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# DISCOVERING COMPLEMENTARY INDEPENDENT TWIN PAIRED DOMINATION NUMBER FOR SOME PRODUCT RELATED GRAPHS

#### G. MAHADEVAN<sup>1</sup> AND M. VIMALA SUGANTHI

ABSTRACT. Recently G. Mahadevan et.al., introduced the concept of "Complementary independent twin paired domination number" of a graph. The set  $S\subseteq V$  is said to be Complementary independent twin paired dominating set, if S is a paired dominating set and < V - S > is a set of independent edges. The minimum cardinality taken over all the Complementary independent twin paired dominating set is called as Complementary independent twin paired domination number and it is denoted by CITD(G). In this paper, we investigate this parameter for some product related graphs

#### 1. Introduction and Preliminaries

The initiation of paired domination was done by T.W. Haynes, et.al., [2,5] in 1998. Which defines as "A paired-dominating set is a set  $S \subseteq V$  such that every vertex is adjacent to some vertex in S and the subgraph < S > induced by S contains a perfect matching. The minimum cardinality taken over all the paired-dominating set is called as paired-domination number and is denoted by  $\gamma_p(G)$ ". The concept of complementary perfect domination number was introduced by Paulraj Joseph [1–4]."A set  $S \subseteq V$  is called a Complementary perfect dominating set, if S is a dominating set of G and the induced subgraph < V - S > has a perfect matching. The minimum cardinality taken over all complementary perfect dominating set is complementary perfect domination number". Inspired by the

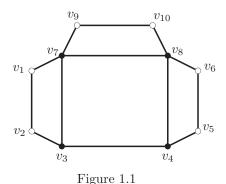
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above definitions, G.Mahadevan et.al., imposed a condition on the complement of paired dominating set and introduce a new concept called complementary independent twin paired domination number.

**Definition 1.1.** [6] The set  $S \subseteq V$  is said to be Complementary independent twin paired dominating set, if S is a paired dominating set and < V - S > is a set of independent edges. The minimum cardinality taken over all the Complementary independent twin paired dominating set is called as Complementary independent twin paired domination number and it is denoted by CITD(G).



**Example 1.** In the above figure 1.1,  $S = \{v_3, v_4, v_7, v_8\}$  is the Complementary independent twin paired dominating set and CITD(G) = 4.

The Cartesian product  $G \times H$  of the graphs G and H is graph such that the vertex set of  $G \times H$  is the Cartesian product of  $V(G) \times V(H)$  and any two vertices (u,u') and (v,v') are adjacent in  $G \times H$  if and only if either u=v and u' is adjacent with v' in H or u'=v' and u is adjacent with v in G.

If  $G_n$  and  $G_m$  be two graphs with n and m vertices respectively, then the Cartesian product  $G_n \times G_m$  has nm vertices.

**Observation 1.1.** As CITD(G) can be found only for even number of vertices, CITD(G) number for Cartesian product is possible only for the following three cases. (i) both n and m are even. (ii) n is odd and m is even. And (iii) n is even and m is odd. Each cases is discussed as a theorem in this paper.

**Theorem 1.1.** [6] If s is odd and n is even, then

$$CITD(P_s@P_n) = \begin{cases} s(2\lfloor \frac{n}{4} \rfloor + 2) & ifn \equiv 2(mod4) \\ s(2\lfloor \frac{n}{4} \rfloor + 2) - (s+1) & ifn \equiv 0(mod4) \end{cases}.$$

**Theorem 1.2.** [6] If s is even and n is odd, then

$$CITD(P_s@P_n) = \begin{cases} s + s(2\lfloor \frac{n-1}{4} \rfloor) & ifn \equiv 1(mod4) \\ \lfloor \frac{s}{3} \rfloor + s(2\lfloor \frac{n-1}{4} \rfloor + 2) & ifn \equiv 3(mod4) \end{cases}.$$

# 2. Complementary independent twin paired domination number for Cartesian Product of Paths

**Theorem 2.1.** For n,m are even,  $CITD(P_n \times P_m) = n(\frac{m}{2})$ .

Proof. Let  $\{v_{ij}: 1 \leq i \leq n, 1 \leq j \leq m\}$  be the vertex set of  $P_n \times P_m$ . Let  $S_1 = \{v_{ij}: i \ and \ j \equiv 1, 3(mod4)\}$  and  $S_2 = \{v_{ij}: i \ and \ j \equiv 0, 2(mod4)\}$ .  $S = S_1 \cup S_2$  is the complementary independent twin paired dominating set whose cardinality  $|S| = n(\frac{m}{2})$ . Clearly  $CITD(P_n \times P_m) \leq |S|$ . If  $T \subseteq S$  is the Complementary independent twin paired dominating set, then T fails to satisfy the definition. Therefore S is the minimum Complementary independent twin paired dominating set. Thus,  $|S| \leq CITD(P_n \times P_m)$  and  $|S| = CITD(P_n \times P_m)$ .

## Example 2.



Figure 2.1

**Illustration.** Here the darkened vertices denote the Complementary independent twin paired dominating set. $CITD(P_n \times P_m) = n(\frac{m}{2})$  implies  $CITD(P_4 \times P_4) = 4(\frac{4}{2}) = 8$ .

**Theorem 2.2.** For  $n \le m$ , n is odd and m is even  $CITD(P_n \times P_m) = (n-1) \lceil \frac{m}{4} \rceil + (n+1) \lfloor \frac{m}{4} \rfloor$ .

Proof. Let  $\{v_{ij}: 1 \leq i \leq n, 1 \leq j \leq m\}$  be the vertex set of  $P_n \times P_m$ . Let  $S_1 = \{v_{ij}: i \ and \ j \equiv 1, 3(mod4)\}$  and  $S_2 = \{v_{ij}: i \ and \ j \equiv 0, 2(mod4)\}$ .  $S = S_1 \cup S_2$  is the complementary independent twin paired dominating set whose cardinality  $|S| = (n-1)\lceil \frac{m}{4} \rceil + (n+1)\lfloor \frac{m}{4} \rfloor$ . Clearly  $CITD(P_n \times P_m) \leq |S|$ .

If  $T \subseteq S$  is the Complementary independent twin paired dominating set, then T fails to satisfy the definition. Therefore S is the minimum Complementary independent twin paired dominating set. Thus,  $|S| \leq CITD(P_n \times P_m)$  and  $|S| = CITD(P_n \times P_m)$ .

# Example 3.



Figue 2.2

**Illustration.** Here the darkened vertices denote the Complementary independent twin paired dominating set.  $CITD(P_n \times P_m) = (n-1)\lceil \frac{m}{4} \rceil + (n+1)\lfloor \frac{m}{4} \rfloor$  implies  $CITD(P_3 \times P_4) = (3-1)\lceil \frac{4}{4} \rceil + (3+1)\lfloor \frac{4}{4} \rfloor = 6$ .

**Theorem 2.3.** For  $n \le m$ , n is even and m is odd  $CITD(P_n \times P_m) = (m-1)\lceil \frac{n}{4} \rceil + (m+1)\lfloor \frac{n}{4} \rfloor$ .

Proof. Let  $\{v_{ij}: 1 \leq i \leq n, 1 \leq j \leq m\}$  be the vertex set of  $P_n \times P_m$ . Let  $S_1 = \{v_{ij}: i \equiv 1, 2 \pmod{4} \text{ and } j \equiv 0, 2 \pmod{4}\}$  and  $S_2 = \{v_{ij}: i \equiv 0, 3 \pmod{4} \text{ and } j \equiv 1, 3 \pmod{4}\}$ .  $S = S_1 \cup S_2$  is the complementary independent twin paired dominating set whose cardinality  $|S| = (m-1)\lceil \frac{n}{4} \rceil + (m+1)\lfloor \frac{n}{4} \rfloor$ . Clearly  $CITD(P_n \times P_m) \leq |S|$ . If  $T \subseteq S$  is the Complementary independent twin paired dominating set, then T fails to satisfy the definition. Therefore S is the minimum Complementary independent twin paired dominating set. Thus,  $|S| \leq CITD(P_n \times P_m)$  and  $|S| = CITD(P_n \times P_m)$ .

3. Complementary independent twin paired domination number for Cartesian product of path with cycle.

**Theorem 3.1.** For n,m are even,

$$CITD(P_n \times P_m) = \begin{cases} n(\frac{m}{2}) & m \equiv 0 \pmod{4} \\ n(\frac{m}{2}) + n & m \equiv 2 \pmod{4}. \end{cases}$$

Proof. Let  $\{v_{ij}: 1 \leq i \leq n, 1 \leq j \leq m\}$  be the vertex set of  $P_n \times C_m$ . Let  $S_1 = \{v_{ij}: i \ and \ j \equiv 1, 3 \ (mod4)\}$ ,  $S_2 = \{v_{ij}: i \ and \ j \equiv 0, 2 \ (mod4)\}$  and  $S_3 = \{v_{ij}: 1 \leq i \leq n \ and \ j = m, \ m-1\}$ . If  $m \equiv 0 \ (mod4)$ , then  $S = S_1 \cup S_2$  is the complementary independent twin paired dominating set whose cardinality  $|S| = n(\frac{m}{2})$ . If  $m \equiv 2(mod4)$ , then  $S = S_1 \cup S_2 \cup S_3$  is the complementary independent twin paired dominating set whose cardinality  $|S| = n(\frac{m}{2}) + n$ . Clearly, in both the cases  $CITD(P_n \times C_m) \leq |S|$ . If  $T \subseteq S$  is the Complementary independent twin paired dominating set, then T fails to satisfy the definition. Therefore S is the minimum Complementary independent twin paired dominating set. Thus,  $|S| \leq CITD(P_n \times C_m)$  and  $|S| = CITD(P_n \times C_m)$ 

## Example 4.



**Illustration.** Here the darkened vertices denote the Complementary independent twin paired dominating set.  $CITD(P_n \times C_m) = n(\frac{m}{2})$  for  $m \equiv 0 mod 4$  implies  $CITD(P_4 \times C_4) = 4(\frac{4}{2}) = 8$ .

### Example 5.

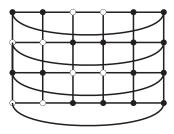


Figure 3.2

**Illustration.** Here the darkened vertices denote the Complementary independent twin paired dominating set.  $CITD(P_n \times C_m) = n(\frac{m}{2}) + n$  for  $m \equiv 0 mod 4$  implies  $CITD(P_4 \times C_6) = 4(\frac{6}{2}) + 4 = 16$ .

**Theorem 3.2.** For  $n \leq m$ , n is odd and m is even,

$$CITD(P_n \times C_m) = \begin{cases} (n-1) \lceil \frac{m}{4} \rceil + (n+1) \lfloor \frac{m}{4} \rfloor & m \equiv 0 \pmod{4} \\ (n-1) \lceil \frac{m}{4} \rceil + (n+1) (\lfloor \frac{m}{4} \rfloor + 1) & m \equiv 2 \pmod{4} \end{cases}$$

Proof. Let  $\{v_{ij}: 1 \leq i \leq n, 1 \leq j \leq m\}$  be the vertex set of  $P_n \times C_m$ . Let  $S_1 = \{v_{ij}: i \ and \ j \equiv 1, 3 \ (mod4)\}$ ,  $S_2 = \{v_{ij}: i \ and \ j \equiv 0, 2 \ (mod4)\}$  and  $S_3 = \{v_{ij}: 1 \leq i \leq n \ and \ j = m, \ m-1\}$ . If  $m \equiv 0 \ (mod4)$ , then  $S = S_1 \cup S_2$  is the complementary independent twin paired dominating set whose cardinality  $|S| = (n-1)\lceil \frac{m}{4} \rceil + (n+1)\lfloor \frac{m}{4} \rfloor$ . If  $m \equiv 2 \ (mod4)$ , then  $S = S_1 \cup S_2 \cup S_3$  is the complementary independent twin paired dominating set whose cardinality  $|S| = (n-1)\lceil \frac{m}{4} \rceil + (n+1)(\lfloor \frac{m}{4} \rfloor + 1)$ . Clearly, in both the cases  $CITD(P_n \times C_m) \leq |S|$ . If  $T \subseteq S$  is the Complementary independent twin paired dominating set, then T fails to satisfy the definition. Therefore S is the minimum Complementary independent twin paired dominating set. Thus,  $|S| \leq CITD(P_n \times C_m)$  and  $|S| = CITD(P_n \times C_m)$ 

**Theorem 3.3.** For  $n \leq m$ , n is even and m is odd,

$$CITD(P_n \times C_m) = \begin{cases} (m-1)\lceil \frac{n}{4} \rceil + (m+1)\lfloor \frac{n}{4} \rfloor & n \equiv 0 \pmod{4} \\ (m-1)\lceil \frac{n}{4} \rceil + (m+1)(\lfloor \frac{n}{4} \rfloor + 1) & n \equiv 2 \pmod{4} \end{cases}.$$

Proof. Let  $\{v_{ij}: 1 \leq i \leq n, 1 \leq j \leq m\}$  be the vertex set of  $P_n \times P_m$ . Let  $S_1 = \{v_{ij}: i \equiv 1, 2 \pmod{4} \text{ and } j \equiv 0, 2 \pmod{4}\}$ ,  $S_2 = \{v_{ij}: i \equiv 0, 3 \pmod{4} \text{ and } j \equiv 1, 3 \pmod{4}\}$ . and  $S_3 = \{v_{ij}: i = n, n-1 \text{ and } 1 \leq j \leq m\}$ . If  $m \equiv 0 \pmod{4}$ , then  $S = S_1 \cup S_2$  is the complementary independent twin paired dominating set whose cardinality  $|S| = (m-1)\lceil \frac{n}{4} \rceil + (m+1)\lfloor \frac{n}{4} \rfloor$ . If  $m \equiv 2 \pmod{4}$ , then  $S = S_1 \cup S_2 \cup S_3$  is the complementary independent twin paired dominating set whose cardinality  $|S| = (m-1)\lceil \frac{n}{4} \rceil + (m+1)(\lfloor \frac{n}{4} \rfloor + 1)$ . Clearly, in both the cases  $CITD(P_n \times C_m) \leq |S|$ . If  $T \subseteq S$  is the Complementary independent twin paired dominating set, then T fails to satisfy the definition. Therefore S is the minimum Complementary independent twin paired dominating set. Thus,  $|S| \leq CITD(P_n \times C_m)$  and  $|S| = CITD(P_n \times C_m)$ .

4. Complementary independent twin paired domination number for Cartesian product of cycles.

**Theorem 4.1.** *For* n,m *are even,* 

$$CITD(C_n \times C_m) = \begin{cases} n(\frac{m}{2}) & m \equiv 0 \ (mod 4) \\ n(\frac{m}{2}) + n & m \equiv 2 \ (mod 4) \end{cases}$$

*Proof.* Is same as theorem 3.1.

**Theorem 4.2.** For n < m, m is even and n is odd,

$$CITD(C_n \times C_m) = \begin{cases} (n+1)(\lceil \frac{m}{4} \rceil + \lfloor \frac{m}{4} \rfloor) & n \equiv 0 \pmod{4} \\ \frac{m-2}{4}(n-1) + m + 2n - 4 & n \equiv 2 \pmod{4} \end{cases}.$$

Proof. Let  $\{v_{ij}: 1 \leq i \leq n, 1 \leq j \leq m\}$  be the vertex set of  $C_n \times C_m$ . Let  $S_1 = \{v_{ij}: i \ and \ j \equiv 1, 3 \ (mod4)\}$ ,  $S_2 = \{v_{ij}: i \ and \ j \equiv 0, 2 \ (mod4)\}$ ,  $S_3 = \{v_{ij}: i = n1 \leq j \leq m\}$  and  $S_4 = \{v_{ij}: 1 \leq i \leq n \ and \ j = m, \ m-1\}$ . If  $m \equiv 0 \ (mod4)$ , then  $S = S_1 \cup S_2 \cup S_3$  is the complementary independent twin paired dominating set whose cardinality  $|S|(n+1)(\lceil \frac{m}{4} \rceil + \lfloor \frac{m}{4} \rfloor)$ . If  $m \equiv 2 \ (mod4)$ , then  $S = S_1 \cup S_2 \cup S_3 \cup S_4$  is the complementary independent twin paired dominating set whose cardinality  $|S| = \frac{m-2}{4}(n-1) + m + 2n - 4$ . Clearly, in both the cases  $CITD(C_n \times C_m) \leq |S|$ . If  $T \subseteq S$  is the Complementary independent twin paired dominating set, then T fails to satisfy the definition. Therefore S is the minimum Complementary independent twin paired dominating set. Thus,  $|S| \leq CITD(C_n \times C_m)$  and  $|S| = CITD(C_n \times C_m)$ .

### Example 6.

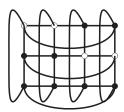


Figure 4.1

**Illustration.** Here the darkened vertices denote the Complementary independent twin paired dominating set.  $CITD(C_n \times C_m) = (n+1)(\lceil \frac{m}{4} \rceil + \lfloor \frac{m}{4} \rfloor)$  for  $m \equiv 0 mod 4$  implies  $CITD(C_3 \times C_4) = (3+1)(\lceil \frac{4}{4} \rceil + \lfloor \frac{4}{4} \rfloor) = 8$ .

**Theorem 4.3.** For  $n \leq m$ , n is even and m is odd,

$$CITD(CnXCm) = \begin{cases} (m+1)(\lceil \frac{n}{4} \rceil + \lfloor \frac{n}{4} \rfloor) & n \equiv 0 \ (mod4) \\ \frac{n-2}{4}(m-1) + n + 2m - 4 & n \equiv 2 \ (mod4) \end{cases}$$

Proof. Let  $\{v_{ij}: 1 \leq i \leq n, 1 \leq j \leq m\}$  be the vertex set of  $P_n \times P_m$ . Let  $S_1 = \{v_{ij}: i \equiv 1, 2 \pmod{4} \text{ and } j \equiv 0, 2 \pmod{4}\}$ ,  $S_2 = \{v_{ij}: i \equiv 0, 3 \pmod{4} \text{ and } j \equiv 1, 3 \pmod{4}\}$ ,  $S_3 = \{v_{ij}: i = n, n-1 \text{ and } 1 \leq j \leq m\}$  and  $S_4 = \{v_{ij}: 1 \leq j \leq m \text{ and } i = n, n-1\}$ . If  $m \equiv 0 \pmod{4}$ , then  $S = S_1 \cup S_2 \cup S_3$  is the complementary independent twin paired dominating set whose cardinality  $|S| = (m+1)(\lceil \frac{n}{4} \rceil + \lfloor \frac{n}{4} \rfloor)$ . If  $m \equiv 2 \pmod{4}$ , then  $S = S_1 \cup S_2 \cup S_3 \cup S_4$  is the complementary independent twin paired dominating set whose cardinality  $|S| = \frac{n-2}{4}(m-1) + n + 2m - 4$ . Clearly, in both the cases  $CITD(C_n \times C_m) \leq |S|$ . If  $T \subseteq S$  is the Complementary independent twin paired dominating set, then T fails to satisfy the definition. Therefore S is the minimum Complementary independent twin paired dominating set. Thus,  $|S| \leq CITD(C_n \times C_m)$  and  $|S| = CITD(C_n \times C_m)$ .

# 5. Complementary independent twin paired domination number for the graphs of the form $C_s@P_n$

The graph  $P_s@P_n$  is formed by pasting the pendent vertices of  $P_n$  (s-copies) to each vertex of the path  $P_s$ . The graph  $C_s@P_n$  is formed by pasting the pendent vertices of  $P_n$  (s-copies) to each vertex of the path  $C_s$ . Complementary independent twin paired domination number for  $C_s@P_n$  can be found only when s is odd and n is even or s is even and n is odd, which is been discussed in the following two theorems

**Theorem 5.1.** *if s is odd and n is even, then* 

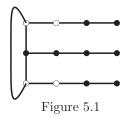
$$CITD(C_s@P_n) = \begin{cases} s(2\lfloor \frac{n}{4} \rfloor + 2) & ifn \equiv 2 \pmod{4} \\ s(2\lfloor n4 \rfloor + 2) - (s+1) & ifn \equiv 0 \pmod{4} \end{cases}.$$

*Proof.* As the Complementary Independent twin paired dominating vertices are choosen from the pendent vertices,

$$CITD(C_s@P_n) = CITD(P_s@P_n)$$
. Hence it follows theorem 1.1

**Observation 5.1.** The above theorem is true when both s and n is even.

### Example 7.



**Illustration.** Here the darkened vertices denote the Complementary independent twin paired dominating set. In figure 5.1, s=3 and n=4,  $CITD(C_s@P_n) = s(2\lfloor n4\rfloor + 2) - (s+1)$  implies  $CITD(C_3 \times C_4) = (3(2\lfloor 44\rfloor + 2) - (3+1)) = 8$ .

**Theorem 5.2.** *if s is odd and n is even, then* 

$$CITD(C_s@P_n) = \begin{cases} s + s(2\lfloor \frac{n-1}{4} \rfloor & ifn \equiv 1 \pmod{4} \\ \lfloor \frac{s}{3} \rfloor + s(2\lfloor \frac{n-1}{4} \rfloor + 2) & ifn \equiv 3 \pmod{4} \end{cases}.$$

*Proof.* As the Complementary Independent twin paired dominating vertices are choosen from the pendent vertices,

$$CITD(C_s@P_n) = CITD(P_s@P_n)$$
. Hence it follows theorem 1.2

## 6. Conclusion

In this paper we have found the Complementary Independent twin paired domination number for Cartesian product of paths and cycle. Also found the result for the graph of the form  $C_s@P_n$ . Future, CITD- number will be compared with other domination parameters which will be reported in subsequent papers.

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