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SOME CHALLENGING TRANSPORTATION PROBLEMS TO THE ASM METHOD

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ABSTRACT. In 1958, Reinfeld and Vogel developed a method known as Vogel's Approximation Method (VAM), which was considered as the most efficient algorithm, for more than five decades, for obtaining an Initial Basic Feasible Solution (IBFS) to the transportation problems (TPs), as it provides a good IBFS. In July 2012, Abdul Quddoos et al. developed a novel method called the ASM method and in June 2016, developed the Revised Version of the ASM method for obtaining the best IBFS to TPs, in the sense that, which is either optimal directly or close to the optimal solution, with minimum effort of mathematical calculations. Although the ASM method produces optimal solution to most of the TPs, in this paper, as an objective, we have identified 10 challenging TPs, in the logic that, as it produces only near optimal solution to the identified problems. Evidently, for evaluating the performance of any new method proposed on TP, the identified 10 challenging problems may be used for testing and validation of the new method proposed. It is the gained advantage of this paper.

1. INTRODUCTION

Transportation problems have been broadly studied in Operations Research and Computer Science. They play a vital role in logistics and supply-chain management for reducing the shipping cost and improving the service. In 1941,

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Hitchcock [6] developed the basic transportation problem along with the constructive method of solution and later in 1949 Koopmans [9] discussed the problem in detail. Again in 1951 Dantzig [4] formulated the transportation problem as linear programming problem and also provided the solution method. During 1960s, quite few methods such as North West Corner (NWC) Method, Least Cost Method (LCM) and Vogel's Approximation Method (VAM) [7, 21, 24, 26] have been established for finding the IBFS.

In the recent years quite a number of methods have been projected by several researchers to find the optimal solution for TPs. directly. But no single method is attaining optimal solution directly to all TPs. Among them, in July 2012, Abdul Quddoos et al. [2] proposed a new method, named ASM method, based on making allocations to zero entry cell of reduced cost matrix, for finding an optimal solution directly for a wide range of TPs. In October 2012, Mohammad Kamrul Hasan [11] proposed that direct methods (including ASM method) for finding optimal solution of a TP do not reflect optimal solution continuously. Murugesan [13] confessed and recognized the statement of Mohammad Kamrul Hasan by testing the ASM method for various benchmark problems. Meanwhile by doing further research, Abdul Quddoos et al.[1] encountered a few problems in which ASM method does not directly provide optimal solution to each and every problem, but provides a best IBFS, which is very close to optimal solution. One basic problem encountered was the unbalanced TP (UTP) in which an IBFS, not optimal but very close to optimal, was obtained. To overcome this problem, in July 2016, Abdul Quddoos et al. [1] presented a Revised Version of the ASM method, which provides optimal solution directly for most of the problems, and if not, it provides best IBFS. Murugesan et al. [14, 15, 16, 17, 18, 19] established Abdul Quddoos et al.'s claim by testing 30 benchmark instances of balanced category and 20 of unbalanced category.

In this paper, we have studied the performance of the Revised Version of ASM method (hereafter it is simply called as the ASM method) in depth and identified 10 classical benchmark problems for which the ASM method has produced only near optimal solutions. By this means, the identified 10 problems are termed as challenging problems to the ASM method.

The paper is organized as follows: Following the brief introduction in Section 1, in Section 2.1 step-by-step algorithm of the ASM method is presented. In Section 3, one benchmark problem from balanced type is illustrated by the ASM method. The identified 10 classical benchmark TPs, 5 from balanced type and another 5 from unbalanced type, of different sizes from some reputed journals published by several authors are shown in Section 4. Section 5 demonstrates the analysis of the results generated by the ASM method with the optimal solution, the number of iterations required to reach the optimal solution and the percentage of deviation of the generated solution from the optimal solution. Finally, in Section 6 conclusions are drawn. Balanced and Unbalanced Transportation Problem.

A transportation problem is said to be balanced if the total supply from all its sources equals the total demand in all its destinations, and is called unbalanced, otherwise.

Feasible Solution

A set of non-negative allocations $X_{ij} \geq 0$, which satisfies the row and column restrictions of a TP is known as a Feasible Solution to the TP.

Basic Feasible Solution

A feasible solution to a m-sources and n-destinations balanced TP (BTP) is said to be a Basic Feasible Solution if the number of positive allocations in it is exactly equal to (m + n -1) in number. In this case, it is called a Non-Degenerate Basic Feasible Solution; otherwise, it is called Degenerate Basic Feasible Solution.

Optimal Solution

A feasible solution (not necessarily basic) of a TP is said to be optimal if it minimizes the overall cost of transportation. There always exists an optimal solution to a balanced TP.

Optimality Test

Optimality test can be performed only if the generated solution is a non-degenerate one. Otherwise, optimality test cannot be performed. In case of the later, it can be made non-degenerate by adding enough number of positive allocations, with allocations say ε (a very small positive quantity), at suitable unallocated cells.

For performing optimality test, two methods namely, Stepping Stone Method and MODI Method [7, 20, 23, 25] are usually used, in which MODI Method is used by and large.

2. METHODOLOGY

As the performance of the ASM method is experienced on some identified classical benchmark problems, in this section, we describe the algorithm of the said method.

2.1. **Algorithm of the ASM method.** The stepwise procedure of the ASM method by Abdul Quddoos et al. [2, 3] is carried out as follows.

Step-1: Construct the transportation tableau from given TP. Check whether the problem is balanced or not. If the problem is balanced, go to Step 4, otherwise go to Step 2.

Step-2: If the problem is not balanced, then anyone of the following two cases may arise: a) If total supply exceeds total demand, introduce an additional dummy column to the transportation table to absorb the excess supply. The unit transportation cost for the cells in this dummy column is set to âĂŸM', where M > 0 is a very large but finite positive quantity.

or

b) If total demand exceeds total supply, introduce an additional dummy row to the transportation table to satisfy the excess demand. The unit transportation cost for the cells in this dummy row is set to âĂŸM', where M>0 is a very large but finite positive quantity.

Step-3:a) In case (a) of Step 2, identify the lowest element of each row and subtract it from each element of the respective row and then, in the resulting tableau, identify the lowest element of each column and subtract it from each element of the respective column and go to Step 5.

or

b) In case (b) of Step 2, identify the lowest element of each column and subtract it from each element of the respective column and then, in the resulting tableau, identify the lowest element of each row and subtract it from each element of the respective row and go to Step 5.

Step-4: Identify the lowest element of each row and subtract it from each element of the respective row and then, in the resulting tableau, identify the lowest element of each column and subtract it from each element of the respective column.

Step-5: In the reduced tableau, each row and each column contains at least one

TABLE 1. The given BTP

Sources	D1	D2	D3	D4	Supply
S1	7	5	9	11	30
S2	4	3	8	6	25
S3	3	8	10	5	20
S4	2	6	7	3	15
Demand	30	30	20	10	

zero. Now, select the first zero (say zero) and count the number of zeros (excluding the selected one) in the row and column and record as a subscript of selected zero. Repeat this process for all zeros in the transportation tableau.

Step-6: Now, choose the cell containing zero for which the value of subscript is minimum and supply maximum possible amount to that cell. If tie occurs for some zeros in Step 5, choose the cell of that zero for breaking tie such that the sum of all the elements in the row and column is maximum. Supply maximum possible amount to that cell.

Step-7: Delete that row (or column) for further consideration for which the supply from a given source is exhausted (or the demand for a given destination is satisfied). If, at any stage, the column demand is completely satisfied and row supply is completely exhausted simultaneously, then delete only one column(or row) and the remaining row (or column) is assigned a zero supply (or demand) in further calculation.

Step-8: Now, check whether the reduced tableau contains at least one zero in each row and each column. If this does not happen, repeat Step 4, otherwise go to Step 9.

Step-9: Repeat Step 5 to Step 8 till all the demands are satisfied and all the supplies are exhausted.

3. NUMERICAL ILLUSTRATION

The above said algorithm, for finding an IBFS to TPs, is illustrated by the following benchmark problem from the literature.

3.1. **Illustration : (Mollah Mesbahuddin Ahmedet al., 2016, [12]).** Consider the following cost minimizing BTP with four sources and four destinations:

TABLE 2. Generated Solution table due to ASM method

Sources	D1	D2	D3	D4	Supply
S1	5 / 7	5 / 5	20 / 9	11	30
S2	4	25 / 3	8	6	25
S3	20 / 3	8	10	5	20
S4	5 / 2	6	7	10 / 3	15
Demand	30	30	20	10	

3.2. **Solution by the ASM Method.** The given BTP has been solved using the procedure of ASM method. The IBFS, thus obtained is shown in Table 3.2. Table 3.2: Generated Solution table due to ASM method **Writing the Allocation Values:**

$X_{11} = 5, X_{12} = 5, X_{13} = 20, X_{22} = 25, X_{31} = 20, X_{41} = 5, X_{44} = 10$ and all other $X_{ij} = 0$. Note that the generated solution is a non-degenerate one as it contains exactly seven (m+n-1 = 4+4-1= 7) allocations.

Computing the Total Transportation Cost:

$$Z = (5 \times 7) + (5 \times 5) + (20 \times 9) + (25 \times 3) + (20 \times 3) + (5 \times 2) + (10 \times 3)$$
$$= 35 + 25 + 180 + 75 + 60 + 10 + 30 = 415.$$

Note that the generated solution by the ASM method is not an optimal one. By applying the MODI method, this solution has been improved towards optimality with Z = $410 in a single iteration. The optimal solution table due to the MODI method is shown in Table 3.3.

Writing the Allocation Values:

$X_{12} = 10, X_{13} = 20, X_{21} = 5, X_{22} = 20, X_{31} = 20, X_{41} = 5, X_{44} = 10$, and all other X_{ij} = 0. Note that the obtained optimal solution is also a non-degenerate one as it contains exactly seven (m+n-1 = 4+4-1= 7) allocations.

Computing the Total Transportation Cost:

$$Z = (10 \times 5) + (20 \times 9) + (5 \times 4) + (20 \times 3) + (20 \times 3) + (5 \times 2) + (10 \times 3)$$
$$= 50 + 180 + 20 + 60 + 60 + 10 + 30 = 410.$$

TABLE 3. Optimal Solution table due to the MODI Method

Sources	D1	D2	D3	D4	Supply
S1	7	10 / 5	20 / 9	11	30
S2	5 / 4	20 / 3	8	6	25
S3	20 / 3	8	10	5	20
S4	5 / 2	6	7	10 / 3	15
Demand	30	30	20	10	

4. NUMERICAL EXAMPLES

To justify the efficiency of the ASM method we have solved a good number of classical benchmark problems from balanced and unbalanced categories in different sizes, from various literature and books. And finally we have identified 10 classical benchmark problems, 5 from balanced category and another 5 from unbalanced category, as challenging problems to the ASM method. Those 10 problems are listed in Table 4.1.

5. RESULT ANALYSIS

For evaluating the performance of the ASM method, simulation experiments were carried out on balanced and unbalanced categories of TPs. The main purpose of the experiment was to evaluate the effectiveness of the IBFS generated by the ASM method by comparing it with the optimal solution. For the identified 10 challenging TPs, the solution generated by the ASM method, the optimal solution obtained through the MODI method, and the number of iterations required to reach the optimal solution are shown in Table 5.

From Table 5, we see that in case of BTPs as well as UTPs, the ASM method has produced only near optimal solution to all the problems tested. The least iteration number to reach the optimality represents the closeness level of the solution generated.

TABLE 4. Some classical and challenging TPs to the ASM method

BTP	UTP
Problem No., (Author(s), Year, [Ref. No.])	**Problem No.,(Author(s), Year, [Ref. No.])**
Problem 1 (Ahmed M.M. et al., 2014, [3]) [C] 4×4= [7 5 9 11; 4 3 8 6; 3 8 10 5; 2 6 7 3] [S] 4×1= [30, 25, 20, 15] [D] 1×4= [30, 30, 20, 10]	**Example 1 (Nagarajan Balakrishnan, 1990 [20])** [C] 3×3= [6 10 14; 12 19 21; 15 14 17] [S] 3×1= [50, 50, 50] [D] 1×3= [30, 40, 55]
Problem 2 (Mhlanga A, 2014, [10]) [C] 4×5= [4 9 8 10 12;6 10 3 2 3;3 2 7 10 3; 3 5 5 4 8] [S] 4×1= [24, 18, 20, 16] [D] 1×5= [10, 20, 10, 18, 20]	**Example 2 (Deshmukh, 2012, [5])** [C] 3×4= [19 30 50 10; 70 30 40 60; 40 8 70 20] [S] 3×1= [7, 9, 18] [D] 1×4= [40, 8, 7, 14]
Problem 3 (Khan A.R. et al., 2015, [8]) [C] 4×5= [25 14 34 46 45; 10 47 14 20 4; 22 42 38 21 46; 36 20 41 38 44] [S] 4×1= [27, 35, 37, 45] [D] 1×5= [22, 27, 28, 33, 34]	**Example 3 (Kanti Swarup et al., 2017, [7])** [C] 3×4= [42 48 38 37; 40 49 52 51; 39 38 40 43] [S] 3×1= [160, 150, 190] [D] 1×4= [80, 90, 110, 160]
Problem 4 (Shweta G.C., et al., 2012, [25]) [C] 5×5= [68 35 4 74 15; 57 88 91 3 8; 91 60 75 45 60; 52 53 24 7 82; 51 18 82 13 7] [S] 5×1= [18, 17, 19, 13, 15] [D] 1×5= [16, 18, 20, 14, 14]	**Example 4 (Abdul Quddoos et al., 2016, [1])** [C] 4×3= [2 7 14; 3 3 1; 5 4 7; 1 6 2] [S] 4×1= [5, 8, 7, 15] [D] 1×3= [7, 9, 18]
Problem 5 (Russell E.J., 1969, [22]) [C] 5×5= [73 40 9 79 20; 62 93 96 8 13; 96 65 80 50 65; 57 58 29 12 87; 56 23 87 18 12] [S] 5×1= [8, 7, 9, 3, 5] [D] 1×5= [6, 8, 10, 4, 4]	**Problem 5 (Sen et al., 2010, [23])** [C] 5×4 = [60 120 75 180; 58 100 60 165; 62 110 65 170; 65 115 80 175; 70 135 85 195] [S] 5×1= [8000, 9200, 6250, 4900, 6100] [D] 1×4= [5000, 2000, 10000, 6000]

TABLE 5. Comparison of solution by ASM method with Optimal Solution.

Balanced TP ←				Prob. No.	→ Unbalanced TP			
Least iteration number	Soln. by ASM	Opt. Soln.	Size		Size	Opt. Soln.	Soln. by ASM	Least iteration number
1	415	410	4 × 4	**1.**	3 × 3	1655	1695	1
3	322	316	4 × 5	**2.**	3 × 4	743	779	1
2	3064	2965	4 × 5	**3.**	3 × 4	17050	17060	1
1	2213	2202	5 × 5	**4.**	4 × 3	75	79	1
1	1103	1102	5 × 5	**5.**	5 × 4	2146750	2164000	4

6. CONCLUSION

In this paper, we have studied the performance of the ASM method in greatness and identified 10 classical benchmark problems for which the ASM method has produced only near optimal solutions. By this way, the identified 10 problems are termed as challenging problems to the ASM method. Therefore, it is established and recognized that the ASM method is the best one for finding a very efficient IBFS only and not a method for generating the optimal solution directly. Further, the most attractive feature of this method is that it requires only simple arithmetical and logical calculations and hence anyone can easily understand and apply it far better than other methods. Also, this method will be more cost-effective for those decision makers who are trading with logistics and supply chain problems. Moreover, for evaluating the performance of any new method proposed on TPs, the listed 10 challenging problems may be used for testing and validation of the new method proposed. It is the gained advantage of this paper.

Other valuable references on the topic are [5,8,10,22].

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