

## HOMOMORPHISM ON BIPOLAR VAGUE NORMAL GROUPS

U. VENKATA KALYANI<sup>1</sup> AND T. ESWARLAL

ABSTRACT. Here we studied the perception of homomorphism and also anti-homomorphism on bipolar- vague groups and bipolar- vague normal groups.

### 1. INTRODUCTION

The fuzzy sets was popularized first by Zadeh [18] in 1965. Suppose  $Z$  is any non-empty set. A mapping  $\gamma : Z \rightarrow [0, 1]$  is known as a Fuzzy subset of  $Z$ . We have many extensions in the fuzzy set theory, such as intuitionistic fuzzy sets, interval - valued fuzzy sets, vague sets etc [17]. The fuzzy set theory govern membership of an element  $z$  only, it means the indication of  $z$  affinity to  $\gamma$ . It does not take care of the indication against  $z$  affinity to  $\gamma$ . To oppose this trouble Gau.W.L and Buehrer.D.J [19] brought in the notion of vague set theory. According to them, a vague set  $\mathcal{A}$  of a non-empty set  $Z$  can be identified by functions  $(t_{\mathcal{A}}, f_{\mathcal{A}})$  where  $t_{\mathcal{A}}$  and  $f_{\mathcal{A}}$  are functions from  $Z$  to  $[0, 1]$  such that  $t_{\mathcal{A}}(x) + f_{\mathcal{A}}(x) \leq 1$  for all  $z \in Z$  where  $t_{\mathcal{A}}$  is called the truth function (or) membership function, which gives indication of how much an element  $z$  belong to  $\mathcal{A}$  and  $f_{\mathcal{A}}$  is called the false function (or) non-membership function, which gives indication of how much an element  $z$  does not belong to  $\mathcal{A}$ . These approaches are being administered in various fields like decision making, fuzzy control etc. In such a way the ideology of vague sets is a generalization of Fuzzy set theory. Ranjit

<sup>1</sup>corresponding author

2010 Mathematics Subject Classification. 16Y60, 16Y99, 03E72.

Key words and phrases. Bipolar vague set, bipolar vague group, bipolar vague normal group, bipolar vague homomorphism, bipolar vague anti homomorphism.

Biswas [17] proposed the theory of vague groups and authors like T. Eswarlal, N. Ramakrishna, Y. Bhargavi, B. Nageswara Rao, S. Ragamayi introduced and studied Boolean vague sets, Vague groups, vague gamma semi rings, translate operators on Vague groups and vague gamma near rings respectively [1–15] and extended the study of vague algebra and its applications. Lee [16] popularized the Bipolar - valued fuzzy sets (BVFS), which are an extension of fuzzy sets. Here the membership degree of these bipolar valued fuzzy sets (BVFS) range is extended from the interval  $[0,1]$  to  $[-1,1]$ . The degree of satisfaction to the property corresponding to a fuzzy set and its counter property are represented by membership degrees of BVFS. This led to a spirited field of research in distinct disciplines like algebraic structures, decision making, graph theory, medical science, machine theory etc.

In this paper we have illustrated homomorphism and anti homomorphism on Bipolar vague normal sub groups (BVNSG) and studied a few of their significant properties.

Suppose  $\eta : K \rightarrow K'$  be a homomorphism from group  $K$  to  $K'$ . Then

- (i) The homomorphic image  $\eta(\mathcal{A})$  of a bipolar vague normal subgroup (BVNSG)  $\mathcal{A}$  of  $K$  is a BVNSG of  $K'$  and
- (ii) The homomorphic preimage  $\eta^{-1}(B)$  of a bipolar vague normal subgroup (BVNSG)  $B$  of  $K'$  is a BVNSG of  $K$ .

## 2. PRELIMINARIES

In this phase we recall a few number of the important standards and definitions, which are probably vital for this paper.

**Definition 2.1.** [18] A mapping  $\gamma : Z \rightarrow [0, 1]$  is referred to as a fuzzy subset of non empty set  $Z$ .

**Definition 2.2.** [17] A vague set  $\mathcal{A}$  in the universe of discourse  $Z$  is a pair  $(t_{\mathcal{A}}, f_{\mathcal{A}})$ , where  $t_{\mathcal{A}} : Z \rightarrow [0, 1]$ ,  $f_{\mathcal{A}} : Z \rightarrow [0, 1]$  are mappings such that  $t_{\mathcal{A}}(z) + f_{\mathcal{A}}(z) \leq 1$ , for all  $z \in Z$ . The functions  $t_{\mathcal{A}}$  and  $f_{\mathcal{A}}$  are referred as true membership function and false membership function respectively.

**Definition 2.3.** [17] Let  $(K, *)$  be a group. A vague set (VS)  $\mathcal{A}$  of  $K$  is termed as a vague group (VG) of  $K$  if for any  $g, h$  in  $K$ , if  $V_{\mathcal{A}}(g * h) \geq \min\{V_{\mathcal{A}}(g), V_{\mathcal{A}}(h)\}$  and  $V_{\mathcal{A}}(g^{-1}) \geq V_{\mathcal{A}}(g)$ , i.e.,

- (i)  $t_{\mathcal{A}}(g * h) \geq \min\{t_{\mathcal{A}}(g), t_{\mathcal{A}}(h)\}$  and  $f_{\mathcal{A}}(g * h) \leq \max\{f_{\mathcal{A}}(g), f_{\mathcal{A}}(h)\}$ ;
- (ii)  $t_{\mathcal{A}}(g^{-1}) \geq t_{\mathcal{A}}(g)$  and  $f_{\mathcal{A}}(g^{-1}) \leq f_{\mathcal{A}}(g)$ .

**Definition 2.4.** [5–8, 10] Consider a group  $(K, \cdot)$  and  $\mathcal{A}$  be a vague group (VG) of  $K$ . A vague left coset (VLC) of  $\mathcal{A}$ , denoted by  $a\mathcal{A}$ , for any  $a \in K$ , and defined by  $V_{a\mathcal{A}}(z) = V_{\mathcal{A}}(a^{-1}(z))$  i.e.  $t_{a\mathcal{A}}(z) = t_{\mathcal{A}}(a^{-1}(z))$  and  $f_{a\mathcal{A}}(z) = f_{\mathcal{A}}(a^{-1}(z))$ .

**Definition 2.5.** [5–8, 10] Consider a group  $(K, \cdot)$  and  $\mathcal{A}$  be a vague group (VG) of  $K$ . A vague right coset (VRC) of  $\mathcal{A}$  is denoted by  $\mathcal{A}a$  and for any  $a$  in  $K$  defined by  $V_{\mathcal{A}a}(z) = V_{\mathcal{A}}((za)^{-1})$  i.e.  $t_{\mathcal{A}a}(z) = t_{\mathcal{A}}((za)^{-1})$  and  $f_{\mathcal{A}a}(z) = f_{\mathcal{A}}((za)^{-1})$ .

**Definition 2.6.** [16] Consider a universal set  $Z$  and  $\mathcal{A}$  be a set over  $Z$  that is defined by a positive membership function,  $\mu_{\mathcal{A}}^+ : Z \rightarrow [0, 1]$  and a negative membership function,  $\mu_{\mathcal{A}}^- : Z \rightarrow [-1, 0]$ . Then  $\mathcal{A}$  is called a bipolar-valued fuzzy set over  $Z$ , and can be written in the form  $\mathcal{A} = \{ \langle z : \mu_{\mathcal{A}}^+(z), \mu_{\mathcal{A}}^-(z) \rangle : z \in Z \}$ .

**Definition 2.7.** [16] Consider a group  $K$ . A bipolar valued fuzzy subset (BVFS)  $B$  of  $K$  is referred as a bipolar valued fuzzy subgroup (BVFSG) of  $K$ , if for all  $g, h$  in  $K$  if

- (i)  $B^+(gh) \geq \min\{B^+(g), B^+(h)\}$ ,
- (ii)  $B^+(g^{-1}) \geq B^+(g)$ ,
- (iii)  $B^-(gh) \leq \max\{B^-(g), B^-(h)\}$ ,
- (iv)  $B^-(g^{-1}) \leq B^-(g)$ .

**Definition 2.8.** [14, 16] Let  $B$  be an object over universe of discourse  $Z$ . Then  $B$  is called a bipolar vague set (BVS) which is of the form:

$$B = \{ \langle z : [t_B^+(z), 1 - f_B^+(z)], [t_B^-(z), 1 - f_B^-(z)] \rangle : z \in Z \},$$

where  $0 \leq t_B^+(z) + f_B^+(z) \leq 1$  and  $-1 \leq t_B^-(z) + f_B^-(z) \leq 0$ . Here  $V_B^+ = [t_B^+, 1 - f_B^+]$  and  $V_B^- = [-1 - f_B^-, t_B^-]$  will be used to denote a bipolar vague set.

**Definition 2.9.** [14, 16] Let  $\mathcal{A}$  be a bipolar-vague set (BVS) in universe of discourse  $Z$ . Then  $\mathcal{A}$  is called a bipolar valued vague group (BVG) of  $Z$  if :

- (i)  $V_B^+(gh) \geq \min\{V_B^+(g), V_B^+(h)\}$  and  $V_B^+(g^{-1}) \geq V_B^+(g)$  and  $V_B^-(gh) \leq \max\{V_B^-(g), V_B^-(h)\}$  and  $V_B^-(g^{-1}) \leq V_B^-(g)$ , i.e.,  $t_B^+(gh) \geq \min\{t_B^+(g), t_B^+(h)\}$  and  $1 - f_B^+(gh) \geq \min\{1 - f_B^+(g), 1 - f_B^+(h)\}$ .
- (ii)  $t_B^+(g^{-1}) \geq t_B^+(g)$  and  $1 - f_B^+(g^{-1}) \geq 1 - f_B^+(g)$ .

- (iii)  $t_B^-(gh) \leq \max\{t_B^-(g), t_B^-(h)\}$  and  $-1 - f_B^-(gh) \leq \max\{-1 - f_B^-(g); -1 - f_B^-(h)\}$ .
- (iv)  $t_B^-(g^{-1}) \leq t_B^-(g)$  and  $-1 - f_B^-(g^{-1}) \leq -1 - f_B^-(g)$ .

**Example 1.** [14, 16] Let  $G = \{1, \omega, \omega^2\}$  where  $\omega$  is the cubic root of unity with the binary operation defined as below: Let  $\mathcal{A} = (Z; V_A^+, V_A^-)$  be a bipolar vague set (BVS)

TABLE 1. composition table of cube roots of unity

*	1	$\omega$	$\omega^2$
1	1	$\omega$	$\omega^2$
$\omega$	$\omega$	$\omega^2$	1
$\omega^2$	$\omega^2$	1	$\omega$

in  $Z$  as defined below: Then  $\mathcal{A} = (Z; V_A^+, V_A^-)$  is a bipolar vague group (BVG) of

TABLE 2. Bipolar vague set in  $Z$

	1	$\omega$	$\omega^2$
$V_A^+$	$[0.9, 0.9]$	$[0.6, 0.8]$	$[0.6, 0.8]$
$V_A^-$	$[-0.4, -0.1]$	$[-0.4, -0.1]$	$[-0.4, -0.1]$

the group  $Z$ .

**Definition 2.10.** [14, 16] Consider a group and  $B$  be a bipolar vague set (BVS) on  $Z$ . Then  $B$  is known as a bipolar vague normal subgroup (BVNSG) over  $Z$  if  $V_B^+(ghg^{-1}) \geq V_B^+(h)$  and  $V_B^-(ghg^{-1}) \leq V_B^-(h)$  for all  $g, h \in Z$ . The set of all bipolar vague normal subgroups on  $Z$  are denoted by  $BVNS(Z)$ .

**Remark 2.1.** [14, 16] Let  $B$  be a bipolar vague set (BVS) on group  $Z$ . Then  $B$  is called a bipolar vague normal subgroup over  $Z$  (BVNS), if  $B(ghg^{-1}) = B(h)$  for all  $g, h \in Z$ .

**Definition 2.11.** Let  $\eta$  be a mapping from a group  $K$  to a group  $K'$  and let  $A = (K; V_A^+, V_A^-)$  bipolar vague subset in  $K$  and  $B = (K'; V_B^+, V_B^-)$  bipolar vague subset in  $\eta(g) = K'$  defined by  $V_B^+(h) = \sup V_A^+(g)$  and  $V_B^-(h) = \inf V_A^-(g)$  where  $g \in \eta^{-1}(h)$  for all  $g \in K$  and  $h \in K'$ .  $A$  is called preimage of  $B$  under  $\eta$  and is denoted by  $\eta^{-1}(B)$  and is defined for  $g \in K$  by

$$(\eta^{-1}(V_B^+(g))) = V_B^+(\eta(g)) \quad (\eta^{-1}(V_B^-(g))) = V_B^-(\eta(g)).$$

**Definition 2.12.** Consider the groups  $K$  and  $K'$ . Then the function  $\eta : K \rightarrow K'$  is an anti-homomorphism if  $\eta(gh) = \eta(h)\eta(g)$  for all  $g$  and  $h$  in  $K$ .

**Theorem 2.1.** [14, 16] Consider two groups  $K$  and  $K'$ . Suppose  $\eta : K \rightarrow K'$  is homomorphism and let  $\mathcal{A} = (K; V_{\mathcal{A}}^+, V_{\mathcal{A}}^-)$  bipolar vague subgroup (BVSG) of  $K$  then  $\eta(\mathcal{A})$ , the image of  $\mathcal{A}$  under  $\eta$  is a bipolar vague subgroup (BVSG) of  $K'$ .

**Theorem 2.2.** [14, 16] Consider two groups  $K$  and  $K'$ . Suppose  $\eta : K \rightarrow K'$  is homomorphism. If  $B$  is (BVSG) i.e bipolar vague sub group in  $K$ , then  $\eta^{-1}(B)$  is a (BVSG) i.e bipolar vague subgroup of  $K$ .

### 3. HOMOMORPHISM AND ANTI-HOMOMORPHISM ON BIPOLAR VAGUE NORMAL GROUPS

**Theorem 3.1.** Let  $\eta : K \rightarrow K'$  be an anti-homomorphism from group  $K$  to a group  $K'$ . Then  $\eta(\mathcal{A})$  is a bipolar vague subgroup (BVSG) of  $K'$  if  $\mathcal{A}$  is a bipolar vague subgroup (BVSG) of  $K$ .

*Proof.* Suppose  $(K, .)$  and  $(K', .)$  be two groups. Let  $\mathcal{A} = (K, V_{\mathcal{A}}^+, V_{\mathcal{A}}^-)$  be a bipolar vague group in  $K$  and given  $\eta : K \rightarrow K'$  is an anti-homomorphism. Let  $B = \eta(\mathcal{A})$ , where  $\mathcal{A}$  is a bipolar vague subgroup (BVSG) of  $K$ . Now we show that  $B$  is a bipolar vague subgroup (BVSG) of  $K'$ . For  $\eta(g), \eta(h)$  in  $K'$ ,

$$\begin{aligned} V_B^+(\eta(g)\eta(h)) &= V_B^+(\eta(hg)) = \sup(V_{\mathcal{A}}^+(hg)) \geq V_{\mathcal{A}}^+(hg) \\ &\geq \min\{V_{\mathcal{A}}^+(g), V_{\mathcal{A}}^+(h)\} = \min\{V_B^+(\eta(g)), V_B^+(\eta(h))\}. \end{aligned}$$

Thus  $V_B^+(\eta(g)\eta(h)) \geq \min\{V_B^+(\eta(g)), V_B^+(\eta(h))\}$ . Now for  $\eta(g)$  in  $K'$ ,

$$\begin{aligned} V_B^+(\eta(g))^{-1} &= V_B^+(\eta(g^{-1})) = \sup V_{\mathcal{A}}^+(g^{-1}) \geq V_{\mathcal{A}}^+(g^{-1}) \\ &\geq V_{\mathcal{A}}^+(g) \geq V_B^+(\eta(g)). \end{aligned}$$

Thus,  $V_B^+(\eta(g))^{-1} \geq V_B^+(\eta(g))$ .

Further, for  $\eta(g), \eta(h)$  in  $K'$ ,

$$\begin{aligned} V_B^-(\eta(g)\eta(h)) &= V_B^-(\eta(hg)) = \inf V_{\mathcal{A}}^-(hg) \leq V_{\mathcal{A}}^-(hg) \\ &\leq \max\{V_{\mathcal{A}}^-(g), V_{\mathcal{A}}^-(h)\} = \max\{V_B^-(\eta(g)), V_B^-(\eta(h))\}. \end{aligned}$$

Therefore,  $V_B^-(\eta(g)\eta(h)) \leq \max\{V_B^-(\eta(g)), V_B^-(\eta(h))\}$ . Now, for  $\eta(g)$  in  $K'$ ,

$$\begin{aligned} V_B^-(\eta(g))^{-1} &= V_B^-(\eta(g^{-1})) = \inf V_{\mathcal{A}}^-(g^{-1}) \\ &\leq V_{\mathcal{A}}^-(g^{-1}) \leq V_{\mathcal{A}}^-(g) \leq V_B^-(\eta(g)). \end{aligned}$$

Thus  $V_B^-(\eta(g))^{-1} \leq V_B^-(\eta(g))$  and  $\eta(\mathcal{A})$  is a bipolar vague subgroup (BVSG) of  $K$  if  $A$  is a bipolar vague subgroup (BVSG) of  $K$ .  $\square$

**Theorem 3.2.** *Let  $\eta : K \rightarrow K'$  be an anti-homomorphism from group  $K$  to a group  $K'$ . Then  $\eta^{-1}(\mathcal{A})$  is a bipolar vague subgroup (BVSG) of  $K$  if  $\mathcal{A}$  is a bipolar vague subgroup (BVSG) of  $K'$ .*

*Proof.* Suppose  $(K, \cdot)$  and  $(K', \cdot)$  be two groups. Let  $\mathcal{A} = (K', V_{\mathcal{A}}^+, V_{\mathcal{A}}^-)$  be a bipolar vague group (BVG) in  $K'$ . Given  $\eta : K \rightarrow K'$  is an anti-homomorphism. Now we show that  $\eta^{-1}(\mathcal{A})$  is a bipolar vague subgroup of  $K$ , and for  $g, h$  in  $K$ ,

$$\begin{aligned} V_{\eta^{-1}(\mathcal{A})}^+(gh) &= \eta^{-1}(V_{\mathcal{A}}^+(gh)) = V_{\mathcal{A}}^+(\eta(gh)), \\ &= V_{\mathcal{A}}^+(\eta(h)\eta(g)) \geq \min\{V_{\mathcal{A}}^+(\eta(g)), V_{\mathcal{A}}^+(\eta(h))\} \\ &= \min\{\eta^{-1}(V_{\mathcal{A}}^+(g)), \eta^{-1}(V_{\mathcal{A}}^+(h))\} = \min\{V_{\eta^{-1}(\mathcal{A})}^+(g), V_{\eta^{-1}(\mathcal{A})}^+(h)\}. \end{aligned}$$

Further, for  $g$  in  $K$ ,

$$\begin{aligned} V_{\eta^{-1}(\mathcal{A})}^+(g^{-1}) &= \eta^{-1}(V_{\mathcal{A}}^+(g^{-1})) = V_{\mathcal{A}}^+(\eta(g^{-1})) \\ &\geq V_{\mathcal{A}}^+(\eta(g)) = V_{\eta^{-1}(\mathcal{A})}^+(g). \end{aligned}$$

Thus  $V_{\eta^{-1}(\mathcal{A})}^+((g^{-1})) \geq V_{\eta^{-1}(\mathcal{A})}^+(g)$ . Now for  $g, h$  in  $K$ ,

$$\begin{aligned} V_{\eta^{-1}(\mathcal{A})}^-(gh) &= \eta^{-1}(V_{\mathcal{A}}^-(gh)) = V_{\mathcal{A}}^-(\eta(gh)), \\ &= V_{\mathcal{A}}^-(\eta(h)\eta(g)) \leq \max\{V_{\mathcal{A}}^-(\eta(g)), V_{\mathcal{A}}^-(\eta(h))\} \\ &= \max\{\eta^{-1}(V_{\mathcal{A}}^-(g)), \eta^{-1}(V_{\mathcal{A}}^-(h))\} = \max\{V_{\eta^{-1}(\mathcal{A})}^-(g), V_{\eta^{-1}(\mathcal{A})}^-(h)\}, \end{aligned}$$

and further  $g$  in  $K$ ,

$$\begin{aligned} V_{\eta^{-1}(\mathcal{A})}^-(g^{-1}) &= \eta^{-1}(V_{\mathcal{A}}^-(g^{-1})) = V_{\mathcal{A}}^-(\eta(g^{-1})) \\ &\leq V_{\mathcal{A}}^-(\eta(g)) = V_{\eta^{-1}(\mathcal{A})}^-(g), \end{aligned}$$

and  $V_{\eta^{-1}(\mathcal{A})}^-((g^{-1})) \geq V_{\eta^{-1}(\mathcal{A})}^-(g)$ . Hence,  $\eta^{-1}(\mathcal{A})$  is a bipolar vague subgroup (BVSG) of  $K$  if  $\mathcal{A}$  is a bipolar vague subgroup (BVSG) of  $K'$ .  $\square$

**Theorem 3.3.** *Let  $\eta : K \rightarrow K'$  be a group homomorphism. Then  $\eta(\mathcal{A})$  is a bipolar vague normal subgroup (BVNSG) of  $K$  if  $\mathcal{A}$  is a bipolar vague normal subgroup (BVNSG) of  $K'$ .*

*Proof.* Let  $(K, \cdot)$  and  $(K', \cdot)$  be any two groups. Let  $\mathcal{A} = (K, V_{\mathcal{A}}^+, V_{\mathcal{A}}^-)$  be a bipolar vague normal group. Let  $B = \eta(\mathcal{A})$ , where  $\mathcal{A}$  is a bipolar vague normal subgroup

(BVNSG) of  $K$ . We prove that  $B$  is a bipolar vague normal subgroup (BVNSG) of  $K'$ . For  $\eta(g), \eta(h)$  in  $K'$ ,

$$\begin{aligned} V_B^+(\eta(g)\eta(h)) &= V_B^+(\eta(gh)) = \sup V_{\mathcal{A}}^+(gh) \\ &\geq V_{\mathcal{A}}^+(gh) = V_{\mathcal{A}}^+(hg) \leq V_B^+(\eta(hg)) \\ &= V_B^+(\eta(h)\eta(g)), \end{aligned}$$

and  $V_B^+(\eta(g)\eta(h)) = V_B^+(\eta(h)\eta(g))$ .

Further,

$$\begin{aligned} V_B^-(\eta(g)\eta(h)) &= V_B^-(\eta(gh)) = \inf V_{\mathcal{A}}^-(gh) \leq V_{\mathcal{A}}^-(gh) \\ &= V_{\mathcal{A}}^-(hg) \geq V_B^-(\eta(hg)) = V_B^-(\eta(h)\eta(g)). \end{aligned}$$

Hence  $V_B^-(\eta(g)\eta(h)) = V_B^-(\eta(h)\eta(g))$ . Thus  $\eta(\mathcal{A})$  is a bipolar vague normal subgroup (BVNSG) of  $K'$ .  $\square$

**Theorem 3.4.** Let  $\eta : K \rightarrow K'$  be a group homomorphism. Then  $\eta^{-1}(\mathcal{A})$  is a bipolar vague normal subgroup (BVNSG) of  $K$  if  $\mathcal{A}$  is a bipolar vague normal subgroup (BVNSG) of  $K'$ .

*Proof.* Let  $(K, \cdot)$  and  $(K', \cdot)$  be any two groups. Let  $\mathcal{A} = (K', V_{\mathcal{A}}^+, V_{\mathcal{A}}^-)$  be a bipolar vague normal subgroup (BVNSG) and Now we show that  $\eta^{-1}(\mathcal{A})$  is a bipolar vague normal subgroup of  $K$ . Now for  $g, h$  in  $K$ ,

$$\begin{aligned} V_{\eta^{-1}(\mathcal{A})}^+(gh) &= \eta^{-1}(V_{\mathcal{A}}^+(gh)) = V_{\mathcal{A}}^+(\eta(gh)) \\ &= V_{\mathcal{A}}^+(\eta(g)\eta(h)) = V_{\mathcal{A}}^+(\eta(h)\eta(g)) \\ &= V_{\mathcal{A}}^+(\eta(hg)) = \eta^{-1}(V_{\mathcal{A}}^+(hg)) = V_{\eta^{-1}(\mathcal{A})}^+(hg), \end{aligned}$$

and hence  $V_{\eta^{-1}(\mathcal{A})}^+(gh) = V_{\eta^{-1}(\mathcal{A})}^+(hg)$ . Now

$$\begin{aligned} V_{\eta^{-1}(\mathcal{A})}^-(gh) &= \eta^{-1}(V_{\mathcal{A}}^-(gh)) = V_{\mathcal{A}}^-(\eta(gh)) \\ &= V_{\mathcal{A}}^-(\eta(g)\eta(h)) = V_{\mathcal{A}}^-(\eta(h)\eta(g)) = V_{\mathcal{A}}^-(\eta(hg)) \\ &= \eta^{-1}(V_{\mathcal{A}}^-(hg)) = V_{\eta^{-1}(\mathcal{A})}^-(hg). \end{aligned}$$

Hence  $V_{\eta^{-1}(\mathcal{A})}^-(hg) = V_{\eta^{-1}(\mathcal{A})}^-(hg)$  and thus  $\eta^{-1}(\mathcal{A})$  is a bipolar vague normal subgroup (BVNSG) of  $K$ .  $\square$

**Theorem 3.5.** Let  $\eta : K \rightarrow K'$  be an anti-homomorphism from group  $K$  to a group  $K'$ . Then  $\eta(\mathcal{A})$  is a bipolar vague normal subgroup (BVNSG) of  $K'$  if  $\mathcal{A}$  is a bipolar vague normal subgroup (BVNSG) of  $K$ .

*Proof.* Suppose  $(K, .)$  and  $(K', .)$  be two groups. Let  $\mathcal{A} = (K, V_{\mathcal{A}}^+, V_{\mathcal{A}}^-)$  be a bipolar vague normal subgroup in  $K$  and  $\eta : K \rightarrow K'$  is an anti-homomorphism.

Let  $B = \eta(\mathcal{A})$ , where  $\mathcal{A}$  is a bipolar vague normal subgroup (BVNSG) of  $K$ .

Now we show that  $B$  is a bipolar vague normal subgroup (BVNSG) of  $K'$ . Now for  $\eta(g), \eta(h)$  in  $K'$ ,

$$\begin{aligned} V_B^+(\eta(g)\eta(h)) &= V_B^+(\eta(hg)) = \sup V_{\mathcal{A}}^+(hg) \\ &\geq V_{\mathcal{A}}^+(hg) = V_{\mathcal{A}}^+(gh) \leq V_B^+(\eta(gh)) \\ &= V_B^+(\eta(h)\eta(g)), \end{aligned}$$

hence  $V_B^+(\eta(g)\eta(h)) = V_B^+(\eta(h)\eta(g))$ . Now

$$\begin{aligned} V_B^-(\eta(g)\eta(h)) &= V_B^-(\eta(hg)) = \inf V_{\mathcal{A}}^-(hg) \leq V_{\mathcal{A}}^-(hg) \\ &= V_{\mathcal{A}}^-(gh) \geq V_B^-(\eta(gh)) = V_B^-(\eta(h)\eta(g)), \end{aligned}$$

hence  $V_B^-(\eta(g)\eta(h)) = V_B^-(\eta(h)\eta(g))$ . Thus the anti-homomorphic image of a bipolar vague normal subgroup (BVNSG) of  $K$  is a bipolar vague normal subgroup (BVNSG) of  $K'$ .  $\square$

**Theorem 3.6.** Let  $\eta : K \rightarrow K'$  be an anti-homomorphism from group  $K$  to a group  $K'$ . Then  $\eta^{-1}(\mathcal{A})$  is a bipolar vague normal subgroup (BVNSG) of  $K$  where  $\mathcal{A}$  is a bipolar vague normal subgroup (BVNSG) of  $K$ .

*Proof.* Suppose  $(K, .)$  and  $(K', .)$  be two groups. Let  $\mathcal{A} = (K, V_{\mathcal{A}}^+, V_{\mathcal{A}}^-)$  be a bipolar vague group in  $K$  and  $\eta : K \rightarrow K'$  is an anti-homomorphism. Now we show that  $\eta^{-1}(\mathcal{A})$  is a bipolar vague subgroup of  $K$ . Now for  $g, h$  in  $K$ ,

$$\begin{aligned} V_{\eta^{-1}(\mathcal{A})}^+(gh) &= \eta^{-1}(V_{\mathcal{A}}^+(gh)) = V_{\mathcal{A}}^+(\eta(gh)), \\ &= V_{\mathcal{A}}^+(\eta(h)\eta(g)) = V_{\mathcal{A}}^+(\eta(g)\eta(h)) = V_{\mathcal{A}}^+(\eta(hg)) \\ &= \eta^{-1}(V_{\mathcal{A}}^+(hg)) = V_{\eta^{-1}(\mathcal{A})}^+(hg), \end{aligned}$$

hence  $V_{\eta^{-1}(\mathcal{A})}^+(gh) = V_{\eta^{-1}(\mathcal{A})}^+(hg)$ . Now

$$\begin{aligned} V_{\eta^{-1}(\mathcal{A})}^-(gh) &= \eta^{-1}(V_{\mathcal{A}}^-(gh)) = V_{\mathcal{A}}^-(\eta(gh)), \\ &= V_{\mathcal{A}}^-(\eta(h)\eta(g)) = V_{\mathcal{A}}^-(\eta(g)\eta(h)) = V_{\mathcal{A}}^-(\eta(hg)) \\ &= \eta^{-1}(V_{\mathcal{A}}^-(hg)) = V_{\eta^{-1}(\mathcal{A})}^-(hg). \end{aligned}$$

Therefore,  $V_{\eta^{-1}(\mathcal{A})}^-(gh) = V_{\eta^{-1}(\mathcal{A})}^-(hg)$ , and further  $\eta^{-1}(\mathcal{A})$  is a bipolar vague normal subgroup (BVNSG) of  $K$ .  $\square$



## 4. CONCLUSION

In this paper we proved some results on homomorphism and anti-homomorphism on bipolar vague subgroups(BVSG) of a group and bipolar vague normal subgroups(BVNSG) of a group. Further, we can extend the concept to prove fundamental theorem of homomorphism and fundamental theorem of isomorphism on bipolar Vague groups(BVG) in our future work.

## ACKNOWLEDGMENT

The authors are thankful to Prof K. L. N Swamy , Dr. Y. Bhargavi and Dr. S. Ragamayi for their precious suggestions.

## REFERENCES

- [1] Y. BHARGAVI, T. ESWARLAL: *Vague semiprime ideals of a Gamma -semiring*, Afrika Matematika, Springer **29**(3-4) (2018), 425–434.
- [2] Y. BHARGAVI: *Vague filters of a Gamma-semiring*, International Journal of Mechanical and Production Engineering Research and Development, **8** (2018), 421–428.
- [3] Y. BHARGAVI, T. ESWARLAL: *Vague magnified translation in  $\gamma$ -semirings*, International Journal of Pure and Applied Mathematics, **106**(2) (2016), 453–460.
- [4] Y. BHARGAVI, T. ESWARLAL: *Application of vague set in medical diagnosis*, International Journal of Chemical Sciences, **14**(2) (2016), 842–846.
- [5] C. MALLIKA, N. RAMAKRISHNA, G. ANANDHA RAO: *Vague Cosets*, World Journal of Research and Review (WJRR), **3**(2) (2016), 28–31.
- [6] T. ESWARLAL: *Vague ideals and Normal vague Ideals in semi rings*, Int. journal of computational cognition, **6**(3) (2008), 33–44.
- [7] B. NAGESWARARAO, N. RAMAKRISHNA, T. ESWARLAL: *Translates of vague  $\beta$ -algebra*, International Journal of Pure and Applied Mathematics, **106**(4)(2016), 989–1002.
- [8] B. NAGESWARARAO, N. RAMAKRISHNA, T. ESWARLAL: *Application of translates of vague sets on electoral-democracy model*, International Journal of Chemical Sciences, **14**(2) (2016), 762–768.
- [9] B. NAGESWARARAO, N. RAMAKRISHNA, T. ESWARLAL: *Application of translates of vague sets in carrier decision making*, International Journal of Chemical Sciences, **14**(1) (2016), 372–380.
- [10] B. NAGESWARARAO, N. RAMAKRISHNA, T. ESWARLAL: *Translates of vague groups*, International Journal of Pure and Applied Mathematical Sciences, **7**(2) (2014), 217–226.
- [11] N. RAMAKRISHNA: *A Characterization of Cyclic Groups in Terms of Vague Groups*, Int. journal of computational cognition, **6**(2) (2008), 106–115.

- [12] S. RAGAMAYI, Y. BHARGAVI: *A Study of Vague Gamma- Nearrings*, International journal of Science and Technology Research, **9**(1) (2020), 3960–3963.
- [13] S. RAGAMAYI, Y. BHARGAVI: *Some Results on Homomorphism of Vague Ideal of A Gamma-Nearring*, International journal of Science and Technology Research, **9**(1) (2020), 3972–3975.
- [14] V. AMARENDRA BABU, T. ANITHA, R. RAMYA SWETHA: *Introduction to vague topology*, Journal of Advanced Research in Dynamical and Control Systems, **9**(18) (2017), 768–780.
- [15] V. AMARENDRA BABU, T. ANITHA: *Vague prime LI-ideals of lattice implication algebras*, International Journal of Pure and Applied Mathematics, **107**(1) (2016), 35–48.
- [16] K. M. LEE: *Bipolar-valued fuzzy sets and their operations*, Proc. Int. conf. on intelligent Technologies, Bangkok, Thailand (2000), 307–312.
- [17] R. BISWAS: *Vague Groups*, Int. journal of computational cognition, **4**(2)(2006), 218–225.
- [18] L. A. ZADEH: *Fuzzy sets*, Information and Control, **8** (1965), 338–353.
- [19] W. L. GAU, D. J. BUEHRER: *Vague Sets*, IEEE Transaction on systems, Man and cybernetics, **23**(2) (1993), 610–614.

DEPARTMENT OF MATHEMATICS  
KONERU LAKSHMAIAH EDUCATION FOUNDATION  
VADDESWAREM, GUNTUR, AP ,INDIA. PINCODE-522502  
E-mail address: u.v.kalyani@gmail.com

DEPARTMENT OF MATHEMATICS  
KONERU LAKSHMAIAH EDUCATION FOUNDATION  
VADDESWAREM, GUNTUR, AP ,INDIA. PINCODE-522502  
E-mail address: eswarlal@kluniversity.in