Advances in Mathematics: Scientific Journal **9** (2020), no.6, 3229–3235
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.6.3

# THE LEAST MONOPOLY DISTANCE ENERGY OF FUZZY GRAPH

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ABSTRACT. In this paper, we present monopoly fuzzy graph, size and cardinality. The idea of least monopoly distance energy of fuzzy graph was discussed and also computed lower and upper bound. Some examples have been illustrated.

## 1. INTRODUCTION

The idea of fuzzy sets and fuzzy relations was introduced by L.A.Zadeh in 1965 [4] and it has found the applications in the analysis of cluster patterns. The spectrum of a graph initially showed up in a paper by Collatz also, Sinogowitz in 1957. The idea of fuzzy graph was presented by Rosenfeld [5] in 1975. The energy of fuzzy graph of a matrix G is the sum of absolute value of eigenvalues.

In 2013 Khoshkhak et al presented the idea of monopoly in graphs. In 1978 I. Gutman presented the idea of energy of graph. A Study of Monopolies in Graphs is presented by K. Khoshkhah, M. Nemati, H. Soltani, M. Zaker [1]. In 2015 [2, 3] Ahmed Mohammed Naji and N. D. Soner described the The Minimum Monopoly Energy of a Graph and also the Independent Monopoly Size in Graphs. A graph G is represented by m edges and n vertices. Let G be a fuzzy graph, a subset D of V is called monopoly fuzzy set in G if every vertex has at

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2010 *Mathematics Subject Classification.* 05C50, 05C72, 05C35.
*Key words and phrases.* Monopoly Set, Monopoly Size, Minimum Monopoly Matrix, Minimum Monopoly Values, Minimum Monopoly Energy of a Graph.

least neighbors in D. The size of monopoly is denoted by mo(G) and the least cardinality of a monopoly fuzzy set among all monopoly fuzzy set in G.

## 2. LEAST MONOPOLY DISTANCE ENERGY OF FUZZY GRAPH

We followed the fuzzy graph as mentioned in [5].

**Definition 2.1.** *The least monopoly distance fuzzy matrix of G is denoted by*

$$A_M(G) = \begin{cases} 1 & \text{if } i = j \text{ and } v_i \in M \\ d(e_{ij}) & \text{if } v_iv_j \in E \\ min(d(e_{ik} + d(e_{kj}) & \text{if } v_iv_j \notin E, v_iv_k \in E \text{ and } v_kv_j \in E \\ 0 & \text{otherwise} . \end{cases}$$

**Example 1.** *Let $v_1, v_2, v_3$ be the set of vertices in fuzzy graph G and $D = v_3$ is the least monopoly set in G and the size of fuzzy monopoly is denoted by $\mu(mo(G)) = 1$.*

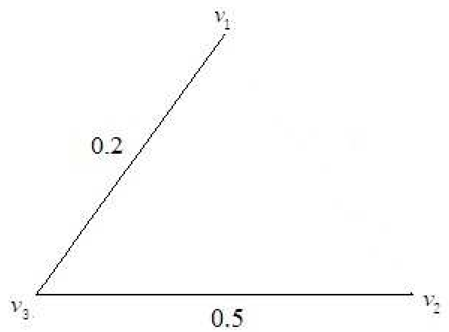


FIGURE 1. Fuzzy graph

$$A_M(G) = \begin{pmatrix} 0 & 0.7 & 0.2 \\ 0.7 & 0 & 0.5 \\ 0.2 & 0.5 & 1 \end{pmatrix}$$

$Spec((\mu_{ij}(G))) = 1.3812, -0.7288, 0.3476$

$E((\mu_{ij})) = 2.4576$

## 3. Bounds on least monopoly distance energy of fuzzy graph

**Theorem 3.1.** *Let G be a connected fuzzy graph with n vertices and m edges. Then*

$$\sqrt{2m+2s+\mu(mo(G))} \leq E(\mu_{ij}(G)) \leq \sqrt{n(2m+2s+\mu(mo(G)))}\,.$$

*Proof.* Examine the Cauchy schwartiz inequality
$(E(\mu_{ij}(G)))^2 = (\sum_{i=1}^{n} a_i b_j)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right)(\sum_{i=1}^{n} b_j^2)$
Let $a_i = 1andb_i = |\zeta_i|$, then:

$$\begin{aligned}
(E(\mu_{ij}(G)))^2 &\leq \left(\sum_{i=1}^{n} 1\right)\left(\sum_{i=1}^{n} |\zeta_i|^2\right)\\
&= n\left(\sum_{i=1}^{n}\sum_{i=1}^{n} a_{ij}a_{ji}\right)\\
&= n\left(\sum_{i=1}^{n} a_{ij}^2 + \sum_{i\neq j}^{n} a_{ij}a_{ji}\right)\\
&= n\left(\sum_{i=1}^{n} a_{ij}^2 + 2\sum_{i<j}^{n} a_{ij}^2\right)\\
&= n\left(\mu(mo(G)) + 2\sum_{1\leq i<j\leq n} d^2(a_i,a_j)\right)\\
&= n\left(\mu(mo(G)) + 2m + 2s\right)\ ,
\end{aligned}$$

where $s = \sum_{i<j, d(a_i,a_j neq 1} d^2(a_i,a_j)$ i.e.,$(E(\mu_{ij}(G)))^2 \leq n(\mu(mo(G)+2m+2s)\,.$
Lower Bound:

$$\begin{aligned}
(E(\mu_{ij}(G)))^2 &= \left(\sum_{i=1}^{n} |\zeta_i|^2\right)\\
&= \sum_{i=1}^{n} |\zeta_i|^2\\
&\geq \mu(mo(G)) + 2m + 2s\,,
\end{aligned}$$

i.e.,$(E(\mu_{ij}(G)))^2 \geq \mu(mo(G)+2m+2s\,.$ □

**Example 2.** *Let G be a fuzzy star graph with 5 vertices and 4 edges.* $D = \{c\}$ *is the*

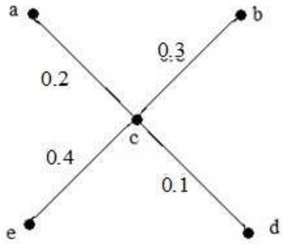


FIGURE 2. Fuzzy star graph

*least monopoly set in G and the size of fuzzy monopoly is denoted by* $\mu(mo(G)) = 1$.

$$A(\mu_{ij}(G)) = \begin{pmatrix} 0 & 0.5 & 0.2 & 0.3 & 0.6 \\ 0.5 & 0 & 0.3 & 0.4 & 0.7 \\ 0.2 & 0.3 & 1 & 0.1 & 0.4 \\ 0.3 & 0.4 & 0.1 & 0 & 0.5 \\ 0.6 & 0.7 & 0.4 & 0.5 & 0 \end{pmatrix}$$

$Spec((\mu_{ij}(G))) = 1.8496, -0.7379, 0.6951, -0.5160, -0.2907$,
$E((\mu_{ij})) = 4.0893$.
*The bounds of fuzzy graph is denoted as* $3.5777 \leq 4.0893 \leq 8$.

**Example 3.** *Let G be a fuzzy double star graph with 8 vertices and 7 edges.* $D = \{c, d\}$ *is the least monopoly set in G and the size of fuzzy monopoly is denoted*

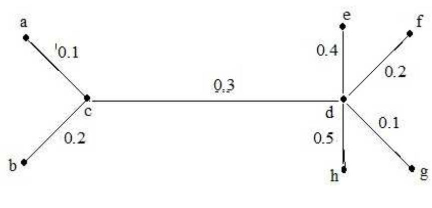


FIGURE 3. Fuzzy double star graph

*by* $\mu(mo(G)) = 2$.

$$A(\mu_{ij}(G)) = \begin{pmatrix} 0 & 0.3 & 0.1 & 0.5 & 0.8 & 0.6 & 0.5 & 0.9 \\ 0.3 & 0 & 0.2 & 0.5 & 0.9 & 0.7 & 0.6 & 1 \\ 0.1 & 0.2 & 1 & 0.3 & 0.7 & 0.5 & 0.4 & 0.8 \\ 0.5 & 0.5 & 0.3 & 1 & 0.4 & 0.2 & 0.1 & 0.5 \\ 0.8 & 0.9 & 0.7 & 0.4 & 0 & 0.6 & 0.5 & 0.9 \\ 0.6 & 0.7 & 0.5 & 0.2 & 0.6 & 0 & 0.3 & 0.8 \\ 0.5 & 0.6 & 0.4 & 0.1 & 0.5 & 0.3 & 0 & 0.6 \\ 0.9 & 1 & 0.8 & 0.5 & 0.9 & 0.8 & 0.6 & 0 \end{pmatrix}$$

$Spec((\mu_{ij}(G))) = 4.0127, -1.09826 + 0.1449i, -1.09826 - 0.1449i, 0.8465,$
$-0.65528, 0.5750, 0.3168, -0.2656$
$E((\mu_{ij})) = 8.8872$ .

*The Bounds of fuzzy graph is denoted as* $6.6483 \leq 8.8872 \leq 18.8042$ .

**Example 4.** *Let G be a fuzzy double star graph with 9 vertices and 8 edges.*
$D = \{b, c, d\}$ *is the least monopoly set in G and the size of fuzzy monopoly is*

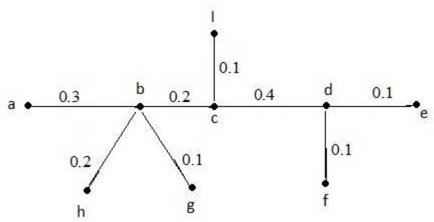


FIGURE 4. Fuzzy caterpillar graph

*denoted by* $\mu(mo(G)) = 3$.

$$A(\mu_{ij}(G)) = \begin{pmatrix} 0 & 0.3 & 0.5 & 0.9 & 1 & 1 & 0.4 & 0.5 & 0.6 \\ 0.3 & 1 & 0.2 & 0.6 & 0.7 & 0.7 & 0.1 & 0.2 & 0.3 \\ 0.5 & 0.2 & 1 & 0.4 & 0.5 & 0.5 & 0.3 & 0.4 & 0.1 \\ 0.9 & 0.6 & 0.4 & 1 & 0.1 & 0.1 & 0.7 & 0.8 & 0.5 \\ 1 & 0.7 & 0.5 & 0.1 & 0 & 0.2 & 0.8 & 0.9 & 0.6 \\ 1 & 0.7 & 0.5 & 0.1 & 0.2 & 0 & 0.8 & 0.9 & 0.6 \\ 0.4 & 0.1 & 0.3 & 0.7 & 0.8 & 0.8 & 0 & 0.3 & 0.4 \\ 0.5 & 0.2 & 0.4 & 0.8 & 0.9 & 0.9 & 0.3 & 0 & 0.5 \\ 0.6 & 0.3 & 0.1 & 0.5 & 0.6 & 0.6 & 0.4 & 0.5 & 0 \end{pmatrix}$$

$Spec((\mu_{ij}(G))) = 4.4855, -2.1090, 0.8653, 0.7285, -0.5862,$
$0.56046, -0.4639, -0.2806, -0.2$
$E((\mu_{ij})) = 10.2794$
*The Bounds of fuzzy graph is denoted as* $7.46993 \leq 10.2794 \leq 22.4098$.

**Example 5.** *Let G be a fuzzy double star graph with 8 vertices and 10 edges.* $D = \{b, c, f, g\}$ *is the least monopoly set in G and the size of fuzzy monopoly is*

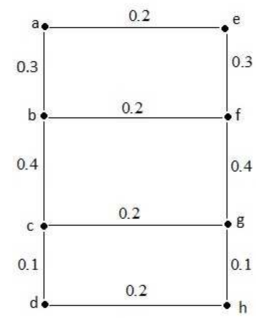


FIGURE 5. Fuzzy ladder graph

*denoted by* $\mu(mo(G)) = 4$.

$$A(\mu_{ij}(G)) = \begin{pmatrix} 0 & 0.3 & 0.7 & 0.8 & 0.2 & 0.5 & 0.9 & 1 \\ 0.3 & 1 & 0.4 & 0.5 & 0.5 & 0.2 & 0.6 & 0.7 \\ 0.7 & 0.4 & 1 & 0.1 & 0.9 & 0.6 & 0.2 & 0.3 \\ 0.8 & 0.5 & 0.1 & 0 & 1 & 0.7 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.9 & 1 & 0 & 0.3 & 0.7 & 0.8 \\ 0.5 & 0.2 & 0.6 & 0.7 & 0.3 & 1 & 0.4 & 0.5 \\ 0.9 & 0.6 & 0.2 & 0.3 & 0.7 & 0.4 & 1 & 0.1 \\ 1 & 0.7 & 0.3 & 0.2 & 0.8 & 0.5 & 0.1 & 0 \end{pmatrix}$$

$Spec((\mu_{ij}(G))) = 4.1176, -1.9488, 1, 0.7403, -0.5403, 0.5248, 0.1063, -1.7064 * 10^{-16}$
$E((\mu_{ij})) = 8.9781$
*The Bounds of fuzzy graph is denoted as* $7.26636 \leq 8.9781 \leq 20.55237$ .

## 4. CONCLUSION

In this paper, we presented the idea of independence monopoly size in fuzzy graphs and also lower and upper bound of least monopoly distance energy of fuzzy graph.

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