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VAGUE ANTI HOMOMORPHISM OF A Γ-SEMIRINGS

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ABSTRACT. In this paper, we introduce and study the concept of vague anti homomorphism of a Γ-semiring and we study the properties of anti homomorphic image and pre-image of a anti vague ideal of a Γ-semiring. Further we establish that the inverse image of an anti right vague ideal of a Γ-semiring is an anti left vague ideal of a Γ-semiring and the anti homomorphic image of an anti left vague ideal of a Γ-semiring is a anti right vague ideal of a Γ-semiring.

1. INTRODUCTION

Semiring is an important algebraic tool in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics, functional analysis and graph theory. M.K.Rao [11] introduced the concept of Γ-semiring as a generalization of semiring as well as Γ-ring. The properties of an ideal in semirings and Γ-semirings were somewhat different from the properties of the usual ring ideals. Moreover the concept of Γ-semiring not only generalizes the concept of semiring and Γ-ring but also the concept of ternary semiring. Zadeh, L.A. [12] introduced the study of fuzzy sets in 1965. Mathematically a fuzzy set on a set X is a mapping μ into [0,1] of real numbers; for x in X, $\mu(x)$ is called the membership of x belonging to X. The membership function gives only an

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approximation for belonging but it does not give any information of not belonging. To counter this problem, Gau, W.L. and Buehrer, D.J. [10] introduced the concept of vague sets. A vague set A of a set X is a pair of functions (t_A, f_A), where t_A and f_A are fuzzy sets on X satisfying $t_A(x) + f_A(x) \geq 1,\ \forall x \in X$. A fuzzy set t_A of X can be identified with the pair $(t_A, 1 - t_A)$. Thus the theory of vague sets is a generalization of fuzzy sets. Later, Bhargavi, Y. and Eswarlal, T. [1]- [9] developed vague sets on Γ-semirings. In this paper, the concept of vague anti homomorphism of Γ-semirings has been introduced and we study the properties of homomorphic, anti homomorphic image and pre-image of vague ideal and anti vague ideal of a Γ-semiring.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. *[11] Let E and Γ be two additive commutative semigroups. Then R is called Γ-semiring if there exists a mapping $R \times \Gamma \times R \to R$ image to be denoted by $a\alpha b$ if it satisfies the following conditions: For all $a, b, c \in R; \alpha, \beta \in \Gamma$.*

(Γ*SR1)* $a\alpha(b + c) = a\alpha b + a\alpha c$;
(Γ*SR2)* $(a + b)\alpha c = a\alpha c + b\alpha c$;
(Γ*SR3)* $a(\alpha + \beta)b = a\alpha b + a\beta b$;
(Γ*SR4)* $a\alpha(b\beta c) = (a\alpha b)\beta c$.

Definition 2.2. *[10] A vague set A in the universe of discourse X is a pair (t_A, f_A), where $t_A : X \to [0, 1]$, $f_A : X \to [0, 1]$ are mappings such that $t_A(x) + f_A(x) \leq 1$, for all $x \in X$. The functions t_A and f_A are called true membership function and false membership function respectively.*

Definition 2.3. *[10] The interval $[t_A(x), 1 - f_A(x)]$ is called the vague value of x in A and it is denoted by $V_A(x)$ i.e., $V_A(x) = [t_A(x), 1 - f_A(x)]$.*

Definition 2.4. *Let f be a mapping from a set X into a set Y. Let A be a vague set in X with vague value V_A. Then the anti image $f(A)$ of A is the vague set in Y defined by*

$$V_{f(A)}(y) = \begin{cases} \inf\limits_{z \in f^{-1}(y)} V_A(z) & \text{if } f^{-1}(y) \neq \phi \\ [0, 0] & otherwise \end{cases}$$

for all $y \in Y$, where $f^{-1}(y) = \{x/f(x) = y\}$.

Definition 2.5. *Let B be a vague set in Y. Then the inverse image of B, $f^{-1}(B)$ is the vague set in X by $V_{f^{-1}(B)}(x) = V_B(f(x))$, for all $x \in X$.*

Definition 2.6. *A vague set A of a Γ-semiring R is said to have the Sup. property if for any subset S of R, there exists $y \in S$ such that $V_A(y) = \sup_{x \in S} V_A(x)$.*

Definition 2.7. *Let R and S be two Γ-semirings. Then $f : R \to S$ is called a homomorphism if $f(x+y) = f(x) + f(y)$ and $f(x\gamma y) = f(x)\gamma f(y)$, $\forall x, y \in R$; $\gamma \in \Gamma$.*

Definition 2.8. *[2] A vague set $A = (t_A, f_A)$ of a Γ-semiring R is said to be left (resp. right) vague ideal of R if it satisfies the following conditions: For all $x, y \in R; \gamma \in \Gamma$,*

(VI1) $V_A(x+y) \geq \min\{V_A(x), V_A(y)\}$;

(VI2) $V_A(x\gamma y) \geq V_A(y)$ (resp. $V_A(x\gamma y) \geq V_\psi(x)$).

If A is both left and right vague ideals of R, then A is called vague ideal of R.

Definition 2.9. *[9] A vague set $A = (t_A, f_A)$ of a Γ-semiring R is said to be anti left (resp. right) vague ideal of R if it satisfies the following conditions: For all $x, y \in R; \gamma \in \Gamma$,*

(VI1) $V_A(x+y) \leq \max\{V_A(x), V_A(y)\}$;

(VI2) $V_A(x\gamma y) \leq V_A(y)$ (resp. $V_A(x\gamma y) \leq V_\psi(x)$).

If A is both anti left and anti right vague ideals of R, then A is called vague ideal of R.

3. Vague Anti Homomorphism of a Γ-Semiring

In this section, we study the concept of anti homomorphism of a Γ-semiring and we prove that the anti homomorphic image and anti pre-image of an anti vague ideal of a Γ-semiring is also an anti vagae ideal of a Γ-semiring.

Theorem 3.1. *Let R and S be Γ-semirings and $f : R \to S$ be an onto homomorphism. If $A = (t_A, f_A)$ is a f-invariant anti left(resp. right) vague ideal of R, then the anti homomorphic image $f(A)$ of A is a anti left(resp. right) vague ideal of S.*

Proof. Suppose A is a anti left vague ideal of R. Let $x, y \in S; \gamma \in \Gamma$. This implies there exists $a, b \in R$ such that $x = f(a)$ and $y = f(b)$. Now, $x = f(a)$ that implies $f^{-1}(x) = a$. Let $t \in f^{-1}(x)$. Then $f(t) = x = f(a)$. Since A is f-invariant, we have $V_A(t) = V_A(a)$. Now, $V_{f(A)}(x) = \inf_{t \in f^{-1}(x)} V_A(t) = V_A(a)$. Therefore $V_{f(A)}(x) = V_A(a)$.

Similarly $V_{f(A)}(y) = V_A(b)$. We have, $x + y = f(a+b)$. Now, $V_{f(A)}(x+y) = V_A(a+b) \leq \max\{V_A(a), V_A(b)\} = \max\{V_{f(A)}(x), V_{f(A)}(y)\}$. Also, $V_{f(A)}(x\gamma y) = V_A(a\gamma b) \leq V_A(b) = V_{f(A)}(y)$. Thus $f(A)$ is a anti left vague ideal of S. Similarly we can prove for right vague ideals also. □

Theorem 3.2. *Let R and S be Γ-semirings and $f : R \to S$ be an onto homomorphism. If B is a anti left(resp. right) vague ideal of S, then the anti pre-image $f^{-1}(B)$ of B is a anti left(resp. right) vague ideal of R.*

Proof. Suppose B is a anti left vague ideal of S. Let $x, y \in R; \gamma \in \Gamma$. Now, $V_{f^{-1}(B)}(x+y) = V_B(f(x+y)) = V_B(f(x)+f(y)) \leq \max\{V_B(f(x)), V_B(f(y))\} = \max\{V_{f^{-1}(B)}(x), V_{f^{-1}(B)}(y)\}$. Also, $V_{f^{-1}(B)}(x\gamma y) = V_B(f(x\gamma y)) = V_B(f(x)\gamma f(y)) \leq V_B(f(y)) = V_{f^{-1}(B)}(y)$. Hence $f^{-1}(B)$ is a anti left vague ideal of R. Similarly we can prove for right vague ideals. □

Definition 3.1. *Let R and S be two Γ-semirings. Then $f : R \to S$ is called an anti homomorpiosm if $f(x+y) = f(x) + f(y)$ and $f(x\gamma y) = f(y)\gamma f(x)$, $\forall x, y \in R$; $\gamma \in \Gamma$.*

Theorem 3.3. *Let R and S be Γ-semirings and $f : R \to S$ be an onto anti homomorphism. If A is a left vague ideal of R with Sup. property, then the homomorphic image $f(A)$ of A is a right vague ideal of S.*

Proof. Suppose A is a left vague ideal of R. Let $x, y \in S; \gamma \in \Gamma$. If either $f^{-1}(x)$ or $f^{-1}(y)$ is empty, then the result is trivially satisfied. Suppose neither $f^{-1}(x)$ nor $f^{-1}(y)$ is non-empty. Let $p \in f^{-1}(x)$ and $q \in f^{-1}(y)$ be such that $V_A(p) = supV_A(a)$ where $a \in f^{-1}(x)$ and $V_A(q) = supV_A(b)$ where $b \in f^{-1}(y)$.

Now:

1. $V_{f(A)}(x+y) = \sup_{z \in f^{-1}(x+y)} V_A(z) \geq V_A(z)$, $z \in f^{-1}(x+y) = V_A(p+q) \geq \min\{V_A(p), V_A(q)\} = \min\{V_{f(A)}(x), V_{f(A)}(y)\}$.
2. $V_{f(A)}(x\gamma y) = \sup_{z \in f^{-1}(x\gamma y)} V_A(z) \geq V_A(z), z \in f^{-1}(x\gamma y) = V_A(q\gamma p), \gamma \in \Gamma \geq V_A(p) = V_{f(A)}(x)$.

Thus $f(A)$ is a anti right vague ideal of S. □

Theorem 3.4. *Let R and S be Γ-semirings and $f : R \to S$ be an anti homomorphism. If A is a right vague ideal of R, then the homomorphic image $f(A)$ of A is a left vague ideal of R.*

Proof. Proof is similar to the above theorem. □

Theorem 3.5. *Let R and S be Γ-semirings and $f : R \to S$ be an onto anti homomorphism. If B is a left vague ideal of S, then the pre-image $f^{-1}(B)$ of B is a right vague ideal of R.*

Proof. Suppose B is a left vague ideal of S. Let $x, y \in R; \gamma \in \Gamma$. Now, $V_{f-1(B)}(x+y) = V_B(f(x+y)) = V_B(f(x) + f(y)) \geq \min\{V_B(f(x)), V_f(B(y))\} = \min\{V_{f-1(B)}(x), V_{f-1(B)}(y)\}$. Also, $V_{f-1(B)}(x\gamma y) = V_B(f(x\gamma y)) = V_B(f(y)\gamma f(x))$ $\geq V_B(f(y)) = V_{f-1(B)}(y)$. Hence $f^{-1}(B)$ is a right vague ideal of R. □

Theorem 3.6. *Let R and S be Γ-semirings and $f : R \to S$ be an onto anti homomorphism. If B is a right vague ideal of S, then the pre-image $f^{-1}(B)$ of B is a left vague ideal of R.*

Proof. Proof is similar to the above theorem. □

Theorem 3.7. *Let R and S be Γ-semirings and $f : R \to S$ be an onto anti homomorphism. If A is a f-infariant anti left vague ideal of R, then the anti homomorphic image $f(A)$ of A is an anti right vague ideal of S.*

Proof. Suopose A is a left vague ideal of R. Let $x, y \in S; \gamma \in \Gamma$. That implies there exists $a, b \in R$ such that $x = f(a)$ and $t = f(b)$. Now, $x = f(a)$ implies $f^{-1}(x) = a$. Let $t \in f^{-1}(x)$. Then $f(t) = x = f(a)$. Since A is f-invariant, we have $V_A(t) = V_A(a)$. Now, $V_{f(A)}(x) = \inf_{t\in f^{-1}(x)} V_A(t) = V_A(a)$. Therefore $V_{f(A)}(A) = V_A(a)$.

Similarly $V_{f(A)}(y) = V_A(b)$. We have, $x + y = f(a + b)$. Now, $V_{f(A)}(x + y) = V_A(a + b) \leq \max\{V_A(a), V_A(b)\} = \max\{V_{f(A)}(x), V_{f(A)}(y)\}$. Also, $V_{V(A)}(x\gamma y) = f_A(b\gamma a) \leq V_A(a) = V_{f(A)}(x)$. Thus $f(A)$ is a anti right vague ideal of S. □

Theorem 3.8. *Let R and S be Γ-semirings and $f : R \to S$ be an onto anti homomorphism. If B is a left vague ideal of S, then the pre-image $f^{-1}(B)$ of B is a right vague ideal of R.*

Proof. Suppose B is left vague ideal of S. Let $x, y \in R; \gamma \in \Gamma$. Now, $V_{f-1(B)}(x + y) = V_B(f(x+y)) = V_B(f(x)+f(y)) \geq \min\{V_B(f(x)), V_B(f(y))\} = \min\{V_{f-1(B)}(x), V_{f-1(B)}(y)\}$. Also, $V_{f-1(B)}(x\gamma y) = V_B(f(x\gamma y)) = V_B(f(y)\gamma f(x)) \geq V_B(f(y)) = V_{f-1(B)}(y)$. Hecne $f^{-1}(B)$ is a right vague ideal of R. □

Definition 3.2. *Let $A = (t_A, f_A)$ and $B = (t_B, f_B)$ be anti vague ideals of R. If there exists $\phi \in Aut(R)$ such that $V_A(x) = V_B(\phi(x))$, $\forall x \in R$. i.e., $t_A(x) = t_B(\phi(x))$ and $f_A(x) = f_B(\phi(x))$, then we say that A and B are homologous anti vague ideals of R. If A and B are homologous, then B amd A are also homologous.*

Theorem 3.9. *Let $B = (t_B, f_B)$ be anti vague ideal of$R$ and $\phi \in Aut(R)$. If $A = (t_A, f_A)$ is a vague set of R such that $V_A(x) = V_B(\phi(x))$, $\forall x \in R$, then A and B are homologous anti vague ideals of R.*

Proof. To prove A and B are homologous, it is enough to prove that A is an anti vague ideal of R. Let $x, y \in R; \gamma \in \Gamma$.
1. $V_A(x+y) = V_B(\phi(x+y)) = V_B(\phi(x)+\phi(y)) \leq \max\{V_B(\phi(x)), V_B(\phi(y))\} = \max\{V_A(x), V_A(y)\}$.
2. $V_A(x\gamma y) = V_B(\phi(x\gamma y)) = V_B(\phi(x)\gamma\phi(y)) \leq V_B(\phi(x)) = V_A(x)$.

Therefore A is an anti vague ideal of R. Hence A and B are homologous anti vague ideals of R. □

Theorem 3.10. *Let $A = (t_A, f_A)$ be an anti vague ideal of R and let $f : R \to R$ be an onto anti homomorphism. Then the vague set $A^f = (t_{A^f}, f_{A^f})$ defined by $V_{A^f}(x) = V_A(f(x))$, $\forall x \in R$ is an anti vague ideal of R.*

Proof. Let $x, y \in R; \gamma \in \Gamma$.
1. $V_{A^f}(x+y) = V_A(f(x+y)) = V_A(f(x)+f(y)) \leq \max\{V_A(f(x)), V_A(f(y))\} = \max\{V_{A^f}(x), V_{A^f}(y)\}$.
2. $V_{A^f}(x\gamma y) = V_A(f(x\gamma y)) = V_A(f(y)\gamma f(x)) \leq V_A(f(x)) = V_{A^f}(x)$.

Hence A^f is an anti vague ideal of R. □

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