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# MULTIOBJECTIVE STOCHASTIC INTERVAL TRANSPORTATION PROBLEM INVOLVING GENERAL FORM OF DISTRIBUTIONS

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ABSTRACT. This paper discusses a multiobjective stochastic transportation problem with interval cost coefficients, in which the supply and demand parameters are probabilistic in nature. These random parameters are assumed to follow any of two general classes of distributions. The multiobjective stochastic interval transportation problem (MOSITP) is converted into an equivalent multiobjective crisp problem and fuzzy programming technique has been used to obtain the pareto optimal solution of the transformed crisp problem. A numerical example is also presented to demonstrate the application of the proposed model.

## 1. INTRODUCTION

All the parameters involved in classical transportation problem are precisely known in advance but in many practical situations it is not always possible to know the precise value of these parameters due to uncertainty of real life. This introduces the some uncertainty in the transportation problem which can be dealt by any of three ways (i) fuzzy (ii) interval and (iii) stochastic. In literature several papers are available where demand and supply parameters follow a specific distribution see [1]- [5]. Quddoos et al. [6] proposed a multichoice stochastic transportation problem where they considered supply and demand

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parameters to follow general classes of distributions. All the work [1]- [5] can be deduced from [6]. Das et al. [7] developed the method to solve interval transportation problem where all the parameters are represented by interval numbers while Roy et al. [9] proposed a multiobjective transportation model where the cost coefficients are assumed to be in the form of interval numbers and demand and supply are assumed to follow log normal distribution.

In this paper, we have developed the frame work for MOSITP in which the supply and demand parameters are assumed to follow any of the two general classes of distributions [10]. The use of general form of distribution enables a decision maker to consider any specific distribution provided in Table 4. The MOSITP has been converted into an equivalent crisp model and solved using fuzzy programming technique [8]. For illustration purpose a numerical example is considered and solved using Lingo 13.0.

## 2. Mathematical Model

**2.1. Mathematical model of MOSITP.** The mathematical model of MOSITP with $k$ interval valued objective functions may be written as follows: Model-I

$$\text{Minimize} Z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}[c^k_{Lij}, c^k_{Rij}]x_{ij}, k = 1, 2, \ldots, K \tag{2.1}$$

Subject to;

$$Pr\Big[\sum_{j=1}^{n} x_{ij} \leq a_i\Big] \geq 1 - \alpha_i, i = 1, 2, \ldots, m \tag{2.2}$$

$$Pr\Big[\sum_{i=1}^{m} x_{ij} \geq b_j\Big] \geq 1 - \beta_j, j = 1, 2, \ldots, n \tag{2.3}$$

$$x_{ij} \geq 0 \ \forall \ i \text{ and } j \tag{2.4}$$

where, $0 < \alpha_i < 1$ and $0 < \beta_j < 1 \ \forall \ i$ and $j$ are aspiration level and $[c^k_{Lij}, c^k_{Rij}]$, is the interval cost of the $k^{th}$ objective functions and $a_i$ and $b_j$ follow any of two general classes of distributions.

**2.2. Construction of crisp constraints.** Let us consider the probabilistic constraints (2.2) and (2.3) of the model-I where $a_i$ and $b_j$ follow any of two general classes of distributions. The constraints (2.2) and (2.3) can be transformed into deterministic constraints (2.5) and (2.6) using Quddoos *et* al. [6].

$$\left\{\left[p_i h\Big(\sum_{j=1}^{n} x_{ij}\Big) + q_i\right]^{r_i}\right\} \text{ or } \left\{1 - e^{p_i h\left(\sum_{j=1}^{n} x_{ij}\right)}\right\} \geq 1 - \alpha_i \tag{2.5}$$

$$\left\{e^{-p'_j g\left(\sum_{i=1}^{m} x_{ij}\right)}\right\} \text{ or } \left\{1 - \left[p'_j g\Big(\sum_{i=1}^{m} x_{ij}\Big) + q'_j\right]^{r'_j}\right\} \geq 1 - \beta_j \tag{2.6}$$

2.3. **Construction of crisp objective function.** The $k^{th}$ interval objective function equation (2.1) of the model-I can be converted into two crisp objective functions using the method proposed by Das et al. [7] which is based on minimizing the expected value and right limit of the interval. The equivalent crisp objective function can be given as follows:

$$\text{Minimize } z_c = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{c_{ij}} x_{ij} \text{ and Minimize } z_R = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{R_{ij}} x_{ij} \tag{2.7}$$

where, $c_c = \left(\dfrac{c_R + c_L}{2}\right)$ and $c_R$ are the center and right-limit of the interval respectively.

2.4. **Equivalent crisp model.** Using equations (2.4), (2.5)-(2.6) and (2.7), the equivalent crisp problem of MOSITP can be written as follows: Model-II

$$\text{Minimize } z_c = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{c_{ij}} x_{ij}, \text{ and Minimize } z_R = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{R_{ij}} x_{ij}$$

Subject to;

$$\left\{\left[p_i h\Big(\sum_{j=1}^{n} x_{ij}\Big) + q_i\right]^{r_i}\right\} \text{ or } \left\{1 - e^{p_i h\left(\sum_{j=1}^{n} x_{ij}\right)}\right\} \geq 1 - \alpha_i,$$

$$\left\{e^{-p'_j g\left(\sum_{i=1}^{m} x_{ij}\right)}\right\} \text{ or } \left\{1 - \left[p'_j g\Big(\sum_{i=1}^{m} x_{ij}\Big) + q'_j\right]^{r'_j}\right\} \geq 1 - \beta_j,$$

$$x_{ij} \geq 0 \;\forall\; i \text{ and } j,\; i = 1, 2, \ldots, m,\; j = 1, 2, \ldots, n$$

## 3. Numerical illustration

Let us consider the transportation problem where the supply parameters $a_i$, $i = 1, 2, 3$ follow Burr-XII distribution and demand parameters $b_j, j = 1, 2, 3, 4$

follow extreme value distribution. The data are given in Table 1, Table 2 and Table 3.

| $[c^1_{Lij}, c^1_{Rij}]$ | | | | $[c^2_{Lij}, c^2_{Rij}]$ | | | |
|---|---|---|---|---|---|---|---|
| [10,12] | [15,16] | [21,24] | [21,25] | [9,11] | [16,17] | [21,24] | [16,18] |
| [15,25] | [10,20] | [9,11] | [18,19] | [9,13] | [14,19] | [16,18] | [19,20] |
| [20,26] | [10,17] | [20,25] | [15,20] | [9,17] | [20,26] | [25,27] | [28,29] |

TABLE 1. Unit interval transportation cost

| $a_i$ | $\alpha_i$ | $k_i$ | $\theta_i$ | $\delta_i$ |
|---|---|---|---|---|
| $a_1$ | 0.01 | 0.002 | 0.73 | 0.7757 |
| $a_2$ | 0.02 | 0.004 | 0.76 | 0.8013 |
| $a_3$ | 0.03 | 0.006 | 0.79 | 0.8267 |

TABLE 2. Specified probability levels and shape parameters of $a_i$

| $b_j$ | $\beta'_j$ | $\gamma'_j$ | $\delta'_j$ |
|---|---|---|---|
| $b_1$ | 0.04 | 600 | 5 |
| $b_2$ | 0.05 | 500 | 4 |
| $b_3$ | 0.06 | 400 | 3 |
| $b_4$ | 0.07 | 300 | 2 |

TABLE 3. Specified probability levels, location and scale parameters of $b_j$

$$\text{Minimize } Z^1 = \sum_{i=1}^{3}\sum_{j=1}^{4}[c^1_{Lij}, c^1_{Rij}]x_{ij},$$

$$\text{Minimize } Z^2 = \sum_{i=1}^{3}\sum_{j=1}^{4}[c^2_{Lij}, c^2_{Rij}]x_{ij}$$

Subject to;

$$\sum_{j=1}^{4} x_{1j} \leq 967.544404, \sum_{j=1}^{4} x_{2j} \leq 762.934875, \sum_{j=1}^{4} x_{3j} \leq 612.817850$$

$$\sum_{i=1}^{3} x_{i1} \geq 615.992671, \sum_{i=1}^{3} x_{i2} \geq 511.880781, \sum_{i=1}^{3} x_{i3} \geq 408.347897$$

$$\sum_{i=1}^{3} x_{i4} \geq 305.246388, x_{ij} \geq 0, i = 1,2,3, j = 1,2,3,4$$

$$\text{minimize } z_R^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{Rij}^1 x_{ij}, \text{minimize } z_c^1 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{cij}^1 x_{ij}$$

$$\text{minimize } z_R^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{Rij}^2 x_{ij}, \text{minimize } z_c^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} c_{cij}^2 x_{ij}$$

where,

$$c_R^1 = \begin{bmatrix} 12 & 16 & 24 & 25 \\ 25 & 20 & 11 & 19 \\ 26 & 17 & 25 & 20 \end{bmatrix}, \qquad c_c^1 = \begin{bmatrix} 11 & 15.5 & 22.5 & 23 \\ 20 & 15 & 10 & 18.5 \\ 23 & 13.5 & 22.5 & 17.5 \end{bmatrix}$$

$$c_R^2 = \begin{bmatrix} 11 & 17 & 24 & 18 \\ 13 & 19 & 18 & 20 \\ 17 & 26 & 27 & 29 \end{bmatrix}, \qquad c_c^2 = \begin{bmatrix} 10 & 16.5 & 22.5 & 17 \\ 11 & 16.5 & 17 & 19.5 \\ 13 & 23 & 26 & 28.5 \end{bmatrix}$$

Using fuzzy programming technique, the pareto optimal solution of the problem is obtained as: $x_{11} = 615.99$, $x_{12} = 241.40$, $x_{14} = 110.14$ , $x_{22} = 159.48$, $x_{23} = 408.34$, $x_{24} = 195.09$, $x_{32} = 110.98$, $Z^1 = [21975.32, 27265.83]$, $Z^2 = [25841.42, 30004.25]$

| $\mathbf{F(y) = 1 - [ph(y) + q]^r, y \in (\xi, \phi)}$ | | | | | |
|---|---|---|---|---|---|
| **Distribution** | $\mathbf{p}$ | $\mathbf{q}$ | $\mathbf{r}$ | $\mathbf{h(y)}$ | $\mathbf{F(y)}$ |
| Exponential | $1$ | $0$ | $\frac{\theta}{k}$ | $e^{-ky}$ | $1 - e^{-y\theta}$ |
| Weibull | $1$ | $0$ | $\frac{\delta-\gamma}{k}$ | $e^{-k(y)^\gamma}$ | $1 - [e^{-y^\gamma \delta^{-\gamma}}]$ |
| Cauchy | $-\frac{1}{\pi}$ | $\frac{1}{2}$ | $1$ | $\tan^{-1}(\frac{y-l}{s})$ | $\frac{1}{2} + \frac{1}{\pi}\tan^{-1}(\frac{y-l}{s})$ |
| Pareto | $d^{-k}$ | $0$ | $-\frac{\theta}{k}$ | $y^k$ | $1 - (d^{-k}y^k)^{\frac{-\theta}{k}}$ |
| Burr-XII | $\theta$ | $1$ | $-k$ | $y^\delta$ | $1 - (\theta y^\delta + 1)^{-k}$ |
| $\mathbf{F(y) = e^{-ph(y)} p \neq 0, y \in (\xi, \phi)}$ | | | | | |
| **Distribution** | $\mathbf{p}$ | – | – | $\mathbf{h(y)}$ | $\mathbf{F(y)}$ |
| Extreme Value | $1$ | – | – | $e^{-\frac{(y-\gamma)}{\delta}}$ | $e^{e^{\frac{-(y-\gamma)}{\delta}}}$ |
| Power Function | $-\theta$ | – | – | $l_n(\frac{y}{d})$ | $e^{\theta l_n(\frac{y}{d})}$ |

TABLE 4. Deduction Table

## 4. Comparison and Conclusion

The aim of this paper is to present a MOSITP with interval cost and random supply and demand parameters which are assumed to follow general classes of distributions. The advantage of using general classes of distributions is much higher than any specific distribution. The Model-II can generate different models for different distributions by setting values of parameters as provided in Table 4. Thus a single model (Model-II) can generate an equivalent crisp model corresponding to every distribution like Exponential, Weibull, Cauchy, Pareto, Burr-XII, Extreme Value, Power Function.

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