e	p_4	e
p_5	p_5	e	e	e	e	p_5

$p_1 +^o p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} = p_1$ (since there are same mapping)

i.e. $p_2 +^o p_2 = p_2,\ p_3 +^o p_3 = p_3,\ p_4 +^o p_4 = p_4,\ p_5 +^o p_5 = p_5$

$$p_1 +^o p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

$$\begin{aligned} p_1(1) +^o p_2(1) &= 2+3 = 5-1 = 4-3 = 1 \\ p_1(2) +^o p_2(2) &= 3+1 = 4-2 = 2 \\ p_1(3) +^o p_2(3) &= 1+2 = 3-3 = 3 \end{aligned}$$

(here 3 is the base element of S_3, so we again eliminate by 3 and we not eliminate mapping fully in the third case)

$$\begin{aligned} p_1 +^o p_2 &= e \\ p_2 +^o p_1 &= e \\ p_1 +^o p_3 &= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} p_1(1) +^o p_3(1) &= 2+1=3 \\ p_1(2) +^o p_3(2) &= 3+3=3 \\ p_1(3) +^o p_3(3) &= 1+2=3 \end{aligned}$$

Here one mapping is same, then we get the identity element.

Therefore $p_1 +^o p_3 = e$ and also, $p_3 +^o p_1 = e$

$$p_1 +^o p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = e$$

$\Rightarrow p_4 +^o p_1 = e$

$$p_1 +^o p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = e$$

$\Rightarrow p_5 +^o p_1 = e$

$$\begin{aligned} p_2 +^o p_3 &= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = e \\ p_3 +^o p_2 &= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = e \\ p_2 +^o p_4 &= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = e \end{aligned}$$

$$\begin{aligned}
p_4 +^o p_2 &= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = e \\
p_2 +^o p_5 &= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = e \\
p_3 +^o p_4 &= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = e
\end{aligned}$$

$p_3(1) +^o p_4(1) = 1 + 3 = 4 - 1 = 3$
$p_3(2) +^o p_4(2) = 3 + 2 = 5 - 2 = 3$
$p_3(3) +^o p_4(3) = 2 + 1 = 3 - 3 = 3.$

Here all the mappings got the same value therefore we choose the identity mapping

$$\begin{aligned}
\Rightarrow p_3 +^o p_4 &= e \\
p_4 +^o p_3 &= e \\
p_3 +^o p_5 &= \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
p_3(1) +^o p_5(1) &= 1 + 2 = 3 - 1 = 2 \\
p_3(2) +^o p_5(2) &= 3 + 1 = 4 - 2 = 2 \\
p_3(3) +^o p_5(3) &= 2 + 3 = 5 - 3 = 2
\end{aligned}$$

Therefore $p_3 +^o p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = e$
$\Rightarrow p_5 +^o p_3 = e$
$p_4 +^o p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} +^o \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$

$$\begin{aligned}
p_4(1) +^o p_5(1) &= 3 + 2 = 5 - 1 = 4 \\
p_4(2) +^o p_5(2) &= 2 + 1 = 3 - 2 = 1 \\
p_4(3) +^o p_5(3) &= 1 + 3 = 4 - 3 = 1
\end{aligned}$$

Therefore $p_4 +^o p_5 = e$
$\Rightarrow p_5 +^o p_4 = e$
It is also a binary operation.

Definition 3.7. *If a symmetric group S_3 with the binary operation-plus circle compo, satisfies the conditions for identity and inverse, then it is called symmetric half group.*

Definition 3.8. *A symmetric group S_3 together with two binary operations $o, +^o$ composition and plus circle compo, respectively, is called S_3^*-(symmetric) ring if satisfies the following conditions:*

(i) (S_3, o) *is a symmetric group*
(ii) $(S_3, +^o)$ *is a symmetric half group.*

Definition 3.9. *Let S_3^* be a symmetric ring. A non empty subset of S_3^* is called a left ideal of S_3^* if*

(i) $e, p_1 \in S \Rightarrow e\ o\ p_1 \in S$
(ii) $e \in S$ *and* $p_1 \in S_3^* \Rightarrow e +^o p_1 \in S.$

and S is called a right ideal of S_3^* if

(i) $e, p_1 \in S \Rightarrow e\ o\ p_1 \in S$
(ii) $e \in S$ *and* $p_1 \in S_3^* \Rightarrow p_1 +^o e \in S.$

S is called an S_3^*- ideal of S_3^* if S is both a left ideal and right ideal.

Verification:

Let $S = e, p_1, p_2$, $S_3 = e, p_1, p_2, p_3, p_4, p_5$

(i) $e\ o\ p_1 = p_1 \in S$
(ii) $e +^o p_1 = p_1 +^o e = p_1 \in S.$

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