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SIGNED DOMINATION NUMBER OF n-STAR GRAPH

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ABSTRACT. The n-star graph S_n is a simple graph whose vertex set is the set of all $n!$ permutations of $\{1, 2, \ldots, n\}$ and two vertices α and β are adjacent if and only if $\alpha(1) \neq \beta(1)$ and $\alpha(i) \neq \beta(i)$ for exactly one i,$i \neq 1$. In this paper we find the signed domination number γ_s of S_n. We also determine the lower bound of the signed domination number γ_s, for the complement of S_n, the lower bound of the sum and product of the signed domination number of n-star graph S_n and its complement.

1. INTRODUCTION

By a graph we mean a finite, undirected, connected graph without loops or multiple edges. Terms not defined here are used in the sense of Haynes et. al. [4] and Harary [3].The n-star graph S_n is first introduced by Akers and Krishnamurthy [1]. The vertex set of S_n is the set of all $n!$ permutations of $\{1, 2, \ldots, n\}$ and two vertices α and β are adjacent if and only if $\alpha(1) \neq \beta(1)$ and $\alpha(i) \neq \beta(i)$ for exactly one i, $i \neq 1$. In this paper we find the signed domination number for odd and even n of S_n. We also obtain lower bound of signed domination number for the complement S_n and sum and product of signed domination number of S_n and its complement. Let G=(V,E) be a graph. For a real valued function $f : V \to R$, the weight of f is $w(f) = \sum_{v \in V} f(v)$ and for $S \subseteq V$ we define $f(S) = \sum_{v \in S} f(v)$. So $w(f) = f(V)$. A signed dominating function is defined as a function $f : V \to \{-1, 1\}$ such that $\sum_{u \in N[v]} f(u) \geq 1$, for all $v \in V$.

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The signed domination number for a graph G is $\gamma_{s(G)} = min\{w(f)|$ f is a signed dominating function on G}. The upper signed domination number for a graph $\Gamma_{s(G)} = max\{w(f)|$ f is a signed dominating function on G}, [2, 4].

Theorem 1.1. *[4] For every k-regular graph G of order n,* $\gamma_s(G) \geq n/(k+1)$.

Theorem 1.2. *[4] For every k-regular graph G of order n, with k odd,*

$$\gamma_s(G) \geq 2n/(k+1).$$

2. Main Results

Theorem 2.1. *For n-star graph, with n odd the signed domination number* $\gamma_s(S_n) = (n-1)!$.

Proof.
Case 1: For $n = 1$.
Since there is only one vertex say $v(1)$ define $f : V(S_1) \to \{-1, 1\}$, such that $f(v_1) = 1$. Then f is the only signed dominating function. The signed domination number of S_1 is $1 = (1-1)!$. Therefore $\gamma_s(S_n) = (n-1)!$, for $n = 1$.
Case 2: For $n > 1$ and odd.
S_n is a $(n-1)$ regular graph. Since n is odd, $(n-1)$ is even. We have for every k-regular graph G of order n, $\gamma_s(G) \geq n/(k+1)$. Therefore S_n,

$$\gamma_s(S_n) \geq n!/(n-1+1) = (n-1)!n/n = (n-1)!.$$

Let $A_i = \{\alpha \in V(S_n) | \alpha(1) = 1, \alpha(1) = 2, \ldots \alpha(1) = i,$ where i $= (n-1)/2\}$.
Define a function $f : V(S_n) \to \{-1, 1\}$ such that

$$f(\alpha) = \begin{cases} -1 & \text{for} \quad \alpha \in A_i \\ 1 & \text{for} \quad \alpha \notin A_i \end{cases}.$$

Then f is a signed dominating function for S_n.

Now for finding the weight of signed dominating function for S_n ,there are $(n-1)!$ elements in each $\alpha(1) = i, i = 1, 2, \ldots, n$. Hence there are $\frac{(n-1)}{2}(n-1)!$ vertices for which $f(\alpha) = -1$. Therefore there are $n! - [\frac{(n-1)}{2}(n-1)!]$ vertices for which $f(\alpha) = 1$. Then weight of the signed dominating function f is

$$w(f) = [\frac{(n+1)}{2}(n-1)!] - [\frac{(n-1)}{2}(n-1)!] = [\frac{(n+1)}{2} - \frac{(n-1)}{2}](n-1)! = (n-1)!.$$

Hence we get a signed dominating function f of S_n with weight $(n-1)!$. But $\gamma_s(S_n) \geq (n-1)!$.

Therefore $\gamma_s(S_n) = (n-1)!$. ☐

Theorem 2.2. *For n-star graph, the signed domination number, $\gamma_s(S_n) = 2(n-1)!$ where n is even.*

Proof.

Case 1: n=2.

There are two vertices in S_2. Define $f : V(S_1) \to \{-1, 1\}$, such that $f(v_1) = 1$ and $f(v_2) = 1$. Then f is the only signed dominating function. The signed domination number of S_2 is $2 = 2(2-1)!$. Therefore $\gamma_s(S_n) = 2(n-1)!$, for $n = 2$.

Case 2: $n > 2$ and even.

S_n is a $(n-1)$ regular graph. Since n is even, $(n-1)$ is odd. We have for every k-regular graph G of order n, with k odd, $\gamma_s(G) \geq 2n/(k+1)$. Therefore for n-star graph,

$$\gamma_s(S_n) \geq 2n!/(n-1+1) = 2(n-1)!n/n = 2(n-1)!.$$

Let $A_i = \{\alpha \in V(S_n) | \alpha(1) = 1, \alpha(1) = 2, \ldots, \alpha(1) = i, \text{where i} = \frac{n}{2} - 1\}$. Define a function $f : V(S_n) \to \{-1, 1\}$, such that

$$f(\alpha) = \begin{cases} -1 & \text{for} \quad \alpha \in A_i \\ \ \ 1 & \text{for} \quad \alpha \notin A_i \end{cases} .$$

Then f is signed dominating function of S_n.

Now for finding the weight of signed dominating function for S_n, there are $(n-1)!$ elements in each $\alpha(1) = i, i = 1, 2, \ldots, n$.

Hence there are $(\frac{n}{2} - 1)(n-1)!$ vertices for which $f(\alpha) = -1$. Therefore there are $n! - [(\frac{n}{2} - 1)(n-1)!] = (\frac{n}{2} + 1)(n-1)!$ vertices for which $f(\alpha) = 1$.Then weight of the signed dominating function f is

$$w(f) = [(\frac{n}{2}+1)(n-1)!] - [(\frac{n}{2}-1)(n-1)!] = [(\frac{n}{2}+1) - (\frac{n}{2}-1)](n-1)! = 2(n-1)!.$$

Hence we get a signed dominating function f of S_n with weight $2(n-1)!$. But $\gamma_s(S_n) \geq 2(n-1)!$.

Hence $\gamma_s(S_n) = 2(n-1)!$. ☐

Theorem 2.3. *The complement $\overline{S_n}$ of n-star graph is $n! - n$ regular.*

Proof. Any two vertices of $\overline{S_n}$ is adjacent if it is not adjacent in S_n. Clearly S_n is $(n-1)$ regular. Also since there are $n!-1$ vertices other than a vertex v_i in $\overline{S_n}$, each vertex v_i in $\overline{S_n}$ is adjacent with $(n!-1)-(n-1) = n!-n$ vertices. Hence $\overline{S_n}$ is $n!-n$ regular. □

Theorem 2.4. *The signed domination number* $\gamma(\overline{S_n}) \geq 2n!/(n!-n+1)$ *if* n *is odd and* $\gamma(\overline{S_n}) \geq n!/(n!-n+1)$ *if* n *is even.*

Proof. $\overline{S_n}$ is n!-n regular. Also if n is odd, then $n!-n$ is odd. We have for every k-regular graph G of order n, with k odd $\gamma(\overline{S_n}) \geq 2n/(k+1)$.
Therefore for the complement of n-star graph $\gamma(\overline{S_n}) \geq 2n!/(n!-n+1)$.

Also if n is even, then $(n!-n)$ is even. We have for every k-regular graph G of order n,$\gamma(\overline{S_n}) \geq n/(k+1)$.

Therefore for the complement of n-star graph $\gamma(\overline{S_n}) \geq n!/(n!-n+1)$. □

Theorem 2.5. *The sum of the signed domination number of* S_n *and its complement* $\overline{S_n}$ *is*

$$\gamma(S_n) + \gamma(\overline{S_n}) \geq \frac{(n-1)!(n!+n+1)}{(n!-n+1)},$$

for odd n.

Proof. By theorem 2.1, $\gamma(S_n) = (n-1)!$. Also by theorem 2.4, $\gamma(\overline{S_n}) \geq 2n!/(n!-n+1)$, if n is odd. Hence it follows that

$$\begin{aligned}\gamma(S_n) + \gamma(\overline{S_n}) &\geq (n-1)! + 2n!/(n!-n+1)\\ &= \frac{(n-1)!(n!-n+1)+2(n-1)!n}{(n!-n+1)} = \frac{(n-1)![n!-n+1+2n]}{(n!-n+1)}\\ &= \frac{(n-1)!((n!+n+1))}{(n!-n+1)}.\end{aligned}$$

Hence $\gamma(S_n) + \gamma(\overline{S_n}) \geq \frac{(n-1)!(n!+n+1)}{(n!-n+1)}$, for n odd. □

Theorem 2.6. *The sum of the signed domination number of n-star graph and its complement* $\overline{S_n}$ *is*

$$\gamma(S_n) + \gamma(\overline{S_n}) \geq \frac{(n-1)!(2n!-n+1)}{(n!-n+1)},$$

for even n.

Proof. By theorem 2.2, $\gamma(S_n) = 2(n-1)!$. Also by theorem 2.4, $\gamma(\overline{S_n}) \geq n!/(n!-n+1)$. Hence it follows that

$$\begin{aligned}
&\gamma(S_n) + \gamma(\overline{S_n}) \geq 2(n-1)! + n!/(n!-n+1)\\
&\quad = \frac{2(n-1)!(n!-n+1) + (n-1)!n}{(n!-n+1)} = \frac{(n-1)![2n!-2n+1+n]}{(n!-n+1)}\\
&\quad = \frac{(n-1)!(2n!-n+1)}{(n!-n+1)}.
\end{aligned}$$

Hence $\gamma(S_n) + \gamma(\overline{S_n}) \geq \dfrac{(n-1)!(2n!-n+1)}{(n!-n+1)}$ for n even. □

Theorem 2.7. *The product of the signed domination number of n-star graph and its complement is*

$$\gamma(S_n)\gamma(\overline{S_n}) \geq \frac{(n-1)!2n!}{(n!-n+1)},$$

for n odd and

$$\gamma(S_n)\gamma(\overline{S_n}) \geq \frac{2(n-1)!n!}{(n!-n+1)},$$

n even.

Proof. By theorem 2.1, $\gamma(S_n) = (n-1)!$ and by theorem 2.4, $\gamma(\overline{S_n}) \geq 2n!/(n!-n+1)$ for odd n. Hence it follows that $\gamma(S_n)\gamma(\overline{S_n}) \geq (n-1)!2n!/(n!-n+1)$, for odd n. Also by theorem 2.2, $\gamma(S_n) = 2(n-1)!$ and by theorem 2.5, $\gamma(\overline{S_n}) \geq n!/(n!-n+1)$ for even n. Hence it follows that for even n

$$\gamma(S_n)\gamma(\overline{S_n}) \geq \frac{2(n-1)!n!}{(n!-n+1)}.$$

□

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