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SOFT A_RS -CLOSED SETS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce new category of soft set called Soft A_RS -Closed sets. Also we study in details the properties of Soft A_RS - Closed sets and its relation with other soft sets. All these findings will provide a base to researchers who want to work in the field of soft topology and will help to establish a general framework for applications in practical fields.

1. INTRODUCTION

Molodtsov introduced the concept of soft sets from which the difficulties of fuzzy sets, intutionistic fuzzy sets, vague sets, interval mathematics and rough sets have been rectified, [8]. A soft set over the universe U is a parametrized family of subsets of the universe U. Application of soft sets in decision making problems has been found by Maji et al. in [7], whereas Chen gave a parametrization reduction of soft sets and a comparison of it with attribute reduction in rough set theory, [3]. Further soft sets are a class of special information.

Shabir and Naz introduced soft topological spaces in 2011 and studied some basic properties of them, [10]. Meanwhile generalized closed sets in topological spaces were introduced by Levine in 1970 and recent survey of them is in which is extended tosoft topological spaces in the year 2012. Further Kannan, [6] and Rajalakshmi have introduced soft $g-$ locally closed sets and soft semi star generalized closed sets. Soft strongly $g-$ closed sets have been studied

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by Kannan, Rajalakshmi and Srikanth. Chandrasekhara Rao and Palaniappan introduced generalized star closed sets in topological spaces and it is extended to the bitopological context by Chandrasekhara Rao and Kannan.

Recently papers about soft sets and their applications in various fields have increased largely. Modern topology depends strongly on the ideas of set theory. Any research work should result in addition to the existing knowledge of a particular concept. Such an effort not only widens the scope of the concept but also encourages others to explore new and newer ideas. Therefore in this work we introduce a new soft generalized set called soft A_RS closed set and its related properties. This may be another starting point for the new soft set mathematical concepts and structures that are based on soft set theoretic operations.

2. Preliminaries

In this section, we present the basic definitions and results of soft theory which may be found in earlier studies. Throughout this work, U refers to an universe, E is a set of parameters, $P(U)$ is the power set of U and $A \subseteq E$.

Definition 2.1. *[8] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x)) : x \in E,\ f_A(x) \in P(U)\}$, where $f_A : E \to P(U)$ such that $f_A(x) = \phi$ if $x \notin A$. Here f_A is called an approximate function of the soft set F_A. The value of f_A may be arbitrary, some of them may be empty, and some may have non empty intersection.*

Example 1. *[10] Suppose there are five cars in the universe.*
Let $U = \{c_1, c_2, c_3, c_4, c_5\}$ under consideration and that
$E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ stand for the parameters expensive, beautiful, manual gear, cheap, automatic gear, in good repair, in bad repair and costly respectively. In this case to define a soft set means to point out expensive cars, beautiful cars and so on. It means that in the mapping f_A given by "cars, (.)" where (.) to be filled in by one of the given parameters $x_i \in E$.

Let $A \subseteq E$, the soft set F_A that describes the "attractiveness in cars" in the opinion of a buyer may be defined like $A = \{x_2, x_3, x_4, x_5, x_7\}$,

$f_A(x_2) = \{c_2, c_3, c_5\}$, $f_A(x_3) = \{c_2, c_4\}$, $f_A(x_4) = \{c_1\}$, $f_A(x_5) = \{U\}$, $f_A(x_7) = \{c_3, c_5\}$.

Then collection of the above approximations is called as soft set

$$F_A = \{(x_2, \{c_2, c_3, c_5\}), (x_3, \{c_2, c_4\}), (x_4, \{c_1\}), (x_5, \{U\}), (x_7, \{c_3, c_5\})\}.$$

Definition 2.2. *[7] A soft set (F, A) over X is said to be Null Soft Set denoted by F_ϕ if for all $e \in A$, $F(e) = \phi$. A soft set (F, E) over X is said to be an Absolute Soft Set denoted by F_X if for all $e \in A,\ F(e) = X$.*

Definition 2.3. *[4] The Union of two soft sets (F, A) and (G, B) over X is the soft set (H, C), where $C = A \cup B$, and for all $e \in C$, $H(e) = F(e)$, if $e \in A\ B$, $H(e) = G(e)$ if $e \in B \setminus A$ and $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$ and is denoted as $(F, A) \tilde{\cap} (G, B) = (H, C)$.*

Definition 2.4. *[10] The Relative Complement of (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in A$.*

Definition 2.5. *[10] The Difference (H, E) of two soft sets (F, E) and (G, E) over X, denoted by $(F, E) \setminus (G, E)$ is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in E$.*

Definition 2.6. *[8] Let (F, A) and (G, B) be soft sets over X, we say that (F, A) is a Soft Subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A) \tilde{\subseteq} (G, B)$.*

Definition 2.7. *[10] Let τ be a collection of soft sets over X with the fixed set E of parameters. Then τ is called a Soft Topology on X if*

(i) $\tilde{\phi}$, $\tilde{X}$ *belongs to* τ.
(ii) *The union of any number of soft sets in τ belongs to τ.*
(iii) *The intersection of any two soft sets in τ belongs to τ.*

The triplet (X, τ, E) is called Soft Topological Spaces over X.

The members of τ are called Soft Open sets in X and complements of them are called Soft Closed sets in X.

Definition 2.8. *[10] Let (X, τ, E) be a Soft Topological Spaces over X. The Soft Interior of (F, E) denoted by $Int(F, E)$ is the union of all soft open subsets of (F, E). Clearly $Int(F, E)$ is the largest soft open set over X which is contained in (F, E).*

The Soft Closure of (F, E) denoted by $Cl(F, E)$ is the intersection of soft closed sets containing (F, E). Clearly (F, E) is the smallest soft closed set containing (F, E).

(i) $Int(F,E) = \tilde{\cup} \{(O.E) : (O,E)$ *is soft open and* $(O,E) \tilde{\subseteq} (F,E)\}$.
(ii) $Cl(F,E) = \tilde{\cap} \{(O.E) : (O,E)$ *is soft closed and* $(F,E) \tilde{\subseteq} (O,E)\}$.

Definition 2.9. *A Subset of a soft topological space* (X,τ,E) *is said to be*

(1) *a soft Semi-Open set, [3], if* $(A,E) \tilde{\subseteq} Cl(int(A,E))$ *and a Soft Semi-Closed set if* $int(Cl(A,E)) \tilde{\subseteq} (A,E))$.
(2) *a soft Pre-Open set, [1], if* $(A,E) \tilde{\subseteq} Int(Cl(A,E))$ *and a Soft Pre-Closed set if* $Cl(int(A,E) \tilde{\subseteq} (A,E)$.
(3) *a soft* α*-Open set, [1], if* $(A,E) \tilde{\subseteq} Int(Cl(int(A,E)))$ *and a Soft* α*-Closed set if* $Cl(int(Cl(A,E))) \tilde{\subseteq} (A,E))$.
(4) *a soft* β*-Open set, [2], if* $(A,E) \tilde{\subseteq} Cl(Int(Cl(A,E)))$ *and a Soft* β*-Closed set if* $Int(Cl(int(A,E)) \tilde{\subseteq} (A,E)$.
(5) *a soft - generalized Closed set (briefly soft* gs *- Closed) if* $Cl(A,E) \tilde{\subseteq} (U,E)$ *whenever* $(A,E) \tilde{\subseteq} (U,E)$ *and* (U,E) *is soft Open in* (X,τ,E). *The complement of a Soft* gs *-Closed set is called a Soft* gs*-Open set.*
(6) *a Soft Semi-generalized Closed set (briefly Soft* $Sg-$*Closed) if* $SCl(A,E) \tilde{\subseteq} (U,E)$ *whenever* $(A,E) \tilde{\subseteq} (U,E)$ *and* (U,E) *is soft semi Open in* (X,τ,E). *The complement of a Soft* $Sg-$ *Closed set is called a Soft* $Sg-$ *Open set.*
(7) *a generalized Soft Semi-Closed set (briefly* gs*-Closed) if* $SCl(A,E) \tilde{\subseteq} (U,E)$ *whenever* $(A,E) \tilde{\subseteq} (U,E)$ *and* (U,E) *is soft Open in* (X,τ,E). *The complement of a Soft* gs*-Closed set is called a Soft* gs*-Open set.*
(8) *a Soft- Closed, [9], if* $Cl(A,E) \tilde{\subseteq} (U,E)$ *whenever* $(A,E) \tilde{\subseteq} (U,E)$ *and* (U,E) *is soft semi Open in* $(X,\tau,)$
(9) *a Soft* ω*-Closed, [9], if* $Cl(A,E) \tilde{\subseteq} (U,E)$ *whenever* $(A,E) \tilde{\subseteq} (U,E)$ *and* (U,E) *is soft semi Open in* (X,τ,E). *Complement of a Soft* ω*-Closed is called a Soft* ω*- open set.*
(10) *a Soft alpha-generalized Closed set (briefly Soft* $\alpha g-$*Closed) if* $\alpha Cl(A,E) \tilde{\subseteq} (U,E)$ *whenever* $(A,E) \tilde{\subseteq} (U,E)$ *and* (U,E) *is soft* α *Open in* (X,τ,E). *The complement of a Soft* $\alpha g-$ *Closed set is called a Soft* $\alpha g-$ *Open set.*
(11) *a Soft generalized alpha Closed set (briefly Soft* $g\alpha$*-Closed) if* $\alpha Cl(A,E) \tilde{\subseteq} (U,E)$ *whenever* $(A,E) \tilde{\subseteq} (U,E)$ *and* (U,E) *is soft Open in* (X,τ,E). *The complement of a Soft* $g\alpha$*-Closed set is called a Soft* $g\alpha$*-Open set.*

(12) *a Soft generalized pre Closed set (briefly Soft gp-Closed), [1], if $pCl(A,E) \tilde{\subseteq} (U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft Open in (X,τ,E). The complement of a Soft gp-Closed set is called a Soft gp-Open set.*

(13) *a Soft generalized pre regular Closed set (briefly Soft gpr-Closed), [5], if $pCl(A,E) \tilde{\subseteq} (U,E)$ whenever $(A,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft regular Open in (X,τ,E). The complement of a Soft gpr-Closed set is called a Soft gpr-Open set.*

3. Soft A_RS -Closed set

Definition 3.1. *Let (X,τ,E) be a soft topological space. A Soft set (F,E) is called soft A_RS -Closed set if $\beta cl(F,E) \tilde{\subseteq} Int(U,E)$ whenever $(F,E) \tilde{\subseteq} (U,E)$ and (U,E) is soft $\omega-$ open. The set of all soft A_RS- closed sets is denoted by $A_RS\ C(X)$.*

Example 2. *Let $X = \{x_1,x_2,x_3\}$, $E = \{e_1,e_2\}$ and $\tau = \{F_4,F_7,F_{15},F_{16}\}$ and $\tau^c = \{F_{12},F_{10},F_{15},F_{16}\}$ then τ defines a soft topology on X where*

$$(F_1,E) = \{\{e_1,x_1\},\{e_2,\phi\}\} \qquad (F_2,E) = \{\{e_1,x_2\},\{e_2,\phi\}\}$$
$$(F_3,E) = \{\{e_1,X\},\{e_2,\phi\}\} \qquad (F_4,E) = \{\{e_2,x_1\},\{e_1,\phi\}\}$$
$$(F_5,E) = \{\{e_2,x_2\},\{e_1,\phi\}\} \qquad (F_6,E) = \{\{e_2,X\},\{e_1,\phi\}\}$$
$$(F_7,E) = \{\{e_1,x_1\},\{e_2,x_1\}\} \qquad (F_8,E) = \{\{e_1,x_1\},\{e_2,x_2\}\}$$
$$(F_9,E) = \{\{e_1,x_2\},\{e_2,x_1\}\} \qquad (F_{10},E) = \{\{e_1,x_2\},\{e_2,x_2\}\}$$
$$(F_{11},E) = \{\{e_1,X\},\{e_2,x_1\}\} \qquad (F_{12},E) = \{\{e_1,X\},\{e_2,x_2\}\}$$
$$(F_{13},E) = \{\{e_1,x_1\},\{e_2,X\}\} \qquad (F_{14},E) = \{\{e_1,x_2\},\{e_2,X\}\}$$
$$(F_{15},E) = \{\{e_1,X\},\{e_2,X\}\} \qquad (F_{16},E) = \{\{e_1,\phi\},\{e_2,\phi\}\}.$$

Here (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E), (F_8,E), (F_9,E), (F_{10},E), (F_{11},E), (F_{12},E), (F_{13},E), (F_{14},E), (F_{15},E), (F_{16},E) are soft sets in (X,τ,E).

Also (F_4,E), (F_7,E), (F_{15},E), (F_{16},E) are soft open sets in (X,τ,E) and (F_{12},E), (F_{10},E), (F_{15},E), (F_{16},E) are soft closed sets in (X,τ,E) then $A_RS\ C = \{F_1,F_2,F_3,F_5,F_6,F_8,F_9,F_{10},F_{11},F_{12},F_{13},F_{14},F_{15},F_{16}\}$.

Lemma 3.1. *[11], A Set is ω- open if and only if $F \subseteq Int(A)$ whenever F is semi closed and $F \subseteq A$.*

Proposition 3.1. *Every soft semi closed set is soft A_RS closed set.*

Proof. Let (F, E) be a soft semi closed set in the soft topological space (X, τ, E) and (U, E) be a soft ω- open set such that $(F, E) \tilde{\subseteq} (U, E)$. Then by Lemma 3.1, $(F, E) = Scl(F, E) \tilde{\subseteq} Int(U, E)$.

Then (F, E) is soft A_RS - closed set. □

Remark 3.1. *The converse of the above theorem need not be true.*

Example 3. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_5, F_{12}, F_{15}, F_{16}\}$ then A_RS*

$$C = \{F_3, F_4, F_6, F_7, F_9, F_{11}, F_{13}, F_{14}, F_{15}, F_{16}\}$$

and soft semi $C = \{F_6, F_{11}, F_9, F_4, F_{15}, F_{16}\}$. Here F_3 is in soft A_RS C but not in soft SC.

Proposition 3.2. *Every soft closed set is soft A_RS closed set.*

Proof. Let (F, E) be a soft closed set in the soft topological space (X, τ, E). Then it is a soft semi closed set. Then by Proposition 3.1, (F, E) is soft A_RS closed set. □

Remark 3.2. *The converse of the above theorem need not be true.*

Example 4. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$, and $\tau = \{F_1, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_2, F_{15}, F_{16}\}$*

then A_RS $C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$.

Here $F_1, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}$ are soft A_RS closed in (X, τ, E) but not in soft closed.

Proposition 3.3. *Every soft α closed set is soft A_RS closed set.*

Proof. Let (F, E) be a soft α closed set in the soft topological space (X, τ, E).

Then it is a soft semi closed set. Then by Proposition 3.1, (F, E) is soft A_RS closed set. □

Remark 3.3. *The converse of the above theorem need not be true.*

Example 5. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_5, F_{12}, F_{15}, F_{16}\}$, $\tau^c = \{F_{11}, F_4, F_{15}, F_{16}\}$,* *then soft* A_RS $C = \{F_3, F_4, F_6, F_7, F_9, F_{11}, F_{13}, F_{14}, F_{15}, F_{16}\}$ *and soft* $\alpha C = \{F_6, F_{11}, F_9, F_4, F_{15}, F_{16}\}$.

Here F_3, F_7, F_{13}, F_{14} *are soft* A_RS *closed in* (X, τ, E) *but not in soft* αC.

Proposition 3.4. *Every soft* JP *closed set is soft* A_RS *closed set.*

Proof. Let (F, E) be a soft JP closed set in the soft topological space (X, τ, E).

Then $Scl(F, E) \tilde{\subseteq} Int(U, E)$.

Therefore $\beta cl(F, E) \tilde{\subseteq} Int(U, E)$.

Hence (F, E) is soft A_RS closed set. □

Remark 3.4. *The converse of the above theorem need not be true.*

Example 6. *In the soft topological space* (X, τ, E)*,* $X = \{x_1, x_2\}$*,* $E = \{e_1, e_2\}$ *and* $\tau = \{F_1, F_4, F_7, F_{13}, F_{15}, F_{16}\}$*,* $\tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_{15}, F_{16}\}$,

then soft A_RS $C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ *and soft* $JPC = \{F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$.

Here F_1 *is in soft* A_RS *closed in* (X, τ, E) *but not in soft JP closed set.*

Proposition 3.5. *Every soft* A_RS *closed set is soft* gsp *closed set.*

Proof. Let (F, E) be a soft A_RS closed set in the soft topological space (X, τ, E) and (U, E) be any soft open set such that $(F, E) \subseteq (U, E)$.

Since every soft open set is soft ω- open, we have $\beta cl(F, E) \tilde{\subseteq} Int(U, E) = (U, E)$. Therefore (F, E) is soft gsp closed. □

Remark 3.5. *The converse of the above theorem need not be true.*

Example 7. *In the soft topological space* (X, τ, E)*,* $X = \{x_1, x_2\}$*,* $E = \{e_1, e_2\}$ *and* $\tau = \{F_1, F_4, F_7, F_{13}, F_{15}, F_{16}\}$*,* $\tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_{15}, F_{16}\}$*, then soft* A_RS $C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ *and soft*

gsp *closed* $= \{F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$.

Here F_7, F_8, F_{13} *are soft* gsp *closed in* (X, τ, E) *but not in soft* A_RS *closed set.*

Remark 3.6. *The collection of soft* A_RS *closed set lies between soft semi closed sets and soft* gsp *closed sets.*

Proposition 3.6. *Arbitrary intersection of soft* A_RS *closed set is soft* A_RS *closed set.*

Proof. Let (F, E) and (G, E) be the two soft $A_R S$ closed set in the soft topological space (X, τ, E). Then $\beta cl(F, E) \subseteq Int(U, E)$ whenever $(F, E) \subseteq (U, E)$, (U, E) is soft ω- open and $\beta cl(G, E) \subseteq Int(U, E)$ whenever $(G, E) \subseteq (U, E), (U, E)$ is soft ω- open.

Hence $\beta cl(F, E) \cap \beta cl(G, E) \subseteq Int(U, E)$ whenever $(F, E) \cap (G, E) \subseteq (U, E)$, (U, E) is soft ω- open.

Therefore intersection of two soft $A_R S$ closed set is again soft $A_R S$ closed set. □

Remark 3.7. *The soft union of two soft $A_R S$ closed sets need not be a soft $A_R S$ closed set.*

Example 8. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_3, F_{11}, F_{15}, F_{16}\}$, $\tau^c = \{F_6, F_5, F_{15}, F_{16}\}$,*

then soft $A_R S$ $C = \{F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$.

Here F_7, F_9 are soft $A_R S$ closed set in (X, τ, E) but $F_7 \cup F_9 = F_{11}$ is not in soft $A_R S$ closed set.

Remark 3.8. *The concepts of soft $A_R S$ closed set and soft strongly $g-$ closed are independent.*

Example 9. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_1, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_2, F_{15}, F_{16}\}$,*

then soft $A_R S$ $C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ and soft $SgC = \{F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$.

Here F_1 is in soft $A_R S$ C but not in soft SgC.

Example 10. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_{14}, F_9, F_{15}, F_{16}\}$, $\tau^c = \{F_1, F_8, F_{15}, F_{16}\}$,*

then soft $A_R S$ $C = \{F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{15}, F_{16}\}$ and soft $SgC = \{F_1, F_3, F_5, F_7, F_8, F_{11}, F_{12}, F_{13}, F_{15}, F_{16}\}$.

Here F_1, F_3, F_7, F_{11} are soft SgC in (X, τ, E) but not in soft $A_R S$ closed set.

Remark 3.9. *The concepts of soft $A_R S$ closed set and soft gs closed sets are independent.*

Example 11. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_1, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_2, F_{15}, F_{16}\}$,* *then soft* $A_RS\ C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ *and soft* $gsC = \{F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$.

Here F_1 *is in soft* $A_RS\ C$ *but not in soft* gs *closed set.*

Example 12. *In the soft topological space* (X, τ, E)*,* $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ *and* $\tau = \{F_{14}, F_9, F_{15}, F_{16}\}$*,* $\tau^c = \{F_1, F_8, F_{15}, F_{16}\}$*,*

then soft $A_RS\ C = \{F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{15}, F_{16}\}$ *and soft* $gsC = \{F_1, F_3, F_7, F_8, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$.

Here $F_1, F_3, F_7, F_{11}, F_{14}$ *are soft* gsC *in* (X, τ, E) *but not in soft* A_RS *closed set.*

Remark 3.10. *The concepts of soft* A_RS *closed set and soft* β *closed sets are independent.*

Example 13. *In the soft topological space* (X, τ, E)*,* $X = \{x_1, x_2\}$*,* $E = \{e_1, e_2\}$ *and* $\tau = \{F_3, F_{11}, F_{15}, F_{16}\}$*,* $\tau^c = \{F_6, F_5, F_{15}, F_{16}\}$*,*

then soft $A_RS\ C = \{F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$ *and soft* $\beta C = \{F_1, F_2, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{13}, F_{14}, F_{15}, F_{16}\}$.

Here F_2, F_{12} *are soft* β *closed set in* (X, τ, E) *but not in soft* A_RS *closed set.*

Example 14. *In the soft topological space* (X, τ, E)*,* $X = \{x_1, x_2\}$*,* $E = \{e_1, e_2\}$ *and* $\tau = \{F_3, F_{11}, F_{15}, F_{16}\}$, $\tau^c = \{F_6, F_5, F_{15}, F_{16}\}$*,*

then soft $A_RS\ C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ *and soft* $\beta C = \{F_1, F_2, F_4, F_5, F_6, F_9, F_{10}, F_{14}, F_{15}, F_{16}\}$.

Here F_3, F_{11}, F_{12} *are soft* A_RS *closed set in* (X, τ, E) *but not in soft* β *closed set.*

Remark 3.11. *The concepts of soft* A_RS *closed set and soft pre closed sets are independent.*

Example 15. *In the soft topological space* (X, τ, E)*,* $X = \{x_1, x_2\}$*,* $E = \{e_1, e_2\}$ *and* $\tau = \{F_1, F_4, F_7, F_{13}, F_{15}, F_{16}\}$*,* $\tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_{15}, F_{16}\}$,

then soft $A_RS\ C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ *and soft pre* $C = \{F_2, F_4, F_5, F_6, F_9, F_{10}, F_{14}, F_{15}, F_{16}\}$.

Here F_1, F_3, F_{11} *are soft pre closed set in* (X, τ, E) *but not in soft* A_RS *closed set.*

Example 16. *In the soft topological space* (X, τ, E)*,* $X = \{x_1, x_2\}$*,* $E = \{e_1, e_2\}$ *and* $\tau = \{F_3, F_{11}, F_{15}, F_{16}\}$*,* $\tau^c = \{F_6, F_5, F_{15}, F_{16}\}$,

then soft $A_RS\ C = \{F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$ *and soft pre* $C = \{F_1, F_2, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{13}, F_{14}, F_{15}, F_{16}\}$.

Here F_{12} *is in soft* A_RS *closed set but not in soft* p *closed set.*

Remark 3.12. *The concepts of soft A_RS closed set and soft g closed sets are independent.*

Example 17. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$,
$E = \{e_1, e_2\}$ and $\tau = \{F_{14}, F_9, F_{15}, F_{16}\}$, $\tau^c = \{F_1, F_8, F_{15}, F_{16}\}$,
then soft A_RS $C = \{F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{15}, F_{16}\}$ and
soft $gC = \{F_1, F_3, F_5, F_7, F_8, F_{11}, F_{12}, F_{15}, F_{16}\}$.
Here F_1, F_3, F_7, F_{11} are soft g closed set in (X, τ, E) but not in soft A_RS closed set.*

Example 18. *In the soft topological space (X, τ, E) , $X = \{x_1, x_2\}$,
$E = \{e_1, e_2\}$ and $\tau = \{F_5, F_{12}, F_{15}, F_{16}\}$, $\tau^c = \{F_{11}, F_4, F_{15}, F_{16}\}$,
then soft A_RS $C = \{F_3, F_4, F_6, F_7, F_9, F_{11}, F_{13}, F_{14}, F_{15}, F_{16}\}$ and
soft $gC = \{F_4, F_6, F_7, F_9, F_{11}, F_{13}, F_{14}, F_{15}, F_{16}\}$.
Here F_3 is in soft A_RS closed set but not in soft g closed set.*

Remark 3.13. *The concepts of soft A_RS closed set and soft ω closed sets are independent.*

Example 19. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$,
$E = \{e_1, e_2\}$ and $\tau = \{F_1, F_4, F_7, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_{15}, F_{16}\}$,
then soft A_RS $C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ and soft $\hat{g}C = \{F_2, F_9, F_{10}, F_{14}, F_{15}, F_{16}\}$.
Here $F_1, F_3, F_4, F_5, F_6, F_{11}, F_{12}$ are soft A_RS closed set in (X, τ, E) but not in soft ω closed set.*

Example 20. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$,
$E = \{e_1, e_2\}$ and $\tau = \{F_{14}, F_9, F_{15}, F_{16}\}$, $\tau^c = \{F_1, F_8, F_{15}, F_{16}\}$,
then soft A_RS $C = \{F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{15}, F_{16}\}$ and
soft $\hat{g}C = \{F_1, F_3, F_5, F_7, F_8, F_{11}, F_{12}, F_{15}, F_{16}\}$.
Here F_1, F_3, F_7, F_{11} are soft $\hat{g}$ closed set in (X, τ, E) but not in A_RS closed set.*

Remark 3.14. *The concepts of soft A_RS closed set and soft αg closed sets are independent.*

Example 21. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$,
$E = \{e_1, e_2\}$ and $\tau = \{F_{14}, F_9, F_{15}, F_{16}\}$, $\tau^c = \{F_1, F_8, F_{15}, F_{16}\}$,
then soft A_RS $C = \{F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{15}, F_{16}\}$ and
soft $\alpha gC = \{F_1, F_3, F_5, F_7, F_8, F_{11}, F_{12}, F_{15}, F_{16}\}$.*

Here F_1, F_3, F_7, F_{11} are soft αg closed set in (X, τ, E) but not in A_RS closed set.

Example 22. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_1, F_4, F_7, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_{12}, F_{10}, F_2, F_{15}, F_{16}\}$,*
then soft A_RS $C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ and soft $\alpha gC = \{F_2, F_9, F_{10}, F_{14}, F_{15}, F_{16}\}$.

Here $F_1, F_3, F_4, F_5, F_6, F_{11}, F_{12}$ are soft A_RS closed set in (X, τ, E) but not in soft αg closed set.

Remark 3.15. *The concepts of soft A_RS closed set and soft gs closed sets are independent.*

Example 23. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_{14}, F_9F_{15}, F_{16}\}$, $\tau^c = \{F_1, F_8, F_{15}, F_{16}\}$,*
then soft A_RS $C = \{F_5, F_6, F_8, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$ and soft $gsC = \{F_1, F_3, F_7, F_8, F_{11}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$.

Here $F_1, F_3, F_7, F_{11}, F_6, F_{11}$ are gs closed set in (X, τ, E) but not in soft A_RS closed set.

Example 24. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_1, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_2, F_{15}, F_{16}\}$,*
then soft A_RS $C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ and soft $gsC = \{F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$.

Here F_1 is in soft A_RS closed set in (X, τ, E) but not in soft gs closed set.

Remark 3.16. *The concepts of soft A_RS closed set and soft gp closed sets are independent.*

Example 25. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_1, F_{13}, F_{15}, F_{16}\}$, $\tau^c = \{F_{14}, F_2, F_{15}, F_{16}\}$,*
then soft A_RS $C = \{F_1, F_2, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}, F_{14}, F_{15}, F_{16}\}$ and soft $gpC = \{F_2, F_{14}, F_{15}, F_{16}\}$.

Here $F_1, F_3, F_4, F_5, F_6, F_9, F_{10}, F_{11}, F_{12}$ are soft A_RS closed sets in (X, τ, E) but not in soft gp closed set.

Example 26. *In the soft topological space (X, τ, E), $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_3, F_{11}, F_{15}, F_{16}\}$, $\tau^c = \{F_6, F_5, F_{15}, F_{16}\}$,*
then soft A_RS $C = \{F_1, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$ and soft gp $C = \{F_1, F_2, F_4, F_5, F_6, F_7, F_8, F_9, F_{10}, F_{12}, F_{13}, F_{14}, F_{15}, F_{16}\}$.

Here F_2 is in soft gp closed set in (X, τ, E) but not in soft A_RS closed set.

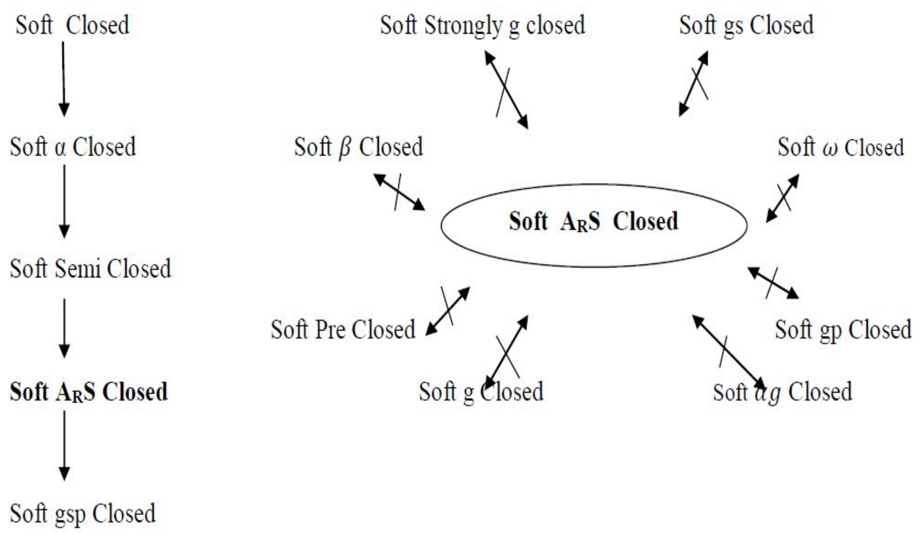

FIGURE 1. Interrelationship

4. SOFT A_RS - OPEN SET

Definition 4.1. *Let (X, τ, E) be a soft topological space. A Soft set (F, E) is called soft A_RS -Closed set if $\beta cl(F, E) \tilde{\subseteq} Int(U, E)$ whenever $(F, E) \tilde{\subseteq} (U, E)$ and (U, E) is soft ω- open. The complement of soft A_RS- closed set is called soft A_RS- open set. The set of all soft A_RS- open sets is denoted by $A_RS\ O(X)$.*

Example 27. *Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{F_1, F_{13}, F_{15}, F_{16}\}$ and $\tau^c = \{F_{14}, F_2, F_{15}, F_{16}\}$, then τ defines a soft topology on X where*

$$(F_1, E) = \{\{e_1, x_1\}, \{e_2, \phi\}\}, \qquad (F_2, E) = \{\{e_1, x_2\}, \{e_2, \phi\}\},$$
$$(F_3, E) = \{\{e_1, X\}, \{e_2, \phi\}\}, \qquad (F_4, E) = \{\{e_2, x_1\}, \{e_1, \phi\}\},$$
$$(F_5, E) = \{\{e_2, x_2\}, \{e_1, \phi\}\}, \qquad (F_6, E) = \{\{e_2, X\}, \{e_1, \phi\}\},$$
$$(F_7, E) = \{\{e_1, x_1\}, \{e_2, x_1\}\}, \qquad (F_8, E) = \{\{e_1, x_1\}, \{e_2, x_2\}\},$$
$$(F_9, E) = \{\{e_1, x_2\}, \{e_2, x_1\}\}, \qquad (F_{10}, E) = \{\{e_1, x_2\}, \{e_2, x_2\}\},$$
$$(F_{11}, E) = \{\{e_1, X\}, \{e_2, x_1\}\}, \qquad (F_{12}, E) = \{\{e_1, X\}, \{e_2, x_2\}\},$$
$$(F_{13}, E) = \{\{e_1, x_1\}, \{e_2, X\}\}, \qquad (F_{14}, E) = \{\{e_1, x_2\}, \{e_2, X\}\},$$
$$(F_{15}, E) = \{\{e_1, X\}, \{e_2, X\}\}, \qquad (F_{16}, E) = \{\{e_1, \phi\}, \{e_2, \phi\}\}.$$

Here (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E), (F_{11}, E), (F_{12}, E), (F_{13}, E), (F_{14}, E), (F_{15}, E), (F_{16}, E) *are soft sets in* (X, τ, E).

Also (F_1, E), (F_{13}, E), (F_{15}, E), (F_{16}, E) *are soft open sets in* (X, τ, E) *and* (F_{14}, E), (F_2, E), (F_{15}, E), (F_{16}, E) *are soft closed sets in* (X, τ, E), *then* $A_R S$ $O = \{F_{14}, F_{13}, F_6, F_{12}, F_{11}, F_3, F_8, F_7, F_5, F_4, F_1, F_{15}, F_{16}\}$.

Proposition 4.1.

(1) *Every soft semi open set is soft* $A_R S$ *open set.*
(2) *Every soft open set is soft* $A_R S$ *open set.*
(3) *Every soft* JP *open set is soft* $A_R S$ *open set.*
(4) *Every soft* α *open set is soft* $A_R S$ *open set.*
(5) *Every soft* $A_R S$ *open set is soft gsp open set.*

Proof. The proof is obvious. □

Remark 4.1. *The converse of the above proposition need not be true.*

Remark 4.2.

(1) *Arbitrary intersection of soft* $A_R S$ *open set is soft* $A_R S$ *open set.*
(2) *The soft union of two soft* $A_R S$ *open sets need not be a soft* $A_R S$ *open set.*
(3) *The collection of soft* $A_R S$ *open set lies between soft semi open sets and soft* gsp *open sets.*
(4) *The concepts of soft* $A_R S$ *open set and soft strongly* $g-$ *open set are independent.*
(5) *The concepts of soft* $A_R S$ *open set and soft* gs *open sets are independent.*
(6) *The concepts of soft* $A_R S$ *open set and soft* β *open sets are independent.*
(7) *The concepts of soft* $A_R S$ *open set and soft pre open sets are independent.*
(8) *The concepts of soft* $A_R S$ *open set and soft* g *open sets are independent.*
(9) *The concepts of soft* $A_R S$ *open set and soft* ω *open sets are independent.*
(10) *The concepts of soft* $A_R S$ *open set and soft* αg *open sets are independent.*
(11) *The concepts of soft* $A_R S$ *open set and soft* gs *open sets are independent.*
(12) *The concepts of soft* $A_R S$ *open set and soft* gp *open sets are independent.*

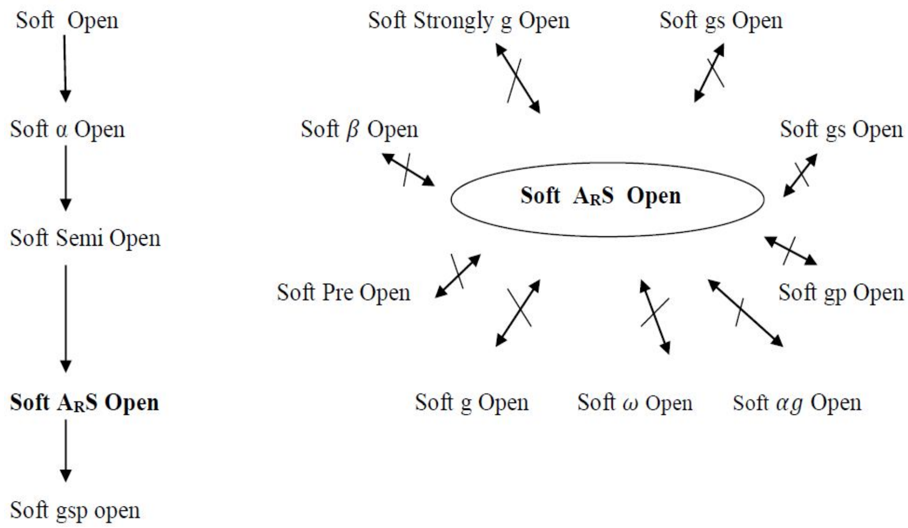

FIGURE 2. Interrelationship

5. CONCLUSION

Thus, we have introduced the concept of soft A_RS closed set in soft topological spaces and studied some basic properties of them. In future, the study on continuous mappings, locally closed sets and separation axioms with the help of soft A_RS closed set may be carried out.

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