

APPLICATIONS OF SOFT DENSE SETS TO SOFT CONTINUITY

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ABSTRACT. In this paper, we introduce the concept of soft dense subsets. We give characterizations of soft continuity in terms of soft dense subsets and in terms of convergent sequences of soft maps taken from soft dense subsets. We obtain (i) an extension theorem for a soft continuous map which is defined on soft dense subsets of the domain (ii) soft continuity of a soft map in terms of the soft continuity of its restrictions to members of a soft cover of the domain having soft dense intersection.

1. INTRODUCTION

Most of the complex problems in engineering, computer sciences, medical sciences, environment etc. have various uncertainties which can not be solved by classical methods. To describe and extract the useful information hidden in uncertain data, researchers in mathematics, computer science and related areas have proposed a number of theories such as intuitionistic fuzzy set theory, fuzzy set theory and rough set theory. But all these theories have their own limitations as pointed out in [6], the inadequacy of the parametrization tool of the theories is possibly, the reason for these limitations. To deal with such type of vagueness and uncertainties, Molodtsov in [6], introduced a new approach called soft set theory which are free from the difficulties affecting existing methods and started to develop the basics of the corresponding theory. There has been a rapid growth

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of interest in soft set theory and its application in recent years. Xiao et al. [14] and Pei and Miao in [10] discussed the relationship between soft sets and information systems. A soft set can be represented by Boolean-valued information system as we may see the structure of a soft set can classify the objects into two classes (1 or 0) and so it can be used to represent a data set. The theory of soft sets has been applied to data analysis and decision support system.

Shabir and Naz [12] initiated the study of soft topological space and later, a lot of work about soft topological spaces has been done by various authors in [12], [1], [5] etc. In these studies, the concept of soft points were studied in various forms. In this paper, we use the concept of soft points defined in [8]. Das and Samanta in [3], introduced the notion of soft real sets and soft real numbers. In [4], they introduced the concept of soft metric spaces by using the notion of soft points and investigated some of its basic properties. In [9], maps between metric spaces and dense subsets in metric spaces are studied. Also, some map gluing theorems which describe the continuity of maps between metric spaces in terms of dense subsets of the domain were obtained. Characterizations of continuity of a map between metric spaces, in terms of convergent sequences taken from dense subset of the domain of the map and in terms of restriction of the map to a dense subset of the domain are given. The above study of maps between metric spaces lead us to study of soft maps and soft dense subsets in soft metric spaces.

In section 2, we recall the basic concept related to soft metric spaces. In section 3, we first define soft dense subsets and give characterization of soft continuous maps between soft metric spaces in terms of soft dense subsets of domain space [Theorem 3.2 below]. We also give characterization of soft continuity in terms of convergent sequence of soft points in soft dense subset of domain of the soft map [Theorem 3.5 below]. As corollaries to Theorem 3.5 we obtain (i) an extension theorem for a soft continuous map which is defined on soft dense subsets of the domain [Corollary 3.1 below], (ii) soft continuity of a soft map in terms of the soft continuity of its restrictions to members of a soft cover of the domain having soft dense intersection [Corollary 3.2].

2. PRELIMINARIES

Zorlutuna, Min and Atmaca studied soft topological spaces in [13] and introduced some important notions related to soft sets which we shall use in our results in this paper. Let A be the set of parameters. A soft set $F : A \rightarrow P(X)$, over X is defined as a parameterized family of subsets of the universe X denoted by (F, A) or F_A and a soft real set is a mapping $F : A \rightarrow \mathfrak{B}(\mathbf{R})$, where $\mathfrak{B}(\mathbf{R})$ be the collection of all non-empty bounded subsets of real numbers \mathbf{R} . Detailed study of soft real numbers and related concepts is given in [3] and [4] where the definition of soft continuity between soft metric spaces is defined in [7]. Throughout the paper, \tilde{X} and $(\tilde{X}, \tilde{d}, A)$ will denote the absolute soft set and soft metric space with soft metric \tilde{d} respectively.

Definition 2.1. [2] A soft metric space $(\tilde{X}, \tilde{d}, A)$ is called soft sequential compact metric space if every soft sequence has a soft subsequence that converges in \tilde{X} .

The following results will be utilized in this paper.

Theorem 2.1. [4] Let $(\tilde{Y}, \tilde{d}_Y, A)$ be a soft metric subspace of a soft metric space $(\tilde{X}, \tilde{d}, A)$ and $P_a^x \tilde{\in} (Y, A)$. Then for any soft open ball $SS(B(P_a^x, \tilde{r}))$ in \tilde{X} , $SS(B(P_a^x, \tilde{r})) \cap (Y, A)$ is soft open ball in $(\tilde{Y}, \tilde{d}_Y, A)$, and also any soft open ball in $(\tilde{Y}, \tilde{d}_Y, A)$ is obtained as the intersection of a soft open ball in \tilde{X} with (Y, A) .

Theorem 2.2. [7] A soft mapping (φ, e) is soft continuous at the soft point $P_a^x \in \tilde{X}$ if and only if for every sequence of soft points $\{P_{a_n}^{x_n}\}_n$ converging to the soft point P_a^x in the soft metric space $(\tilde{X}, \tilde{d}, A)$, the sequence $\{(\varphi, e)(P_{a_n}^{x_n})\}_n$ in $(\tilde{Y}, \tilde{\rho}, B)$ converges to a soft point $(\varphi, e)(P_a^x)$ in \tilde{Y} .

Theorem 2.3. [11] Let $(\tilde{X}, \tilde{d}, A)$ be a soft metric space. A soft point $P_a^x \tilde{\in} \tilde{X}$ is a soft limit point of (F, A) if and only if there is a soft sequence of soft points in (F, A) converging to P_a^x .

3. RESULTS

In this section, we study soft continuity of a soft map between soft metric spaces in terms of soft dense subsets of the domain space. We begin by introducing the following definition of soft dense subset of \tilde{X} . Throughout this paper, (Z, A) will denote an arbitrarily fixed soft dense subset of \tilde{X} .

Definition 3.1. A soft set (Z, A) of \tilde{X} will be called soft dense in \tilde{X} if $\overline{(Z, A)} = \tilde{X}$. In other words, for every $P_a^x \in \tilde{X}$ and $\tilde{\epsilon} \succ \bar{0}$ there exists soft point $P_b^y \in (Z, A)$ such that $\tilde{d}(P_b^y, P_a^x) \prec \tilde{\epsilon}$.

Our first theorem follows directly from Theorem 2.3 above:

Theorem 3.1. A soft set (Z, A) of \tilde{X} is said to be soft dense in \tilde{X} if and only if for every $P_a^x \in \tilde{X}$ there exists a sequence $\{P_{a_n}^{d_n}\}_n$ of soft points in (Z, A) converging to P_a^x .

Following theorem gives characterization of soft continuous maps in soft metric spaces in terms of soft dense subsets of domain space:

Theorem 3.2. Let $(\tilde{X}, \tilde{d}, A)$ and $(\tilde{Y}, \tilde{\rho}, B)$ be two soft metric spaces. A soft mapping $(\varphi, e) : (\tilde{X}, \tilde{d}, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ is soft continuous if and only if $(\varphi, e)|_{(Z, A)}$ is soft continuous and (φ, e) is soft continuous at each point of $(Z, A)^c$ where (Z, A) is soft dense in \tilde{X} .

Proof. Let $P_a^x \in (Z, A)$ and suppose (φ, e) is not soft continuous at P_a^x . Then $(\varphi, e)|_{(Z, A)}$ is soft continuous implies there exists an $\tilde{\epsilon} \succ \bar{0}$ and a sequence $\{P_{a_n}^{x_n}\}_n$ in $(Z, A)^c$ such that $P_{a_n}^{x_n} \rightarrow P_a^x$ but $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_a^x)) \not\prec \tilde{\epsilon}$ for every n . As (Z, A) is soft dense in \tilde{X} , for every $P_a^x \in \tilde{X}$ and $\tilde{\epsilon} \succ \bar{0}$ there exist $P_b^y \in (Z, A)$ such that $\tilde{d}(P_b^y, P_a^x) \prec \tilde{\epsilon}$. Now since (φ, e) is soft continuous at each point of $P_{a_n}^{x_n} \in (Z, A)^c$ and (Z, A) is soft dense in \tilde{X} then for each n , there exists a soft point $P_{b_n}^{y_n} \in (Z, A)$ such that $\tilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \prec \frac{\bar{1}}{n}$ and $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n})) \prec \frac{\bar{1}}{n}$. By $P_{a_n}^{x_n} \rightarrow P_a^x$ and $\tilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \prec \frac{\bar{1}}{n}$, we get $P_{b_n}^{y_n} \rightarrow P_a^x$. Since $(\varphi, e)|_{(Z, A)}$ is soft continuous at P_a^x , it follows that $(\varphi, e)(P_{b_n}^{y_n}) \rightarrow (\varphi, e)(P_a^x)$ and so $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n})) \prec \frac{\bar{1}}{n}$ implies that $(\varphi, e)(P_{a_n}^{x_n}) \rightarrow (\varphi, e)(P_a^x)$. This contradicts our assumption that $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_a^x)) \not\prec \tilde{\epsilon}$ for every n . Hence (φ, e) is soft continuous at P_a^x and therefore, (φ, e) is soft continuous. \square

The proof of the following theorem, on soft continuity of soft maps between soft metric space, follows from above theorem and easily proved fact that if $(\varphi, e)|_{(F, A)}$ is soft continuous and (F, A) is soft open then (φ, e) is soft continuous at each soft point of (F, A) :

Theorem 3.3. Let $(\tilde{X}, \tilde{d}, A)$ and $(\tilde{Y}, \tilde{\rho}, B)$ be two soft metric spaces. Let $(\varphi, e) : (\tilde{X}, \tilde{d}, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ be a soft mapping and $\tilde{X} = (F, A) \cup (G, A)$ where (F, A) is

soft open and (G, A) is soft dense in \tilde{X} . Then (φ, e) is soft continuous if $(\varphi, e)|_{(F, A)}$ and $(\varphi, e)|_{(G, A)}$ are both soft continuous.

The following theorem is a map gluing theorem on soft continuity of a soft map between soft metric spaces:

Theorem 3.4. Let $(\tilde{X}, \tilde{d}, A)$ and $(\tilde{Y}, \tilde{\rho}, B)$ be two soft metric spaces. Let $(\varphi, e) : (\tilde{X}, \tilde{d}, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ be a soft mapping and $\tilde{X} = (M, A) \tilde{\cup} (N, A)$ where $(M, A) \tilde{\cap} (N, A)$ is soft dense in \tilde{X} . Then (φ, e) is soft continuous if $(\varphi, e)|_{(M, A)}$ and $(\varphi, e)|_{(N, A)}$ are both soft continuous.

Proof. Let $P_a^x \tilde{\in} \tilde{X}$. Without loss of generality, we may assume that $P_a^x \tilde{\in} (M, A)$. Let (φ, e) is not soft continuous at P_a^x then there exist $\tilde{\epsilon} \succ \bar{0}$ and a sequence $\{P_{a_n}^{x_n}\}_n$ in \tilde{X} such that $P_{a_n}^{x_n} \rightarrow P_a^x$ but $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_a^x)) \not\geq \tilde{\epsilon}$ for every n . Since, $(\varphi, e)|_{(M, A)}$ is soft continuous at P_a^x , this sequence $\{P_{a_n}^{x_n}\}_n$ can be taken to be in (N, A) . Now as $(\varphi, e)|_{(N, A)}$ is soft continuous at each $P_{a_n}^{x_n}$ and $(M, A) \tilde{\cap} (N, A)$ is soft dense in \tilde{X} , there exists, for each n , a soft point $P_{b_n}^{t_n} \tilde{\in} (M, A) \tilde{\cap} (N, A)$ such that $\tilde{d}(P_{a_n}^{x_n}, P_{b_n}^{t_n}) \prec \frac{\bar{1}}{n}$ and $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{t_n})) \prec \frac{\bar{1}}{n}$. Therefore, $P_{a_n}^{x_n} \rightarrow P_a^x$ and $\tilde{d}(P_{a_n}^{x_n}, P_{b_n}^{t_n}) \prec \frac{\bar{1}}{n}$ implies $P_{b_n}^{t_n} \rightarrow P_a^x$. Again since $(\varphi, e)|_{(M, A)}$ is soft continuous at P_a^x , $(\varphi, e)(P_{b_n}^{t_n}) \rightarrow (\varphi, e)(P_a^x)$ and therefore by, $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{t_n})) \prec \frac{\bar{1}}{n}$ we get $(\varphi, e)(P_{a_n}^{x_n}) \rightarrow (\varphi, e)(P_a^x)$, which contradicts our assumption $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_a^x)) \not\geq \tilde{\epsilon}$ for every n . Hence (φ, e) is soft continuous at P_a^x . \square

Next, we give the following characterization of soft continuity of a soft map in terms of convergent sequence of soft points taken from a soft dense subset of domain of the soft map:

Theorem 3.5. Let $(\tilde{X}, \tilde{d}, A)$ and $(\tilde{Y}, \tilde{\rho}, B)$ be two soft metric spaces. For a soft mapping, $(\varphi, e) : (\tilde{X}, \tilde{d}, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$, the following conditions are equivalent.

- (1) (φ, e) is soft continuous.
- (2) $(\varphi, e)|_{(Z, A)} : (\tilde{Z}, \tilde{d}_Z, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ is soft continuous and (φ, e) is soft continuous at each point of $(Z, A)^c$.
- (3) for any sequence of soft points $\{P_{a_n}^{x_n}\}_n$ in (Z, A) converging to a soft point $P_a^x \tilde{\in} \tilde{X}$ implies $\{(\varphi, e)(P_{a_n}^{x_n})\}_n$ converges to $(\varphi, e)(P_a^x)$.

Proof. (1) \Leftrightarrow (2) by Theorem 3.2

(2) \Rightarrow (3) is obvious.

(3) \Rightarrow (1) Suppose $\{P_{a_n}^{x_n}\}_n$ be a sequence of soft points in \tilde{X} such that $P_{a_n}^{x_n} \rightarrow P_a^x$ where $P_a^x \in \tilde{X}$. Since (Z, A) is soft dense in \tilde{X} , for each n , there is sequence $\{P_{a_{n_k}}^{d_{n_k}}\}$ of soft points in (Z, A) converging to $P_{a_n}^{x_n} \in \tilde{X}$. Therefore by (3), $(\varphi, e)(P_{a_{n_k}}^{d_{n_k}}) \rightarrow (\varphi, e)(P_{a_n}^{x_n})$. Then for each n , there exist a positive integers $k_1(n)$ and $k_2(n)$ such that $\tilde{d}(P_{a_{n_k}}^{d_{n_k}}, P_{a_n}^{x_n}) < \frac{1}{n}$ for all $k \geq k_1(n)$ and $\tilde{\rho}((\varphi, e)(P_{a_{n_k}}^{d_{n_k}}), (\varphi, e)(P_{a_n}^{x_n})) < \frac{1}{n}$ for all $k \geq k_2(n)$. Let $k(n) = \max\{k_1(n), k_2(n)\}$, $\tilde{d}(P_{a_{n_k(n)}}^{d_{n_k(n)}}, P_{a_n}^{x_n}) \rightarrow 0$ and $\tilde{\rho}((\varphi, e)(P_{a_{n_k(n)}}^{d_{n_k(n)}}), (\varphi, e)(P_{a_n}^{x_n})) \rightarrow 0$ as $n \rightarrow \infty$. Now as $P_{a_n}^{x_n} \rightarrow P_a^x$, we get $P_{a_{n_k(n)}}^{d_{n_k(n)}} \rightarrow P_a^x$. Therefore, by (3) again, $(\varphi, e)(P_{a_{n_k(n)}}^{d_{n_k(n)}}) \rightarrow (\varphi, e)(P_a^x)$ and so $\tilde{\rho}((\varphi, e)(P_{a_{n_k(n)}}^{d_{n_k(n)}}), (\varphi, e)(P_{a_n}^{x_n})) \rightarrow 0$ implies that $(\varphi, e)(P_{a_n}^{x_n}) \rightarrow (\varphi, e)(P_a^x)$. Hence (φ, e) is soft continuous. \square

By using above theorem we get the following corollary.

Corollary 3.1. Let $(\varphi, e) : (\tilde{Z}, \tilde{d}_z, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ be a soft continuous mapping where (Z, A) is soft dense in \tilde{X} . Then there exist a largest soft subset (W, A) of \tilde{X} , $(Z, A) \subseteq (W, A)$ such that (φ, e) can be extended to a soft map $(f, e) : (\tilde{W}, \tilde{d}_w, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ which is soft continuous.

Proof. Let $(Z', A) = (Z, A) \cup P_a^x$ and $(W, A) = \bigcup_{P_a^x \in S} P_a^x$ where S be set of all soft points P_a^x in \tilde{X} for which there exist a soft continuous extension of $(\varphi, e), (f_x, e) : (\tilde{Z}', \tilde{d}_{z'}, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$. Obviously, $(Z, A) \subseteq (W, A)$. Let $(f, e) : (\tilde{W}, \tilde{d}_w, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ defined by $(f, e)(P_a^x) = (f_x, e)(P_a^x)$ is the extension of (φ, e) to (W, A) and $\{P_{a_n}^{d_{n_k}}\}_n$ be a sequence of soft points converging to P_a^x where $P_{a_n}^{d_{n_k}} \in (Z, A)$. As (f_x, e) is soft continuous, $(f, e)(P_{a_n}^{d_{n_k}}) = (f_x, e)(P_{a_n}^{d_{n_k}}) \rightarrow (f_x, e)(P_a^x) = (f, e)(P_a^x)$. By Theorem 3.5 (3), (f, e) is soft continuous as (Z, A) is soft dense in \tilde{X} . Now, Let (W_0, A) be another soft set such that $(Z, A) \subseteq (W_0, A)$ and $(f_0, e) : (\tilde{W}_0, \tilde{d}_{w_0}, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ be a soft continuous extension of (φ, e) then $(\varphi, e)|_{(Z', A)} : (\tilde{Z}', \tilde{d}_{z'}, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ is soft continuous for all $P_a^x \in (W_0, A)$ and so $(W_0, A) \subseteq (W, A)$ then (W, A) is largest soft subset of \tilde{X} where $(Z, A) \subseteq (W, A)$. \square

In the following corollary of above proved Theorem 3.2, we obtain the soft continuity of a soft map in terms of the soft continuity of its restrictions to members of a soft cover of the domain having soft dense intersection:

Corollary 3.2. Let $\{(F, A)_\alpha \mid \alpha \in \Lambda\}$ be soft cover of \tilde{X} , that is $\tilde{X} = \bigcup_{\alpha \in \Lambda} (F, A)_\alpha$ such that $\tilde{\cap}_\alpha (F, A)_\alpha$ is soft dense in \tilde{X} then a map $(\varphi, e) : (\tilde{X}, \tilde{d}, A) \rightarrow (\tilde{Y}, \tilde{\rho}, B)$ is soft continuous if $(\varphi, e)|_{(F, A)_\alpha}$ is soft continuous.

Proof. By Theorem 3.5, it is sufficient to prove that if the sequence $\{P_{a_n}^{d_n}\}_n$ of soft points in $\tilde{\cap}_\alpha (F, A)_\alpha$ converging to $P_a^x \tilde{\in} \tilde{X}$ then $(\varphi, e)(P_{a_n}^{d_n}) \longrightarrow (\varphi, e)(P_a^x)$. As $\{(F, A)_\alpha \mid \alpha \in \Lambda\}$ be soft cover of \tilde{X} , $P_a^x \tilde{\in} (F, A)_\alpha$ for some α , result follows from soft continuity of $(\varphi, e)|_{(F, A)_\alpha}$ at P_a^x . \square

4. CONCLUSION

In this paper, we give applications of soft dense sets to soft continuous mappings between soft metric spaces. Soft metric spaces provides a powerful tool to study the optimization and approximation theory, variational inequalities and so on.

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