Advances in Mathematics: Scientific Journal **9** (2020), no.6, 4203–4212
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.6.102 Spec. Issue on ICIGA-2020

AT MOST TWIN EXTENDABLE SEPARATED DOMINATION NUMBER OF A GRAPH

G. MAHADEVAN[1] AND S. ANUTHIYA

ABSTRACT. In this paper, we introduce the concept of At most twin extendable separated domination number of a graph. A set S $\subseteq V$ is said to be At most twin extendable separated dominating set, if for every vertex v $\in V-S$, $1 \leq |N(V) \cap S| \leq 2$ and $< V - S >$ is a perfect matching. The minimum cardinality taken over all At most twin extendable separated dominating sets is called At most twin extendable separated domination number of a graph and it is denoted by ATES(G). In this paper, we initiate this parameter and investigate this number for many graphs.

1. INTRODUCTION

The concept of complementary perfect domination number was introduced by Paulraj Joseph et.al., [1]. A set $S \subseteq V$ is called Complementary perfect dominating set, if S is a dominating set of G and the induced sub graph $< V - S >$ has a perfect matching. The minimum cardinality taken over all complementary perfect dominating sets is called the complementary perfect domination number and is denoted by $\gamma_{cp}(G)$. In [2], by Xiaojing Yang and Baoyindureng Wu, the study of this parameter was extended. A vertex set S in graph G is a [1, 2]-domination set, if $1 \leq |N(V) \cap S| \leq 2$ for every vertex $v \in V - S$, that is, every vertex $v \in V - S$, is adjacent to either one or two vertices in S. The

[1]*corresponding author*

2010 *Mathematics Subject Classification.* 05C69.
Key words and phrases. Perfect matching, Complementary perfect matching, At most twin outer perfect domination number.

minimum cardinality of a [1, 2]-set of G is denoted by $\gamma_{[1,2]}(G)$ and is called [1, 2]-domination number of G. A matching is a subset of edges, in which no two edges are adjacent. A matching that contains all the vertices is called perfect matching. In all the above, discussion of the concept of complementary perfect set is missing. We impose the condition of complementary perfect step in the sub graph induced by $< V - S >$ is maintaining [1, 2]- dominating set. Motivated by the above, in this paper we introduce the concept of at most twin extendable separated domination number of a graph and this number for some standard classes of graph and special types of graphs.

For $n \geq 3$, wheel $W_{1,n}$ is defined to be the graph $K_1 + C_n$. For $n \geq 3$, helm graph H_n is obtained from $W_{1,n}$ by attaching a pendent edge at each vertex of the n-cycle. $F_{m,n}$ is a fire cracker graph obtained by attaching any one of pendent vertex of m copy of the star $K_{1,n-1}$. Web graph is obtain from prism by attaching pendent vertex to each vertex of the outer cycle and is denoted by W_n. Barbell graph is connecting two copies of complete graph K_n by an edge and it is denoted by $K_n \cup K_n + e$. Flower graph Fl_n is the graph obtained from the helm H_n by joining each pendant vertex to the apex of the helm. Windmill graph $Wd(k, j)$ is an undirected graph constructed for $j, k \geq 2$ by joining j copies of the complete graph K_k at a shared vertex. The Corona product $G_1 \odot G_2$ is defined as the graph G obtained by taking one copy of G_1 of order n and n copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The Cartesian product of two graphs G and H with disjoint set of vertices V_m and V_n is the graph with vertex set $V(G \times H)$ and edge set $E(G \times H)$ such that any two vertices (u_1, u_2) and (v_1, v_2) are adjacent in $G \times H$ if and only if either (i) $u_1 = v_1$ and u_2 is adjacent to v_2 in H. (ii) $u_2 = v_2$ and u_1 is adjacent to v_1 in G, where $G \times H$ denotes the Cartesian product of graphs G and H.

Notation 1. *ATES(G) - At most twin extendable separated domination number of a graph.*

Definition 1.1. *A set $S \subseteq V$ is said to be At most twin extendable separated dominating set, if for every vertex v $\in V - S$, $1 \leq |N(V) \cap S| \leq 2$ and $< V - S >$ is a perfect matching. The minimum cardinality taken over all At most twin extendable separated dominating sets is called At most twin extendable separated domination number of a graph and it is denoted by ATES(G).*

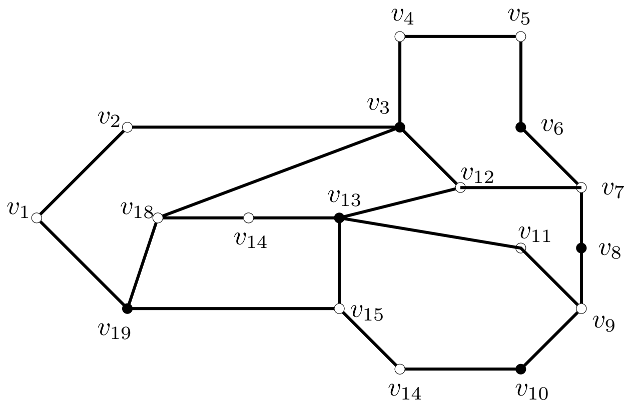

Figure 1.1

Example 1. *In figure 1.1.* S=$\{v_3, v_6, v_8, v_{10}, v_{13}, v_{18}\}$ *At most twin extendable separeded dominating set. Hence ATES(G)=6.*

Observation 1. *For any connected graph G,* $\gamma(G) \leq \gamma_{cp}(G) \leq \gamma_{atop}(G) \leq ATES(G)$.

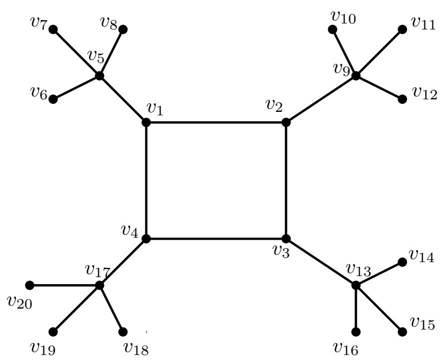

Figure 1.2

Example 2. *In figure 2.2,* S_1= $\{v_5, v_9, v_{13}, v_{17}\}$ *is a domination set so that* $\gamma(G) = 4$*,* S_2=$\{v_6, v_7, v_8, v_{10}, v_{11}, v_{12}, v_{14}, v_{15}, v_{16}, v_{18}, v_{19}, v_{20}\}$ *is a complementary perfect domination set so that* $\gamma_{cp}(G) = 12$*,*
S_3=$\{v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}, v_{19}, v_{20}\}$ *is an at most twin outer perfect domination set so that* $\gamma_{atop}(G) = 16$*,*
S_4=$\{v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}, v_{19}, v_{20}\}$ *is an at most twin outer perfect domination set so that ATES(G)=18.*

Observation 2. *For any connected graph G,* $\gamma(G) = \gamma_{cp}(G) = \gamma_{atop}(G) = ATES(G)$.

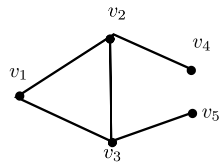

Figure1.3

Example 3. *In figure1.3, S=*$\{v_1, v_4, v_5, \}$ *is a dominating set, complementary perfect dominating set, at most twin outer perfect dominating set and ATES-set. Hence* $\gamma(G) = \gamma_{cp}(G) = \gamma_{atop}(G) = ATES(G)$.

Observation 3. *The complementary of the ATES-set need not be a ATES-set.*

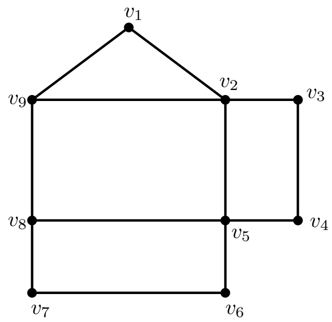

Figure 1.4

Example 4. *In figure 1.4., S=*$\{v_2, v_5, v_8, \}$ *is an ATES-set and <v-s> is not an ATES-set.*

Observation 4. *For any graph G,* $\gamma(G) \leq ATES(G)$.

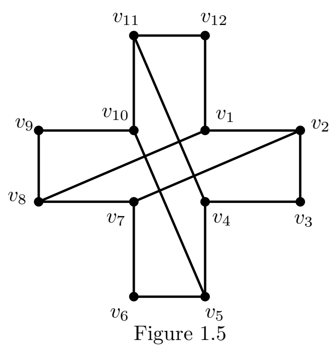

Figure 1.5

Example 5. *In figure 1.5., S=* $\{v_1, v_4, v_7, v_{10}\}$ *is a dominating set, at most twin extendable separated dominating set.*

Observation 5. *ATES Dominating set does not exists for the following graphs: Complete graph, Bipartite graph, star graph, Helm graph, Web graph, $P_n \times P_m$,(for $n \geq 3$), $C_n \times C_m$(for $n \geq 3$),Windmill graph, Barbell graph, $P_m \odot K_n^c$, Triangular snake graph. Since the above graphs doesn't satisfy the condition "for every vertex $v \in V - S, 1 \leq |N(V) \cap S| \leq 2$" it is not a ATES Dominating set.*

2. Extact values of At most Twin Exrendable Separated Domination number for a some graphs:

The at most twin extendable separated domination number for some graphs are given below:

(1) $ATES(P_2 \times P_m) = \begin{cases} n & if\ n\ is\ even \\ n-1 & if\ n\ is\ odd \end{cases}$
(2) $ATES(P_2 \odot P_n) = n.$
(3) $ATES(Q_n) = n.$
(4) $ATES(F_n) = n.$
(5) $ATES(D(Q_n)) = n.$
(6) $ATES(A(DQ_n)) = n.$

3. At most twin extendable separated domination number for some standard graphs:

Theorem 3.1. *If $G = P_n$, for $n \geq 4$ then,*

$$ATES(G) = \begin{cases} \lceil \frac{n}{3} \rceil + 2 & if\ n \equiv 0(mod 3) \\ \lceil \frac{n+1}{3} \rceil & if\ n \equiv 1(mod 3) \\ \lceil \frac{n}{3} \rceil + 1 & if\ n \equiv 2(mod 3) \end{cases}.$$

Proof. Let $\{v_1, v_2, v_3, \dots, v_n\}$ be the set of all vertices of the path P_n. Let $S = \{v_i; i \equiv 1(mod 3)\}$. If $n \equiv 0(mod 3)$, then $\{S \cup \{v_{n-1}, v_n\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(P_n) = \lceil \frac{n}{3} \rceil + 2$.If $n \equiv 1(mod 3)$, then $\{S\}$ is the At most twin extendable separated domination set implies $|S| = ATES(P_n) = \lceil \frac{n+1}{3} \rceil$. If $n \equiv 2(mod 3)$, then $\{S \cup \{v_n\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(P_n) = \lceil \frac{n}{3} \rceil + 1$. In all the above cases $\gamma \leq |S|$, we have to prove that $\gamma \geq |S|$. Suppose that we take $T \subseteq S$ such that T is an At most twin extendable separated dominating set contradicting the definition. Hence $\gamma \geq |S|$ which means $ATES(G) = |S|$. □

Example 6.

Figure 3.1

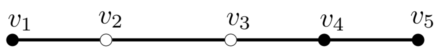

Figure 3.2

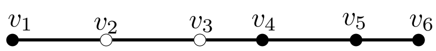

Figure 3.3

Illustration Here, darkened vertices are ATES-dominatinf set, denoted by S
In figure 3.1, $|S| = 2$
As $n = 4, ATES(P_n) = \lceil \frac{n+1}{3} \rceil$ implies $ATES(P_4) = \lceil \frac{4+1}{3} \rceil = 2$.
In figure 3.2,$|S| = 3$
As $n = 5, ATES(P_n) = \lceil \frac{n}{3} \rceil + 1$ implies $ATES(P_5) = \lceil \frac{5}{3} \rceil = 3$.
In figure 3.3,$|S| = 4$
As $n = 6, ATES(P_n) = \lceil \frac{n}{3} \rceil + 2$ implies $ATES(P_6) = \lceil \frac{6}{3} \rceil + 2 = 4$.

Theorem 3.2. *If $G = C_n$, for $n \geq 3$ then,*

$$ATES(G) = \begin{cases} \lceil \frac{n}{3} \rceil & if\ n \equiv 0, 1 \ \ (mod 3) \\ \lceil \frac{n}{3} \rceil + 1 & if\ n \equiv 2 \ \ (mod 3) \end{cases} .$$

Proof. Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be the set of all vertices of the cycle C_n. Let $S = \{v_i; i \equiv 1(mod 3)\}$. If $n \equiv 0(mod 3)$, then $\{S\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(C_n) = \lceil \frac{n}{3} \rceil$. If $n \equiv 1(mod 3)$, then $\{S \cup \{v_n\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(C_n) = \lceil \frac{n}{3} \rceil$. If $n \equiv 2(mod 3)$, then $\{S \cup \{v_{n-1}, v_n\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(C_n) = \lceil \frac{n}{3} \rceil + 1$. In all the above cases $\gamma \leq |S|$, we have to prove that $\gamma \geq |S|$. Suppose that we take $T \subseteq S$ such that T is an At most twin extendable separated dominating set contradicting the definition. Hence $\gamma \geq |S|$ which means $ATES(G) = |S|$. □

Example 7.

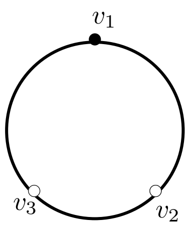

Figure 3.4

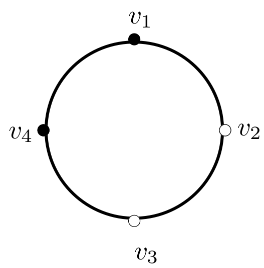

Figure 3.5

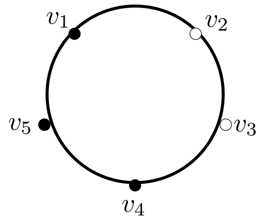

Figure 3.6

Illustration Here, darkened vertices are ATES-dominatinf set, denoted by S
In figure 3.4, $|S| = 1$
As $n = 3, ATES(C_n) = \lceil \frac{n}{3} \rceil$ implies $ATES(C_3) = \lceil \frac{3}{3} \rceil = 1$.
In figure 3.5,$|S| = 2$
As $n = 4, ATES(C_n) = \lceil \frac{n}{3} \rceil$ implies $ATES(C_4) = \lceil \frac{4}{3} \rceil = 2$.
In figure 3.6,$|S| = 3$
As $n = 5, ATES(C_n) = \lceil \frac{n}{3} \rceil + 1$ implies $ATES(C_5) = \lceil \frac{5}{3} \rceil + 1 = 3$.

Theorem 3.3. *If $G = W_{1,n}$, for $n \geq 3$ then,*

$$ATES(G) = \begin{cases} \lceil \frac{n}{3} \rceil + 1 & if\ n \equiv 0, 1\ \ (mod 3) \\ \lceil \frac{n}{3} \rceil + 2 & if\ n \equiv 2\ \ (mod 3) \end{cases}.$$

Proof. Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be the set of all vertices of the Wheel $W_{1,n}$. Let $S = \{v_i; i \equiv 1(mod 3)\} \cup \{v_0\}$. If $n \equiv 0(mod 3)$, then $\{S\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(W_{1,n}) = \lceil \frac{n}{3} \rceil + 1$. If $n \equiv 1(mod 3)$, then $\{S \cup \{v_n\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(W_{1,n}) = \lceil \frac{n}{3} \rceil + 1$. If $n \equiv 2(mod 3)$, then $\{S \cup \{v_{n-1}, v_n\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(W_{1,n}) = \lceil \frac{n}{3} \rceil + 2$. In all the above cases $\gamma \leq |S|$, we have to prove that $\gamma \geq |S|$. Suppose that we take $T \subseteq S$ such that T is an At most twin extendable separated dominating set contradicting the definition. Hence $\gamma \geq |S|$ which means $ATES(G) = |S|$. □

Example 8.

Illustration Here, darkened vertices are ATES-dominatinf set, denoted by S
In figure 3.7, $|S| = 2$

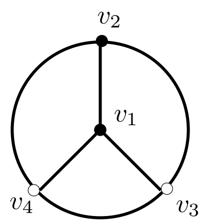

Figure 3.7

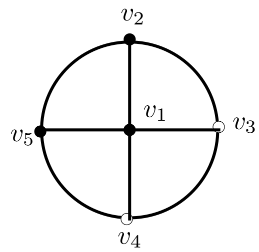

Figure 3.8

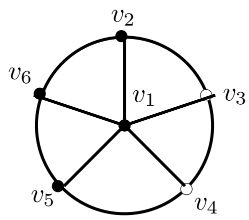

Figure 3.9

As $n = 3, ATES(W_{1,n}) = \lceil \frac{n}{3} \rceil + 1$ implies $ATES(W_{1,3}) = [\frac{3}{3}] = 2$.
In figure 3.8,$|S| = 3$
As $n = 4, ATES(W_{1,n}) = \lceil \frac{n}{3} \rceil + 1$ implies $ATES(W_{1,4}) = \lceil \frac{4}{3} \rceil + 1 = 3$.
In figure 3.9,$|S| = 4$
As $n = 5, ATES(W_{1,n}) = \lceil \frac{n}{3} \rceil + 2$ implies $ATES(W_{1,5}) = \lceil \frac{5}{3} \rceil + 2 = 4$.

Theorem 3.4. *If $G = S_n$, for $n \geq 3$ then,*

$$ATES(G) = \begin{cases} \lceil \frac{n}{3} \rceil + n & if\ n \equiv 0, 1 \ \ (mod 3) \\ \lceil \frac{n}{3} \rceil + n + 1 & if\ n \equiv 2 \ \ (mod 3) \end{cases}.$$

Proof. consider the sunlet graph S_n with en vertices. Let S_n be a graph obtained by attaching the pendent vertices to all the vertices of the cycle C_n. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices of the cycle C_n and the pendent vertices are $u_1, u_2, v_3, \ldots, u_n$. Now ataching the pendent vertices of $v_1 to u_1, v_2 to u_2, \ldots, v_n to u_n$ respectively. Let $S = \{v_i; i \equiv 1(mod 3)\} \cup \{u_i; i = 1 to n\}$. If $n \equiv 0(mod 3)$, then $\{S\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(S_n) = \lceil \frac{n}{3} \rceil + n$. If $n \equiv 1(mod 3)$, then $\{S \cup \{v_n\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(S_n) = \lceil \frac{n}{3} \rceil + n$. If $n \equiv 2(mod 3)$, then $\{S \cup \{v_{n-1}, v_n\}\}$ is the At most twin extendable separated domination set implies $|S| = ATES(S_n) = \lceil \frac{n}{3} \rceil + n + 1$. In all the above cases $\gamma \leq |S|$, we have to prove that $\gamma \geq |S|$. Suppose that we take $T \subseteq S$ such that T is an At most twin extendable separated dominating set contradicting the definition. Hence $\gamma \geq |S|$ which means $ATES(G) = |S|$. □

Example 9.

Illustration Here, darkened vertices are ATES-dominatinf set, denoted by S
In figure 3.10, $|S| = 4$
As $n = 3, ATES(S_n) = \lceil \frac{n}{3} \rceil + n$ implies $ATES(S_3) = \lceil \frac{3}{3} \rceil + 3 = 4$.

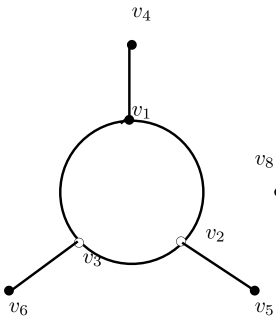

Figure 3.10

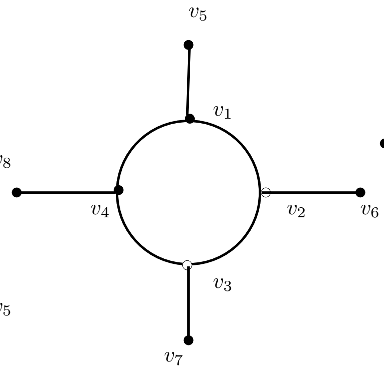

Figure 3.11

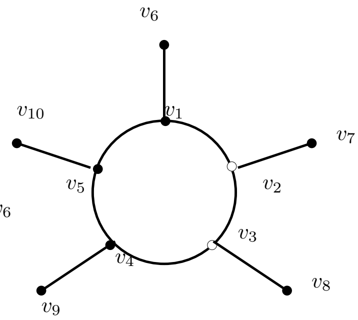

Figure 3.12

In figure 3.11,$|S| = 6$
As $n = 4, ATES(S_n) = \lceil \frac{n}{3} \rceil + n$ implies $ATES(S_n) = \lceil \frac{4}{3} \rceil + 4 = 6.$
In figure 3.12,$|S| = 8$
As $n = 5, ATES(S_n) = \lceil \frac{n}{3} \rceil + n + 1$ implies $ATES(S_n) = \lceil \frac{5}{3} \rceil + 5 + 1 = 8.$

4. CONCLUSION

In this paper we introduced the new parameter called At most twin extendable separated domination number of a graph. we investigate this parameter for standard graphs and its various derived graph. The authors investigate this number for many product related graph like Cartesian, Strong, Tensor, Lexico graph, Semi product, corona product and some special types like prism, web, tadpole, lollipop, step ladder, umbrella, silicon net work, which will be reported in the sub sequent papers.

ACKNOWLEDGMENT

The research work was supported by DSA (Departmental special assistance), Department of Mathematics, Gandhigram Rural Institute- Deemed to be University, Gandhigram, under University Grants Commission- New Delhi .

REFERENCES

[1] J. PAULRAJ JOSEPH, G. MAHADEVAN: *On Complementary perfect domination number of a graph,* Acta Cienia Indica, An International Journal of Physical Sciences, **31**(2) (2006), 847–853.

[2] X. YANG, B. WU: *Domination in graphs,* Discrete Applied Mathematics, **175** (2014), 79–86.

DEPARTMENT OF MATHEMATICS
THE GANDHIGRAM RURAL INSTITUTE - DEEMED TO BE UNIVERSITY
Email address: drgmaha2014@gmail.com

DEPARTMENT OF MATHEMATICS
THE GANDHIGRAM RURAL INSTITUTE - DEEMED TO BE UNIVERSITY
Email address: anuthiya96@gmail.com