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SOME NEW SETS ON GENERALIZED TOPOLOGY

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ABSTRACT. In this paper g^* sets, ω^* sets, $g^*\omega^*$ sets are introduced and properties are studied. Properties of continuity is studied using these sets.

1. INTRODUCTION

Csaszar introduced generalized topology, generalized open sets, generalized closed sets in [1, 2] in 2002. Mathematicians studied further and introduced semi open sets, semi closed sets, g-closed sets, g-open sets, ω-closed sets and ω-open sets, [5]. Further various continuous functions were introduced using the above sets, [3, 4].

In this paper g^* sets, ω^* sets, $g^*\omega^*$ sets are introduced and properties are studied.

Also some different types of continuous functions are introduced. Also a decomposition is introduced.

2. PRELIMINARIES

(1) Generalized Topology : Let X be a non empty set. Let $\mu \subset P(X)$. μ is called a generalized topology if $\Phi \in X$ and μ is closed under arbitrary union. Elements of μ are called open sets.
(2) Semi open set : A subset A of X is called semi open set if $A \subset cl(intA)$.

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(3) g-closed set: A is called g-closed set if $c(A) \subset U$, whenever U is an open set containing A. The complement of g-closed set is g-open set.
(4) ω-Closed set : A is called ω-Closed set if $c(A) \subset U$, whenever U is a semi open set containing A. The complement of ω-closed set is ω-open set.
(5) Every open set is ω-open.
(6) Every ω-open is g-open.

3. New sets

Definition 3.1. *Let X be generalized topological space. Let $A \subset X$. A is called a $D(\mu, \omega(\mu))$ set or ω^* set if $i_\mu(A) = i_\omega(A)$.*

Definition 3.2. *Let X be generalized topological space. Let $A \subset X$. A is called a $D(\mu, g(\mu))$ set or g^* set if $i_\mu(A) = i_g(A)$.*

Definition 3.3. *Let X be generalized topological space. Let $A \subset X$. A is called $D(\omega(\mu), g(\mu))$ set or $g^*\omega^*$ set if $i_\omega(A) = i_g(A)$.*

Example 1. *Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{b\}, \{a, c\}, \{b, c\}, X\}$. Then $A = \{c\}$ is ω-open set but neither ω^* set nor g^* set, $B = \{a, b\}$ is ω^* set but not ω-open set, $C = \{a\}$ is g^* set but not ω-open set.*

Example 2. *Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then $A = \{c\}$ is g-openset but neither g^* set nor $g^*\omega^*$ set, $B = \{b, d\}$ is g^* set but not g-open set and $C = \{a, d\}$ is $g^*\omega^*$ set but not g-open set.*

Example 3. *Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{b\}, \{a, c\}, \{b, c\}, X\}$. Then in (X, μ), $A = \{a, c\}$ and $B = \{b, c\}$ are ω^* sets and g^* sets but $A \cap B = \{c\}$ is neither ω^* sets nor g^* sets.*

Example 4. *Let $X = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, d\}, \{a, b, c, d\}\}$. Then $A = \{a, b, d, e\}$ and $B = \{b, c, d, e\}$ are $g^*\omega^*$ sets but $A \cap B = \{b, d, e\}$ is not a $g^*\omega^*$ set.*

Example 5. *Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. Then $A = \{d\}$ is g^* set but not μ-open set.*

Example 6. *Let $X = \{a, b, c\}$, $\mu = \{\emptyset, \{b\}, \{a, c\}, \{b, c\}, X\}$. Then $A = \{c\}$ is ω-open set but neither g^* set nor $g^*\omega^*$ set, $B = \{a\}$ is g^* set but not ω-open set.*

Example 7. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. *Then* $A = \{c\}$ *is* ω^* *set but not* $g^*\omega^*$ *set.*

Example 8. *Let* $X = \{a, b, c\}$, $\mu = \{\emptyset, \{b\}, \{a, c\}, \{b, c\}, X\}$. *Then* $A = \{c\}$ *is* $g^*\omega^*$ *set but not* ω^* *set.*

Example 9. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. *Then* $A = \{c\}$ *is g -open set but not* $g^*\omega^*$ *set and* $B = \{a, d\}$ *is* $g^*\omega^*$ *set but not g -openset.*

Example 10. *Let* $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$. *Then* $A = \{c\}$ *is* ω^* *set but not* g^* *set.*

Example 11. *Let* $X = \{a, b, c\}$, $\mu = \{\emptyset, \{b\}, \{a, c\}, \{b, c\}, X\}$. *Then* $A = \{a\}$ *is* ω^* *set but not* μ*-open set.*

4. CHARACTERIZATIONS

Theorem 4.1. *In* (X, μ) *the following are true.*

(1) *Every open set is a* g^* *set.*
(2) *Every* g^* *set is a* ω^* *set.*
(3) *Every open set is* ω^* *set.*
(4) *Every open set is* $g^*\omega^*$ *set.*
(5) *Every* g^* *set is* $g^*\omega^*$ *set.*
(6) *Every* ω *open set is* $g^*\omega^*$ *set.*

Proof.

(1) Let A be an open set. Then $A = i(A)$. Every open set is g open set. We have $i(A) \subset i_g(A)$. Always it is true that $i_g(A) \subset A$. Therefore $i(A) = i_g(A)$. Hence A is a g^* set.
(2) Let A be a g^* set. Then $i(A) = i_g(A)$. Every ω open set is g open set. We have $i_\omega(A) \subset i_g(A)$. Therefore $i_\omega(A) \subset i_g(A) = i(A)$. Every open set is ω open set. Therefore $i(A) \subset i_\omega(A)$. We have $i(A) = i_\omega(A)$. Hence A is ω^* set.
(3) Let A be an open set. Then $i(A) = A$. Every open set is ω open set. We have $i(A) \subset i_\omega(A)$. Always $i_\omega(A) \subset i(A)$. Therefore $i(A) = i_\omega(A)$. Hence A is a ω^* set.

(4) Let A be an open set. Then $i(A) = (A)$. Every open set is ω^* set. We have $i(A) = i_\omega(A)$. Every open set is a g^* set. Therefore $i(A) = i_g(A)$. Therefore $i_\omega(A) = i_g(A)$. Hence A is $g^*\omega^*$ set.
(5) Let A be a g^* set. Then $i(A) = i_g(A)$. Every g^* set is a ω^* set. Therefore A is ω^* set. Therefore $i(A) = i_\omega(A)$. Hence $i_\omega(A) = i_g(A)$. Hence A is $g^*\omega^*$ set.
(6) Let A be an ω open set. Then $A = i_\omega(A)$. Every ω open set is g open set. Therefore A is g open set. We have $A = i_g(A)$. Therefore $i_\omega(A) = i_g(A)$. Hence A is a $g^*\omega^*$ set.

□

Result 1. *In each case converse is not true. This is seen from the above examples.*

Theorem 4.2. *Let (X, μ) be a generalized topological space. A subset A is open iff A is both ω open and ω^* set.*

Proof. Let A be an open set. Hence i(A)=A. Every open set is ω open set. $i(A) \subset i_\omega(A)$. Always $i_\omega(A) \subset i(A)$. Hence $i(A) = i_\omega(A)$ and $i_\omega(A) = A$. Therefore A is both ω open and ω^* set.
Conversely, let A be both ω open and ω^* set. Now A is ω open implies $i_\omega(A) = A$. Also A is ω^* set implies $i_\omega(A) = i(A)$. Hence i(A)=A. Therefore A is open. □

Theorem 4.3. *Let (X, μ) be a generalized topological space. A subset A is open iff A is both g open and g^* set.*

Proof. Proof is similar. □

Theorem 4.4. *Let (X, μ) be a generalized topological space. A subset A is open iff A is both ω open and g^* set.*

Proof. Proof is similar. □

Theorem 4.5. *Let (X, μ) be a generalized topological space. A subset A is ω open A iff A is g open and$g^*\omega^*$ set.*

Proof. Proof is similar. □

Theorem 4.6. *Let (X, μ) be a generalized topological space. A subset A is g^* open iff A is both $g^*\omega^*$ set and ω^* set.*

Proof. Proof is similar. □

Theorem 4.7. *Let (X, μ) be a generalized topological space. A subset A is open iff A is g open, $g^*\omega^*$ set and ω^* set.*

Proof. Proof is similar. □

Theorem 4.8. *For a subset A of (X, μ) the following conditions are equivalent:*

(1) *A is μ-open.*
(2) *A is ω-open and a g^* set.*
(3) *A is g-open and a g^* set.*
(4) *A is ω-open and ω^* set.*

Proof. Proof follows from above theorems. □

Remark 4.1. (1) *The notions of ω-open sets and ω^* sets are independent,*
(2) *The notions of ω-open sets and g^* sets are independent,*
(3) *The notions of g-open sets and g^* sets are independent,*
(4) *The notions of g-opensets and $g^*\omega^*$ sets are independent,*
(5) *The notions of ω^* sets and $g^*\omega^*$ sets are independent.*

5. Decompositions of continuity

The following definitions are given in [6–8].

Definition 5.1. *A function $f : X \to Y$ is said to be g-continuous, if for each open set U in Y, $f^1(U)$ is g open set in X.*

Definition 5.2. *A function $f : X \to Y$ is said to be ω-continuous, if for each open set U in Y, $f^1(U)$ is ω open set in X.*

Definition 5.3. *A function $f : X \to Y$ is said to be g^*-continuous, if for each open set U in Y, $f^1(U)$ is g^* set in X.*

Definition 5.4. *A function $f : X \to Y$ is said to be ω^*-continuous, if for each open set U in Y, $f^1(U)$ is ω^* set in X.*

Definition 5.5. *A function $f : X \to Y$ is said to be $g^*\omega^*$-continuous ,if for each open set U in Y, $f^1(U)$ is $g^*\omega^*$ set in X.*

Theorem 5.1. *Let $f : (X, \mu) \to (Y, \lambda)$. Then the following conditions are equivalent:*

(1) *f is -continuous,*
(2) *f is ω-continuous and g^*-continuous,*
(3) *f is g-continuous and g^*-continuous.*
(4) *f is ω-continuous and ω^*-continuous.*

Proof. Proof follows from Theorem 4.8. □

Theorem 5.2. *Let $f : X \to Y$ is ω-continuous if and only if it is g-continuous and $g^*\omega^*$-continuous.*

Proof. Proof follows from Theorem 4.6. □

Theorem 5.3. *Let $f : X \to Y$ is g^*-continuous if and only if it is $g^*\omega^*$-continuous and ω^*-continuous.*

Proof. Proof follows from Theorem 4.7. □

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