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DECOMPOSITION OF NANO α - CONTINUITY AND NANO $\ddot{G}_\alpha$ - CONTINUITY

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ABSTRACT. The main purpose of this paper is to introduce the concepts of N$\eta^\sharp$-sets, N$\eta^{\sharp\sharp}$-sets, N$\eta^\sharp$-continuity and N$\eta^{\sharp\sharp}$-continuity and to obtain decomposition of Nano α-continuity and N$\ddot{g}_\alpha$-continuity in nano topological spaces.

1. INTRODUCTION

Jayalakshmi and Janaki [8] introduced and studied the notions of Nt-sets, NA-sets and NB-sets in nano topological spaces. Recently, Ganesan [7] introduced and studied the notions of NαB-sets, Nη-sets and N$\eta\zeta$-sets in Nano topological spaces, to obtain a decomposition of nano continuity. In this paper, we introduce the notions of N$\eta^\sharp$-sets, N$\eta^{\sharp\sharp}$-sets, N$\eta^\sharp$-continuity and N$\eta^{\sharp\sharp}$-continuity and obtain decomposition of Nano α-continuity and N$\ddot{g}_\alpha$-continuity. Moreover the study of N$\eta^\sharp$-sets, N$\eta^{\sharp\sharp}$-sets led to some decomposition of nano continuity which is extensively developed and used in computer science and digital topology.

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2. Preliminaries

Definition 2.1. *[9] Let U be a non-empty finite set of objects, called the universe, and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible one with another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(1) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L_R(X). That is, L_R(X) = $\bigcup x_\epsilon$ U $\{R(X) : R(X) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X.*
(2) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to $R4$ and it is denoted by U_R(X). That is, $U_R(X) = \bigcup x_\epsilon U\{R(X) : R(X) \cap X \neq \emptyset\}$.*
(3) *The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Proposition 2.1. *[9] If (U, R) is an approximation space and $X, Y \subseteq U$, then:*

(1) L_R(X) $\subseteq$ X $\subseteq$ U_R(X).
(2) $L_R(\emptyset)$ = $U_R(\emptyset)$ = $\emptyset$, L_R(U) = U_R(U) = U.
(3) U_R(X $\cup$ Y) = U_R(X) $\cup$ U_R(Y).
(4) U_R(X $\cap$ Y) $\subseteq$ U_R(X) $\cap$ U_R(Y).
(5) L_R(X $\cup$ Y) $\supseteq$ L_R(X) $\cup$ L_R(Y).
(6) L_R(X $\cap$ Y) = L_R(X) $\cap$ L_R(Y).
(7) L_R(X) $\subseteq$ L_R(Y) and U_R(X) $\subseteq$ U_R(Y) whenever X $\subseteq$ Y .
(8) $U_R(X^c\ ) = [L_R((X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.
(9) U_R (U_R (X)) = L_R (U_R (X)) = U_R (X).
(10) L_R (L_R (X)) = U_R(L_R (X))=L_R (X) .

Definition 2.2. *[9] Let U be an universe, R be an equivalence relation on U and $\tau U_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then by Property 2.1, $\tau U_R(X)$ satisfies the following axioms:*

(1) *U , $\emptyset \in \tau U_R(X)$;*
(2) *the union of the elements of any sub-collection of $\tau U_R(X)$ is in $\tau U_R(X)$;*
(3) *the intersection of the elements of any finite sub collection of $\tau U_R(X)$ is in $\tau U_R(X)$.*

Then $\tau U_R(X)$ is called the Nano topology on U with respect to X. The space (U, $\tau U_R(X)$) is the Nano topological space. The elements of are called Nano open sets.

Definition 2.3. *[9] If $(U, \tau U_R(X))$ is the Nano topological space with respect to X where $X \subseteq U$ and if $M \subseteq U$, then:*

(1) *The Nano interior of the set M is defined as the union of all Nano open subsets contained in M and it is denoted by $NInte(M)$. That is, $NInte(M)$ is the largest Nano open subset of M.*
(2) *The Nano closure of the set M is defined as the intersection of all Nano closed sets containing M and it is denoted by $NClo(M)$. That is, $NClo(M)$ is the smallest Nano closed set containing M.*

Definition 2.4. *A subset M of a space $(U, \tau U_R(X))$ is called:*

(1) *Nano α-open set [9] if $M \subseteq$ Ninte(Ncl(Nint(M))).*
(2) *Nano semi-open set [9] if $M \subseteq$ Ncl(Nint(M)).*
(3) *Nano pre-open set [9] if $M \subseteq$ Nint(Ncl(M)).*
(4) *Nano regular-open set [9] if M = Nint(Ncl(M)).*

The complements of the above mentioned Nano open sets are called their respective Nano closed sets.

The Nano α-closure [5] (resp. Nano semi-closure [2, 3], Nano pre-closure [1]) of a subset M of U, denoted by Nαcl(M) (resp.Nscl(M), Npcl(M)) is defined to be the intersection of all Nano α-closed (resp. Nano semi-closed, Nano pre closed) sets of $(U, \tau U_R(X))$ containing M.

Definition 2.5. *A subset M of a space $(U, \tau U_R(X))$ is called:*

(1) *a Nt-set [8] if Nint(Ncl(M)) = Nint(M).*
(2) *an NA-set [8] if $M = S \cap G$ where S is Nano open and G is a Nano regular closed set.*
(3) *a NB-set [8] if $M = S \cap G$ where S is Nano open and G is a Nt-set.*
(4) *a Nano locally closed set [4] if $M = S \cap G$ where S is Nano open and G is Nano closed.*
(5) *an NαB-set [7] if $M = S \cap G$ where S is Nano α-open and G is a Nt-set.*
(6) *an Nη-set [7] if $M = S \cap G$ where S is Nano open and G is an Nano α-closed set.*

The collection of Nt-sets (resp. NA-sets, NB-sets, locally closed sets, NαB-set, Nη-set) in U is denoted by Nt(U) (resp. NA(U), NB(U), NLC(U), NαB(U), Nη(U)).

Definition 2.6. *A subset M of a space $(U, \tau U_R(X))$ is called*

(1) *a Nano semi generalized closed (briefly Nsg-closed) set [2] if Nscl(M) $\subseteq$ T whenever $M \subseteq T$ and T is Nano semi-open in $(U, \tau U_R(X))$.*
(2) *Nano $\ddot{g}_\alpha$-closed (briefly N$\ddot{g}_\alpha$-closed) set [5] if Nαcl(M) $\subseteq$ T whenever $M \subseteq$ T and T is Nsg-open in $(U, \tau U_R(X))$. The complement of N$\ddot{g}_\alpha$-closed set is called N$\ddot{g}_\alpha$-open set.*
(3) *Nano $\ddot{g}_p$-closed (briefly N$\ddot{g}_p$-closed) set [5] if Npcl(M) $\subseteq$ T whenever $M \subseteq$ T and T is Nsg-open in $(U, \tau U_R(X))$. The complement of N$\ddot{g}_p$-closed set is called N$\ddot{g}_p$-open set.*

The collection of all N$\ddot{g}_\alpha$-closed (resp. N$\ddot{g}_p$-closed) sets is denoted by:

$$N\ddot{g}_\alpha c((\tau U_R(X))(resp. N\ddot{g}_p c((\tau U_R(X)))$$

Proposition 2.2. *[6] In a space U, the following statements hold:*

(1) *Every Nano α-open set is N$\ddot{g}_\alpha$-open but not conversely.*
(2) *Every N$\ddot{g}_\alpha$-open set is N$\ddot{g}_p$-open but not conversely.*
(3) *Every N$\ddot{g}_\alpha$-continuous map is N$\ddot{g}_p$-continuous but not conversely.*

Theorem 2.1. (1) *Every Nano closed is Nt-set but not conversely [8].*
(2) *Every Nano α-closed set is Nano semi-closed but not conversely [11].*
(3) *Every Nt-set is NB-set but not conversely [8].*

Theorem 2.2. *[8] In a space U, the following statements hold:*

(1) *M is Nt-set if and only if it is Nano semi closed.*
(2) *If M and N are two Nt-sets, then $M \cap N$ is a Nt-set.*

3. N$\eta^\sharp$-SETS AND N$\eta^{\sharp\sharp}$-SETS

In this section we introduce and study the notions of N$\eta^\sharp$-sets and N$\eta^{\sharp\sharp}$-sets in nano topological spaces.

Definition 3.1. *A subset M of a space U is called:*

(1) *an $N\eta^\sharp$-set if $M = S \cap G$ where S is Nsg-open and G is Nano α-closed in U.*

(2) *an* $N\eta^{\sharp\sharp}$*-set if* $M = S \cap G$ *where* S *is* $N\ddot{g}_{\alpha}$*-open and* G *is a Nt-set in* U.

The collection of all N$\eta^{\sharp}$*-sets (resp. N*$\eta^{\sharp\sharp}$*-sets) in* U *will be denoted by N*$\eta^{\sharp}$*(U) (resp. N*$\eta^{\sharp\sharp}$*(U))*

Proposition 3.1. *Every N*η*-set is N*$\eta^{\sharp}$*-set but not conversely.*

Proof. Let A be Nη-set. Then $A = S \cap G$, where S is Nano open and G is Nano α-closed set. Since every Nano open set is Nsg-open set, S is Nsg-open set. Hence A is N$\eta^{\sharp}$-set. □

Example 1. *Let* $U = \{1, 2, 3, 4\}$ *with* $U/R = \{\{3\}, \{4\}, \{1, 2\}\}$ *and* $X = \{2\}$. *The Nano topology* $\tau U_R(X) = \{\emptyset, \{1, 2\}, U\}$. *Then* $N\eta^{\sharp}$*-sets are* $\emptyset$, U, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{1, 2\}$, $\{3, 4\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$ *and N*η*-set are* $\emptyset$, U, $\{3\}$, $\{4\}$, $\{1, 2\}$, $\{3, 4\}$. *It is clear that* $\{1, 2, 3\}$ *is N*$\eta^{\sharp}$*-set but it is not N*η*-set.*

Proposition 3.2. *Every N*α*B-set is N*$\eta^{\sharp\sharp}$*-set but not conversely.*

Proof. Let A be NαB-set. Then $A = S \cap G$, where S is Nano α-open and G is Nt set. Since every Nano α-open set is N$\ddot{g}_{\alpha}$-open set, S is N$\ddot{g}_{\alpha}$-open set. Hence A is N$\eta^{\sharp\sharp}$-set. □

Example 2. *Let* U *and* $\tau U_R(X)$ *as in the Example 1. Then N*$\eta^{\sharp\sharp}$*-set are* $\emptyset$, U, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{1, 2\}$, $\{3, 4\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$ *and N*α*B-set are* ϕ, U, $\{3\}$, $\{4\}$, $\{1, 2\}$, $\{3, 4\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$. *It is clear that* $\{2\}$ *is N*$\eta^{\sharp\sharp}$*-set but it is not N*α*B-set.*

Proposition 3.3. *Every N*$\ddot{g}_{\alpha}$*-open set is N*$\eta^{\sharp\sharp}$*-set but not conversely.*

Proof. It follows form Definition 2.6 (2) and Definition 3.1 (2). □

Example 3. *Let* U *and* $\tau U_R(X)$ *as in the Example 2. Then N*$\ddot{g}_{\alpha}$*-open set are* $\emptyset$, U, $\{1\}$, $\{2\}$, $\{1, 2\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$. *It is clear that* $\{4\}$ *is N*$\eta^{\sharp\sharp}$*-set but it is not N*$\ddot{g}_{\alpha}$*-open set.*

Remark 3.1.

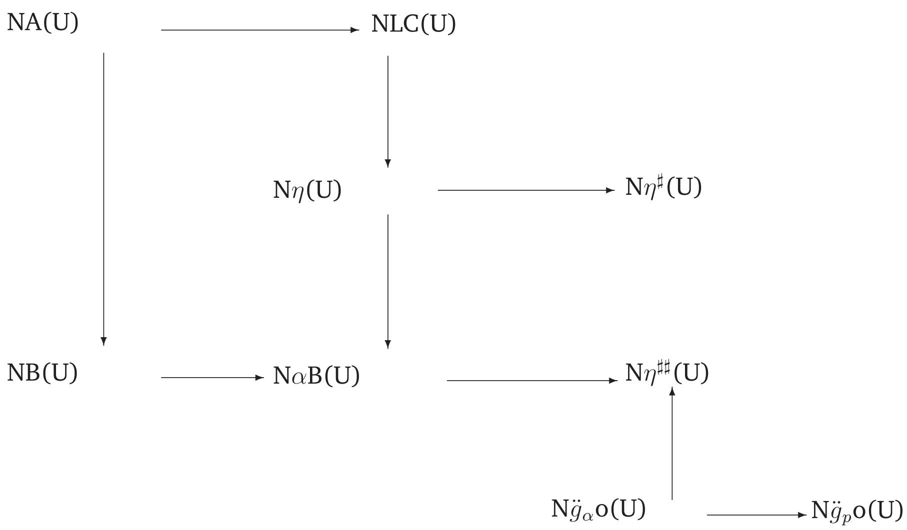
(1) *The notions of N*$\eta^{\sharp}$*-sets and N*$\ddot{g}_{\alpha}$*-closed sets are independent.*
(2) *The notions of N*$\eta^{\sharp\sharp}$*-sets and N*$\ddot{g}_{p}$*-closed sets are independent.*

Example 4.

(1) *Let U and $\tau U_R(X)$ are as in the Example 1. Then N$\ddot{g}_\alpha$-closed sets $\emptyset$, U, $\{3\}$, $\{4\}$, $\{3,4\}$, $\{1,3,4\}$, $\{2,3,4\}$. The set $\{1,3,4\}$ is N$\ddot{g}_\alpha$-closed but not an N$\eta^\sharp$-set and also the set $\{1,2,3\}$ is an N$\eta^\sharp$-set but not a N$\ddot{g}_\alpha$-closed in $(U, \tau U_R(X))$.*

(2) *Let U and $\tau U_R(X)$ as in the Example 2. Then N$\ddot{g}_p$-closed sets are $\emptyset$, U, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{1,3\}$, $\{1,4\}$, $\{2,3\}$, $\{2,4\}$, $\{3,4\}$, $\{1,3,4\}$, $\{2,3,4\}$. The set $\{1,3\}$ is N$\ddot{g}_p$-closed but not an N$\eta^{\sharp\sharp}$-set and also the set $\{1,2,4\}$ is an N$\eta^{\sharp\sharp}$-set but not a N$\ddot{g}_p$-closed in $(U, \tau U_R(X))$.*

Remark 3.2. *We have the following implications.*



where none of these implications is reversible as shown in [7].

Theorem 3.1. *For a subset M of a space U, the following are equivalent:*

(1) *M is an N$\eta^\sharp$-set.*
(2) *$M = S \cap$ Nαcl(M) for some Nsg-open set S.*

Proof. (1) $\Rightarrow$ (2) Since M is an N$\eta^\sharp$-set, then $M = S \cap G$, where S is Nsg-open and G is Nano α-closed. So, $M \subset S$ and $M \subset G$. Hence Nαcl(M) $\subset$ Nαcl(G). Therefore $M \subset S \cap$ Nαcl(M) $\subset S \cap$ Nαcl(G) $= S \cap G = M$. Thus, $M = S \cap N\alpha cl(M)$..

(2) $\Rightarrow$ (1) It is obvious because Nαcl(M) is Nano α-closed. (Since A is Nano α-closed if and only if $A = N\alpha cl(A)$). □

Remark 3.3. *In a space U, the intersection of two N$\eta^\sharp$-sets is an N$\eta^\sharp$-set.*

Remark 3.4. *Union of two N$\eta^\sharp$-sets need not be an N$\eta^\sharp$-set as seen from the following example.*

Example 5. *Let U and $\tau U_R(X)$ are as in the Example 1. The sets $\{1\}$, $\{3\}$ are N$\eta^{\sharp\sharp}$-sets in $(U, \tau U_R(X))$ but their union $\{1, 3\}$ is not an N$\eta^\sharp$-set in $(U, \tau U_R(X))$.*

Theorem 3.2. *For a subset M of a space U, the following are equivalent:*

(1) *M is Nano α-closed.*
(2) *M is an N$\eta^\sharp$-set and Nano N$\ddot{g}_\alpha$-closed.*

Proof. (1) $\Rightarrow$ (2) This is obvious.

(2) $\Rightarrow$ (1) Since M is an N$\eta^\sharp$-set, then according to Theorem 3.1, $M = S \cap N\alpha cl(M)$ where S is Nsg-open in U. So, $M \subset S$ and since M is N$\ddot{g}_\alpha$-closed, then $N\alpha cl(M) \subset S$. Therefore, Nαcl(M) $\subset S \cap N\alpha cl(M) = M$. Hence, M is Nano α-closed. □

Remark 3.5. *In a space U, the intersection of two N$\eta^{\sharp\sharp}$-sets is an N$\eta^{\sharp\sharp}$-set.*

Remark 3.6. *Union of two N$\eta^{\sharp\sharp}$-sets need not be an N$\eta^{\sharp\sharp}$-set as seen from the following example.*

Example 6. *Let U and $\tau U_R(X)$ are as in the Example 2. The sets $\{2\}$, $\{3, 4\}$ are N$\eta^{\sharp\sharp}$-sets in $(U, \tau U_R(X))$ but their union $\{2, 3, 4\}$ is not an N$\eta^{\sharp\sharp}$-set in $(U, \tau U_R(X))$.*

Theorem 3.3. *For a subset M of a space U, the following are equivalent.*

(1) *M is N$\ddot{g}_\alpha$-open.*
(2) *M is an N$\eta^{\sharp\sharp}$-set and N$\ddot{g}_p$-open.*

Proof. Necessity: It follows from Remark 1.4 (3) and Definition 2.1 (2).

Sufficiency: Assume that M is N$\ddot{g}_p$-open and an N$\eta^{\sharp\sharp}$-set in U. Then $M =$ A $\cap$ B where A is N$\ddot{g}_\alpha$-open and B is a Nt-set in U. Let $F \subset M$, where F is Nsg-closed in U. Since M is N$\ddot{g}_p$-open in U, $F \subset$ Npint(M) = $M \cap$ Nint(Ncl(M)) = (A $\cap$ B) $\cap$ Nint[Ncl(A $\cap$ B)] $\subset$ A $\cap$ B $\cap$ Nint(Ncl(A)) $\cap$ Nint(Ncl(B)) = A $\cap$ B $\cap$ Nint(Ncl(A)) $\cap$ Nint(B), since B is a Nt-set. This implies, F $\subset$ Nint(B). Note that M is N$\ddot{g}_\alpha$-open and that F$\subset$ A. So, F $\subset$ Nαint(A). Therefore, F $\subset$ Nαint(A) $\cap$ Nint(B) = Nαint(S). Hence S is N$\ddot{g}_\alpha$-open. □

4. Nη-CONTINUITY, N$\eta^\sharp$-CONTINUITY AND N$\eta^{\sharp\sharp}$-CONTINUITY

Definition 4.1. *A map f : $(U, \tau U_R(X)) \rightarrow$ (L, $\tau U'_R(Y)$) is called:*

1. *A-continuous [7] if f^{-1}(V) is a NA-set in U for every Nano open set V of L.*
2. *B-continuous [7] if f^{-1}(V) is a NB-set in U for every Nano open set V of L.*
3. *Nano α-continuous [10] if f^{-1}(V) is a Nano α-open set in U for every Nano open set V of L.*
4. *Nano LC-continuous [4] if f^{-1}(V) is a Nano locally closed set in U for every Nano open set V of L.*
5. *NαB-continuous [7] if f^{-1}(V) is a NαB-set in U for every Nano open set V of L.*
6. *Nη-continuous [7] if f^{-1}(V) is a Nη-set in U for every Nano open set V of L.*
7. *N$\ddot{g}_\alpha$-continuous [6] (resp. N$\ddot{g}_p$-continuous [6]) if f^{-1}(V) is an N$\ddot{g}_\alpha$-open set (resp. N$\ddot{g}_p$-open set) in U for every Nano open set V of L.*

Definition 4.2. *A map f : $(U, \tau U_R(X)) \rightarrow (L, \tau U'_R(Y))$ is said to be N$\eta^\sharp$-continuous (resp. N$\eta^{\sharp\sharp}$-continuous) if f^{-1}(V) is an N$\eta^\sharp$-set (resp. an N$\eta^{\sharp\sharp}$-set) in U for every Nano open subset V of L.*

Definition 4.3. *A map f : $(U, \tau U_R(X)) \rightarrow (L, \tau U'_R(Y))$ is said to be N$^\sharp\eta^\sharp$-continuous if f^{-1}(V) is an N$\eta^\sharp$-set in U for every Nano closed subset V of L.*

Remark 4.1. *It is clear that, a map f : $(U, \tau U_R(X)) \rightarrow (L, \tau U'_R(Y))$ is Nano α-continuous if and only if f^{-1}(V) is an Nano α-closed set in U for every Nano closed set V of L.*

Proposition 4.1. *Every Nη-continuous is N$\eta^\sharp$-continuous but not conversely.*

Proof. It is follows from Proposition 3.1. □

Example 7. *Let $U = \{1, 2, 3\}$, with $U/R = \{\{1\}, \{2, 3\}\}$ and $X = \{1\}$. Then the Nano topology is $\tau U_R(X) = \{\emptyset, \{1\}, U\}$. Let $L = \{1, 2, 3\}$ with $L/R' = \{\{2\}, \{1, 3\}, \{3, 1\}\}$ and $Y = \{1, 3\}$. Then Nano topology is $\tau U'_R(Y) = \{\emptyset, \{1, 3\}, L\}$. The $N\eta^\sharp$ -sets are $\emptyset, U, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$ and $N\eta$-sets are $\emptyset$, U, $\{1\}, \{2\}, \{3\}, \{2, 3\}$. Let $f : (U, \tau U_R(X)) \rightarrow (L, \tau U'_R(Y))$ be the identity map. Then it is $N\eta^\sharp$-continuous but not $N\eta$-continuous, since $f^{-1}(\{1, 3\}) = \{1, 3\}$ is not $N\eta$-set.*

Proposition 4.2. *Every NαB-continuous is N$\eta^{\sharp\sharp}$-continuous but not conversely.*

Proof. It is follows from Proposition 3.2. □

Example 8. *Let* $U = \{1,2,3\}$*, with* $U/R = \{\{3\},\{1,2\},\{2,1\}\}$ *and* $X = \{1,2\}$*. Then the Nano topology is* $\tau U_R(X) = \{\emptyset,\{1,2\},U\}$*. Let* $L = \{1,2,3\}$ *with* $L/R' = \{\{1\},\{2,3\}\}$ *and* $Y = \{1\}$*. Then Nano topology is* $\tau U'_R(Y) = \{\emptyset,\{1\},L\}$*. The* $N\eta^{\sharp\sharp}$*-sets are* $\emptyset, U, \{1\},\{2\},\{3\},\{1,2\}$ *and* $N\alpha B$*-sets are* $\emptyset, U, \{3\},\{1,2\}$*. Define* $f : (U,\tau U_R(X)) \to (L,\tau U'_R(Y))$ *be the identity map. Then it is* $N\eta^{\sharp\sharp}$*-continuous but not* $N\alpha B$*-continuous, since* $f^{-1}(\{1\}) = \{1\}$ *is not* $N\alpha B$*-set.*

Proposition 4.3. *Every* $N\ddot{g}_\alpha$*-continuous is* $N\eta^{\sharp\sharp}$*-continuous but not conversely.*

Proof. It is follows from Proposition 3.3. □

Example 9. *Let* $U, \tau U_R(X)$ *and* f *be as in the Example 8. Let* $L = \{1,2,3\}$ *with* $L/R' = \{\{3\},\{1,2\}\}$ *and* $Y = \{3\}$*. Then Nano topology is* $\tau U'_R(Y) = \{\emptyset,\{3\},L\}$*. The* $N\ddot{g}_\alpha$*-open sets are* $\emptyset, U, \{1\},\{2\},\{1,2\}$*. Define* $f : (U,\tau U_R(X)) \to (L,\tau U'_R(Y))$ *be the identity map. Then it is* $N\eta^{\sharp\sharp}$*-continuous but not* $N\ddot{g}_\alpha$*-continuous, since* $f^{-1}(\{3\}) = \{3\}$ *is not* $N\ddot{g}_\alpha$*-open set.*

Remark 4.2. *The following examples show that the concepts of*

(1) *$N\ddot{g}_p$-continuity and $N\eta^{\sharp\sharp}$-continuity are independent.*
(2) *$N\ddot{g}_\alpha$-continuity and $N^\sharp\eta^\sharp$-continuity are independent.*
(3) *$N\eta^\sharp$-continuity and $N^\sharp\eta^\sharp$-continuity are independent.*

Example 10. *Let U,* $\tau U_R(X)$*, L,* $\tau U'_R(Y)$ *and f as in the Example 9. Then* $N\eta^{\sharp\sharp}$*-sets are* $\emptyset$*, U,* $\{1\}$*,* $\{2\}$*,* $\{3\}$*,* $\{1,2\}$ *and* $N\ddot{g}_p$*-open sets are* $\emptyset$*, U,* $\{1\}$*,* $\{2\}$*,* $\{1,2\}$*,* $\{1,3\}$*,* $\{2,3\}$*. Define f :* $(U,\tau U_R(X)) \to (L,\tau U'_R(Y))$ *be the identity map. Then it is* $N\eta^{\sharp\sharp}$*-continuous but not* $N\ddot{g}_p$*-continuous, since* $f^{-1}(\{3\}) = \{3\}$ *is not* $N\ddot{g}_p$*-open set.*

Example 11. *Let* $U = \{1,2,3\}$*, with* $U/R = \{\{2\},\{1,3\},\{3,1\}\}$ *and* $X = \{1,3\}$*. Then the Nano topology is* $\tau U_R(X) = \{\emptyset,\{1,3\},U\}$*. Let* $L = \{1,2,3\}$ *with* $L/R' = \{\{3\},\{1,2\},\{2,1\}\}$ *and* $Y = \{1,2\}$*. Then Nano topology is* $\tau U'_R(Y) = \{\emptyset,\{1,2\},L\}$*. The* $N\eta^{\sharp\sharp}$*-sets are* $\emptyset, U, \{1\},\{2\},\{3\},\{1,3\}$ *and* $N\ddot{g}_p$*-open sets are* $\emptyset, U,$ $\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\}$*. Then* $N\ddot{g}_p$*-continuous but not* $N\eta^{\sharp\sharp}$*-continuous, since* $f^{-1}(\{1,2\}) = \{1,2\}$ *is not* $N\eta^{\sharp\sharp}$*-set.*

Example 12. *Let* $U = \{1,2,3\}$*, with* $U/R = \{\{1\},\{2,3\},\{3,2\}\}$ *and* $X = \{2,3\}$*. Then the Nano topology is* $\tau U_R(X) = \{\emptyset,\{2,3\},U\}$*. Let* $L, \tau U'_R(Y)$*, and* f *as in the Example 9. Then* $N\ddot{g}_\alpha$*-open sets are* $\emptyset, U, \{2\},\{3\},\{2,3\}$ *and* $N\eta^\sharp$*-sets are*

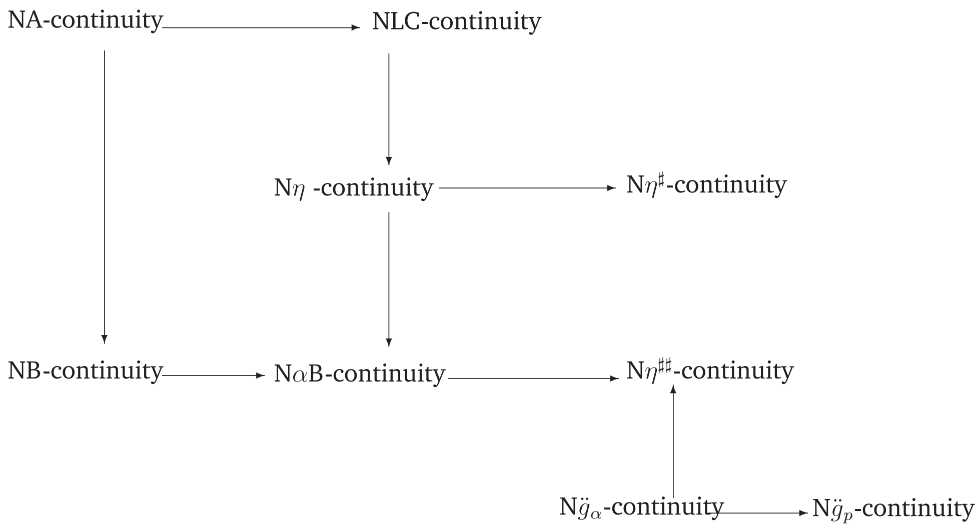
$\emptyset, U, \{1\}, \{2\}, \{3\}, \{2,3\}$. *Then* $N\ddot{g}_\alpha$*-continuous but not* $N^\sharp\eta^\sharp$*-continuous, since* $f^{-1}(\{1,2\}) = \{1,2\}$ *is not* $N^\sharp\eta^\sharp$*-set.*

Example 13. *Let* $U, \tau U_R(X)$*, and* f *as in the Example 9. Let* $L = \{1,2,3\}$ *with* $L/R' = \{\{1\},\{2,3\},\{3,2\}\} and Y = \{2,3\}$. *Then Nano topology is* $\tau U'_R(Y) = \{\emptyset, \{2,3\}, L\}$. *The* $N\eta^\sharp$*-sets are* $\emptyset, U, \{1\}, \{2\}, \{3\}, \{1,2\}$ *and* $N\ddot{g}_\alpha$*-open sets are* $\emptyset, U, \{1\}, \{2\}, \{1,2\}$. *Then* $N^\sharp\eta^\sharp$*-continuous but not* $N\ddot{g}_\alpha$*-continuous, since* $f^{-1}(\{2,3\})$ $= \{2,3\}$ *is not* $N\ddot{g}_\alpha$*-open set.*

Example 14. *Let* $U, \tau U_R(X), L, \tau U'_R(Y)$ *and f be as in the Example 12. Then* $N\eta^\sharp$*-continuous but not* $N^\sharp\eta^\sharp$*-continuous, since* $f^{-1}(\{1,2\}) = \{1,2\}$ *is not* $N^\sharp\eta^\sharp$*-set.*

Example 15. *Let* $U, \tau U_R(X), L, \tau U'_R(Y)$ *and* f *as in the Example 11. Then* $N\eta^\sharp$*-sets are* $\emptyset, U, \{1\}, \{2\}, \{3\}, \{1,3\}$. *Then* $N^\sharp\eta^\sharp$*-continuous but not* $N\eta^\sharp$*-continuous, since* $f^{-1}(\{1,2\}) = \{1,2\}$ *is not* $N\eta^\sharp$*-set.*

Remark 4.3. *From the above discussions we obtain the following diagram where* $A \to B$ *represents A implies B, but not conversely.*



Theorem 4.1. *For a map f :* $(U, \tau U_R(X)) \to (L, \tau U'_R(Y))$*, the following are equivalent.*

(1) *f is Nano* α*-continuous.*
(2) *f is* $N^\sharp\eta^\sharp$*-continuous and* $N\ddot{g}_\alpha$*-continuous.*

Proof. According to Definition 4.1 (7), Definition 4.3, Remark 4.4 and Theorem 3.2, the proof is immediate. □

Theorem 4.2. *For a map f :* $(U, \tau U_R(X)) \to (L, \tau U'_R(Y))$*, the following are equivalent.*

(1) *f is N$\ddot{g}_\alpha$-continuous.*
(2) *f is N$\eta^{\sharp\sharp}$-continuous and N$\ddot{g}_p$-continuous.*

Proof. According to Theorem 3.3, the proof is immediate. □

5. CONCLUSION

General topology plays vital role in many fields of applied sciences as well as in all branches of mathematics. In reality it is used in data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, digital topology, information systems, particle physics and quantum physics etc. In this paper, we have defined and studied the notions of N$\eta^\sharp$-sets, N$\eta^{\sharp\sharp}$-sets, N$\eta^\sharp$-continuous and N$\eta^{\sharp\sharp}$-continuous map in nano topology and discussed their properties. Also we have discussed the relationships between the other existing continuities. Finally, we have found a decomposition of nano α-continuity and N$\ddot{g}_\alpha$-continuity. In future, we plan to extend this work in various nano topological fields.

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