ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.6, 3653–3662
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.6.43

CHARACTERIZATION OF FOREMOST IDEAL OF A CONVENTIONAL SEMILATTICE WITH FUZZY RELATIONS

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ABSTRACT. An Upper fuzzy semilattice is the fuzzy total ordered set (χ , $\Re$), where every ordered pair (x, y) of χ contains lub(A Least Upper Bound) $x \vee y$ in χ. A Conventional semilattice is defined with fuzzy relations and an Upper fuzzy semilattice. In this manuscript, the characterization of Conventional semilattice is done with Fuzzy relations and also proved that a if an Ideal I is maximal then it is an foremost Ideal in upper fuzzy semilattice. The characterization theorem of conventional semilattice is obtained with minimal foremost Ideals which are co-maximal. An essential and acceptable condition for a upper fuzzy distributive semilattice to conventional semilattice is provided.

1. INTRODUCTIOIN

The study of Fuzzy relations assumed greater importance as the Fuzzy relations are applicable in almost all disciplines in the field of research. If every prime ideal of a lattice contains a unique minimal prime ideal then that lattice is a normal lattice, defined by Cornish [5] and Zaanen [1]. The concepts of modular and distributive semilattices are studied by Rhodes [4], Gratzer [3] and Hickman [6] et.al. Using the definition of Fuzzy compatibility, the concept of Fuzzy partial congruence is determined by Hariprakash [2].Using the set of

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2010 *Mathematics Subject Classification.* 06A06, 06A12, 06B10, 06D72.
Key words and phrases. Semilattice, Upper fuzzy semilattice, minimal foremost ideal, Upper fuzzy- distributive semilattice, co-maximal.

all Fuzzy partial congruences on a semigroup, a complete Lower fuzzy semilattice and a complete Upper fuzzy semilattice are defined. In this manuscript, the characterization of Conventional semilattice is done with Fuzzy relations and also obtained that if Upper fuzzy Ideal is maximal then it is an Upper fuzzy foremost Ideal. The characterization of Conventional semilattice is obtained with minimal Upper fuzzy foremost Ideals which are co-maximal. An essential and acceptable condition for a Conventional semilattice is that, any two distinct Upper fuzzy foremost Ideals are co-maximal is obtained. And also determined an essential and acceptable condition for a upper fuzzy distributive semilattice to conventional semilattice.

2. Preliminaries

Definition 2.1. *(Fuzzy-relation) Fuzzy-relation indicates the strength or association among the elements of n-tuple $(x1,\ x2,\ x3, .., \ xn)$. A fuzzy-relation for $n = 2$ is called a fuzzy-binary relation.*

Definition 2.2. *(Fuzzy-reflexive relation) If $\Re$ is a fuzzy-binary relation on a nonempty set χ then the relation $\Re$ is known as fuzzy-reflexive relation, when $\Re(x,\ x) = 1$ for every $x \in \chi$.*

Example 1. *If $\chi = \{a,\ b,\ c\}$ and, if $\Re \subseteq \chi \times \chi$ then* $\Re = \begin{bmatrix} 1 & 0.6 & 0.3 \\ 0.6 & 1 & 0.4 \\ 0.3 & 0.4 & 1 \end{bmatrix}$ *is a fuzzy-reflexive relation.*

Definition 2.3. *(Fuzzy-symmetric relation) If $\Re$ is a fuzzy-binary relation on a nonempty set χ then the relation $\Re$ is known as fuzzy-symmetry relation, when $\Re(x1,\ x2) = \Re(x2,\ x1)$ for all $x,\ y \in \chi$.*

Example 2. *If $\chi = \{a1,\ a2,\ a3\}$ and, if $\Re \subseteq \chi \times \chi$ then* $\Re = \begin{bmatrix} 1 & 0.5 & 0.3 \\ 0.5 & 1 & 0.6 \\ 0.3 & 0.6 & 1 \end{bmatrix}$ *is a fuzzy-symmetric relation.*

Definition 2.4. *(Fuzzy-Anti symmetric relation) Let $\Re$ be a fuzzy-binary relation on a nonempty set χ then the relation $\Re$ is known as fuzzy-anti symmetric relation, when $\Re(x1,\ x2) \neq \Re(x2,\ x1)$ implies $x1 \neq x2$, for all $x1,\ x2 \in \chi$.*

Example 3. *If* χ =$\{a,\ b,\ c\}$ *and, if* $\Re \subseteq \chi \times \chi$ *then* $\Re = \begin{bmatrix} 0 & 0 & 0.7 \\ 0.2 & 0 & 0.3 \\ 0.3 & 0.2 & 0 \end{bmatrix}$ *is a fuzzy-anti symmetric relation.*

Definition 2.5. *(Fuzzy-Transitive relation) Let* $\Re$ *be a fuzzy-binary relation on a nonempty set* χ *then the relation* $\Re$ *is known as fuzzy-transitive relation if* $\Re(a,\ c) \geq \max\ (\min\{\ \Re(a,\ b)\ ,\ \Re(b,\ c)\})$ *for all* $a,\ b,\ c$ *in* χ.

Example 4. *Let* $\Re = \begin{bmatrix} 0.5 & 0.1 \\ 0.6 & 0.3 \end{bmatrix}$ *then* $\Re^2 = \Re \circ \Re$ *(Composition of* $\Re$ *with* $\Re$*) is determined.*

$$\Re^2 = \begin{bmatrix} 0.5 & 0.1 \\ 0.6 & 0.3 \end{bmatrix} \circ \begin{bmatrix} 0.5 & 0.1 \\ 0.6 & 0.3 \end{bmatrix}$$
$$= \begin{bmatrix} \max\{\min(0.5, 0.5), min(0.1, 0.6)\} & \max\{\min(0.5, 0.1), \min(0.1, 0.3)\} \\ \max\{\min(0.6, 0.5), \min(0.3, 0.6)\} & \max\{\min(0.6, 0.1), min(0.3, 0.3)\} \end{bmatrix}$$
$$= \begin{bmatrix} 0.5 & 0.1 \\ 0.5 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.1 \\ 0.6 & 0.3 \end{bmatrix} \leq \Re,$$

therefore the relation $\Re$ *is fuzzy-transitive relation. If* $\Re = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.5 \end{bmatrix}$ *and* $\Re^2$ *is the composition of* $\Re$ *and* $\Re$*, then* $\Re^2 = \begin{bmatrix} 0.2 & 0.2 \\ 0.3 & 0.5 \end{bmatrix}$*; hence* $\Re^2 \not\leq \Re$*. Therefore the relation* $\Re$ *is not fuzzy-transitive.*

Definition 2.6. *(Fuzzy-Partial ordering relation) A relation* $\Re$ *is said to be a Fuzzy-Partial order relation on* χ *if it is fuzzy-reflexive, fuzzy-anti symmetric and fuzzy-transitive relations.*

Definition 2.7. *(Fuzzy-Partial ordered set) A Fuzzy-Partial ordering relation* $\Re$ *on* χ *is a Fuzzy-Partial ordered set, denoted by* $(\chi,\ \Re)$.

Definition 2.8. *(Fuzzy-Total ordered set) A Fuzzy-Partial ordering relation* $\Re$ *on* χ *is a Fuzzy-Total ordered set if,* $x\ \Re\ y$ *or* $y\ \Re\ x$ *for all* $x,\ y \in \chi$ *and, then* $(\chi,\ \Re)$ *is Fuzzy-Total order set.*

Definition 2.9. *(An Upper bound of fuzzy-poset) Let*$(\chi,\ A)$ *be a fuzzy-poset and let* $\mathbb{Y}$ *be a nonempty subset of* χ*, then an element* $u \in \chi$ *is an upper bound of* $\mathbb{Y}$ *if* *and only if* $A(y,u) \in \chi$*, for all* $y \in \mathbb{Y}$*. The upper bound* u_o *for* $\mathbb{Y}$ *is, an lub (a Least Upper Bound) of* $\mathbb{Y}$ *if and only if* $A(u_o, u) \in \chi$*, to each upper bound* u *of* $\mathbb{Y}$*.*

Definition 2.10. *(A Complete Upper fuzzy-semilattice) If* $x \vee y$ *(least upper bound) exists in* χ *to each* x*,* y *in* χ*, then a fuzzy-totally order set (*χ*,* $\Re$*) is a complete upper fuzzy-semilattice.*

Example 5. *Let* $\chi = \{x1, y1, z1, w1\}$ *and Let* $\Re : \chi \times \chi \rightarrow [0,1]$ *be a fuzzy relation such that* $\Re = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.3 & 1 & 0 & 0 \\ 0.5 & 0.2 & 1 & 0 \\ 0.8 & 0.4 & 0.1 & 1 \end{bmatrix}$ *where* $\Re(x1, x1) = \Re(y1, y1) = \Re(z1, z1) = \Re(w1, w1) = 1$ *and* $\Re(x1, y1) = \Re(x1, z1) = \Re(x1, w1) = \Re(y1, z1) = \Re(z1, w1) = 0$*,* $\Re(y1, x1) = 0.3$*,* $\Re(z1, x1) = 0.5$*,* $\Re(w1, x1) = 0.8$*,* $\Re(z1, y1) = 0.2$*,* $\Re(w1, y1) = 0.4$ *and* $\Re(w1, z1) = 0.1$*, then it is easy to check that the relation* $\Re$ *is a fuzzy-total order relation and also* $x1 \vee y1 = x1$*,* $x1 \vee z1 = x1$*,* $x1 \vee w1 = x1$*,* $y1 \vee z1 = y1$*,* $y1 \vee w1 = y1$ *and* $z1 \vee w1 = z1$*. Hence the fuzzy-total ordered set (*χ*,* $\Re$*) is a complete upper fuzzy-semilattice.*

Definition 2.11. *(Complete Lower fuzzy-semilattice) If the G.L.B (Greatest Lower Bound) of* x *and* y *is denoted by* $x \wedge y$ *and is exists for every* x *and* y *of a non empty set* χ*, then the Fuzzy-totally ordered set (*χ*,* $\Re$*) is called a complete lower fuzzy-semilattice.*

Definition 2.12. *(Fuzzy-Ideal) A fuzzy subset I of a complete upper fuzzy-semilattice (*$\mathbb{S}$*,* $\leq$*) is an upper fuzzy-Ideal if for* $p \in I$ *, such as* $q \leq p$ *for* q *in* $\mathbb{S}$*, and* $p \vee q \in I$*, for all* p*,* $q \in I$*.*

Example 6. *In the above example 5 ,* $\mathbb{S}$ *= (*X*,* R*) be the Upper fuzzy-semilattice defined, then a fuzzy set* $I = \{(x,0.0), (y,0.3), (z,0.5), (w,0.8)\}$ *is a fuzzy-Ideal of* $\mathbb{S}$*.*

Definition 2.13. *(fuzzy-dual ideal) A sub set* D *of* $\mathbb{S}$ *is called an upper fuzzy-dual ideal whenever, for* $a \in D$ *and such that* $a \leq x$*, thus* $x \in D$ *and for* a*,* b *in* D *there exists a lower bound* d *of* a*,* b *in* D*.*

Definition 2.14. *(fuzzy-foremost ideal) An upper fuzzy-Ideal* I *of* $\mathbb{S}$ *is called an upper fuzzy-foremost if and only if* $\mathbb{S} \setminus I$ *is an upper fuzzy dual Ideal.*

Definition 2.15. *(Distributive fuzzy-semilattice) An upper fuzzy-semilattice $\mathbb{S}$ is called distributive fuzzy-semilattice, if $x \leq a \vee b$, implies that there exist c and d in $\mathbb{S}$ as $c \leq a$, $d \leq b$ such that $x = c \vee d$.*

Definition 2.16. *(Fuzzy subset of Upper fuzzy-semilattice) Let $\mathbb{S}$ be an upper fuzzy-semilattice with 1, we denote the subset $P1$, and defined as set $P1$ =$\{p \notin P \mid p \vee q$ $= 1$ for $p \in P\}$ is called fuzzy subset of $\mathbb{S}$.*

3. CHARACTERIZATION OF CONVENTIONAL SEMILATTICE

Theorem 3.1. *Suppose P is a fuzzy-Ideal of distributive fuzzy- semilattice $\mathbb{S}$ with 1. Then the set $P1$ is a fuzzy-dual Ideal of $\mathbb{S}$, where $P1 = \{y \notin P \mid x \vee y = 1$ for some $x \in P\}$.*

Proof. If P is an fuzzy-Ideal of a distributive fuzzy-semilattice $\mathbb{S}$ with 1, then $P1$ is a subset of an upper fuzzy-semilattice $\mathbb{S}$ with 1, and is defined as $P1 = \{y \notin P \mid x \vee y = 1$ for some $x \in P\}$. Now, to prove that $P1$ is a fuzzy-dual ideal of $\mathbb{S}$. Let $t \in P1$ such that $t \leq z$ for $z \in P$, since $t \in P1$, we have $z \vee t = 1$, also if $t \leq z$, then $t \vee z = z$, this implies that, $1 = z$, thus $1 \vee z = 1$, this implies $z \in P1$. Let $a, b \in P1$, then $x \vee a$ =1 and $x \vee b = 1$ for some $x \in P$. Let $d \in P1$, then $x \vee d$ =1 for $x \in P$. Thus $x \vee d = x \vee a$, which gives $d \leq a$. Similarly, $x \vee d = x \vee b$, which gives $d \leq b$. Therefore d is an lower bound of both a and b in $P1$. Since $\mathbb{S}$ is upper fuzzy -distributive semilattice, for $d \leq a \vee b$, there exists $x1 \leq a$ and $x2$ $\leq b$, such that $d = x1 \vee x2$, for which $x1 \vee a = a$ and $x2 \vee b$ =b, if and only if, $d \vee a = x1 \vee x2 \vee a = x1 \vee a \vee x2 = a \vee x2$,which gives $d \leq x2 \leq b$. Similarly, if $d \leq x1 \leq a$, we can obtain that $d \leq a$ and $d \leq b$. Hence $P1$ is an upper fuzzy-dual Ideal of $\mathbb{S}$ □

Definition 3.1. *(fuzzy-minimal foremost Ideal) An upper fuzzy-foremost ideal A is said to be minimal, if there is an upper fuzzy-foremost ideal B, with $B \subseteq A$, then $B = A$.*

Theorem 3.2. *A fuzzy-ideal P of a distributive fuzzy-semilattice $\mathbb{S}$ with 1 is maximal if and only if $\mathbb{S} \setminus P = P1$, where $P1 = \{ j \notin P \mid i \vee j = 1$ for some $i \in P\}$. Also if P is maximal, then P is a fuzzy-foremost ideal.*

Proof. Let P be a fuzzy-ideal of $\mathbb{S}$ and consider that, P is maximal. Define a subset $P1$ as $P1 = \{j \notin P \mid i \vee j = 1$ for $i \in P \}$. To prove that $\mathbb{S} \setminus P = P1$, where $P1 = \{j \notin P \mid i \vee j = 1, \text{ for } i \in P\}$. Clearly $P1 \subseteq \mathbb{S} \setminus P$. Let $a \in \mathbb{S} \setminus P$, which implies $a \in \mathbb{S}$ and $a \notin P$. Thus $P \vee (a] = \mathbb{S}$. This implies $i \vee a = 1$ for some i in P. Therefore $a \in P1$. Therefore $\mathbb{S} \setminus P \subseteq P1$. Hence $\mathbb{S} \setminus P = P1$. Now, conversely suppose $\mathbb{S} \setminus P = P1$ where P is an upper fuzzy-ideal of $\mathbb{S}$. Let us assume, P is not maximal, then $P \subseteq Q$ for some fuzzy-Ideal Q of $\mathbb{S}$. Let $t \in Q \setminus P$, then $t \notin P$. This implies $P \vee (t] = \mathbb{S}$ and $t \in Q$. Thus, $t \in \mathbb{S}$ and $t \notin P$. So, we have that $t \in \mathbb{S} \setminus P$. This implies $t \in P1$. Then $t \vee y$ =1 for some $y \in P \subseteq Q$. This shows that $t \vee y = 1$ for some $y \in Q$. This leads to $t \notin Q$, which is a contradiction. Therefore, the assumption that $P \subseteq Q$ is incorrect. Thus P should be maximal. Now to prove that, if P is maximal then, P is an upper fuzzy-foremost Ideal. Since we have $\mathbb{S} \setminus P = P1$, where $P1$ is a fuzzy -dual Ideal of $\mathbb{S}$ and if $\mathbb{S} \setminus P$ is an fuzzy-dual Ideal then by definition, P is an fuzzy-foremost ideal. □

We make use of the following generalization of Stone's Separation theorem.

Theorem 3.3. *Consider an Ideal M and a dual ideal N of a distributive join semilattice $\mathbb{S}$ and, If $M \cap N = \Phi$, then we can find a prime ideal Z of $\mathbb{S}$ such that $M \subseteq Z$ and $Z \bigcap N = \Phi$.*

Definition 3.2. *(Co-maximal Ideals) Two upper fuzzy-Ideals I and J of a semilattice $\mathbb{S}$ are called co-maximal Ideals of $\mathbb{S}$, whenever $I \bigvee J$ =$\mathbb{S}$.*

Definition 3.3. *(Conventional semilattice) An upper fuzzy-semilattice $\mathbb{S}$ is called Conventional if every fuzzy-foremost Ideal contains a unique minimal fuzzy-foremost ideal.*

Theorem 3.4. *A distributive fuzzy-semilattice $\mathbb{S}$ is Conventional if and only if any two distinct fuzzy-foremost Ideals are co-maximal.*

Proof. Let us suppose that $\mathbb{S}$ be a Conventional Semilattice and, $X1$ and $X2$ be two distinct fuzzy-foremost Ideals. To show that $X1$ and $X2$ are co-maximal. Let us suppose that $X1 \bigvee X2 \neq \mathbb{S}$. Then we can find an element a, as such $a \in \mathbb{S}$ and $a \notin X1 \bigvee X2$. This implies $(X1 \bigvee X2) \bigcap [a) = \Phi$ (whose membership function value is zero, in fuzzy approach). Then by generalized theorem of Stone's separation (by above Theorem: 3.3), we can find a foremost Ideal I such as $X1 \bigvee X2 \subseteq I$ and $I \bigcap [a) = \Phi$. This implies $X1 \subseteq I$ and $X2 \subseteq I$. As $\mathbb{S}$ is a conventional semilattice, every fuzzy-foremost Ideal contains unique minimal fuzzy-foremost Ideal, where $X1$ and $X2$ are distinct fuzzy-foremost Ideals of $\mathbb{S}$, thus there exists unique minimal upper fuzzy-foremost ideal M in $X1$ and N in $X2$, such that $M \subseteq X1$ and $N \subseteq X2$. Now by using Stone's separation theorem, we have $X1 \bigcap [a) = \Phi$ and $X2 \bigcap [a) = \Phi$ (as $(X1 \bigvee X2) \bigcap [a) = \Phi$, implies $(X1 \bigcap [a)) \bigvee (X2 \bigcap [a)) = \Phi$) and $M \bigcap [a) = \Phi$; $X2 \bigcap [a) = \Phi$ which implies $M \bigcap [a) = X2 \bigcap [a)$, implies $M = X2$, this contradicts the definition of Conventional semilattice. Thus $X1 \bigvee X2 = \mathbb{S}$. Hence the two distinct upper fuzzy-foremost Ideals are co-maximal. Conversely, suppose that, two upper fuzzy- foremost Ideals P and $X2$ are co-maximal. Then $X1 \bigvee X2 = \mathbb{S}$. To prove that $\mathbb{S}$ is a conventional semilattice, where $\mathbb{S}$ is distributive fuzzy-semilattice. Let X, Y, Z and A, B be fuzzy-foremost ideals of $\mathbb{S}$. As $\mathbb{S}$ is a distributive fuzzy-semilattice and if $X \subseteq A \bigvee B$, then we find, $Y \subseteq A$, $Z \subseteq B$, because $X = Y \vee Z$. Thus Y and Z are two minimal fuzzy-foremost Ideals contained in fuzzy-foremost Ideals A and B respectively, where $Y \neq Z$. If Y and Z are co-maximal fuzzy-foremost Ideals, then we have $Y \bigvee Z = \mathbb{S}$. Then we have that $\mathbb{S} \subseteq A \bigvee B$, which, contradicts the fact, that Y and Z are two minimal fuzzy-foremost Ideals contained in fuzzy-foremost Ideals A and B respectively, such that $Y \neq Z$. Therefore $Y = Z$. Hence there exists a unique minimal foremost Ideal in every fuzzy-foremost Ideal. □

Theorem 3.5. *Let $\mathbb{S}$ be a distributive fuzzy-semilattice, then the given statements are equivalent:*

(I) *Semilattice $\mathbb{S}$ is conventional.*
(II) *Any two minimal fuzzy-foremost Ideals are co-maximal.*
(III) *Every maximal fuzzy-Ideal contains a unique minimal fuzzy-foremost Ideal.*

Proof. To prove that $(I) \Rightarrow (II)$: Let a distributive fuzzy- semilattice $\mathbb{S}$ be Conventional. Then by previous theorem (3.4), a distributive fuzzy-semilattice $\mathbb{S}$ is conventional if and only if any two distinct fuzzy-foremost Ideals are co-maximal. Therefore, $(I) \Rightarrow (II)$ holds good.
To prove that $(II) \Rightarrow (III)$: Let any two minimal fuzzy-foremost Ideals are co-maximal, by theorem (3.2), a fuzzy-Ideal P of a distributive fuzzy-semilattice $\mathbb{S}$ with 1 is maximal implies and implies by $\mathbb{S} \setminus P = P1$, where $P1 = \{v \notin P \mid u \vee v = 1$ for some $u \in P\}$. Let $Z1$ and $Z2$ be two minimal fuzzy-foremost Ideals of P, then by hypothesis $Z1$ and $Z2$ are co-maximal, that is $Z1 \bigvee Z2 = P$, then $\mathbb{S} \setminus (Z1 \bigvee Z2) = P1$ where $P1 = \{v \notin P \mid u \vee v = 1$ for some $u \in P\} = \{v \notin Z1 \bigvee$

$Z2 \mid u \vee v = 1$ for some $u \in Z1 \bigvee Z2\}$, which implies that $u \in Z1$, $u \in Z2$ and $v \notin Z1$ and $v \notin Z2$,but $u \vee v$ =1, which shows$Z1 = Z2$. Hence, every maximal upper fuzzy-ideal contains a unique minimal fuzzy-foremost Ideal.
To prove that $(III) \Rightarrow (I)$: Let every maximal fuzzy-Ideal contains a unique minimal fuzzy foremost Ideal. To prove that $\mathbb{S}$ is conventional semilattice. Let T be a fuzzy-foremost Ideal containing two minimal fuzzy-foremost Ideals A and B. Then $A \subseteq T$ and $B \subseteq T$. Since, every maximal fuzzy-Ideal may contain a unique minimal fuzzy-foremost Ideal, thus T is present in some maximal fuzzy-Ideal W of $\mathbb{S}$. As T is minimal fuzzy-foremost Ideal, we have $A = B$. Hence fuzzy-foremost Ideal, T contains a unique minimal fuzzy-foremost Ideal. □

Theorem 3.6. *If $\mathbb{S}$, is a fuzzy-semilattice with 0, then the following relations hold for the statements given below (in the manner: $(i) \Rightarrow (ii)$; $(ii) \Leftrightarrow (iii)$ and $(ii) \Rightarrow (iv)$).*

(i) *For every fuzzy-foremost Ideal T, the set $K(T) = \{u \mid u \wedge v$ =0 for some $v \in \mathbb{S} \setminus T\}$ is a fuzzy-foremost Ideal of $\mathbb{S}$.*
(ii) *If $u \wedge v = 0$ (that is, the membership function value of u and v is almost zero), then lub (A Least Upper Bound) of ($(u]^*$, $(v]^*$) = $\mathbb{S}$, where $(u]^* = \{a \mid a \wedge u$ =0$\}$.*
(iii) *For each u and v in $\mathbb{S}$, we have ($((u] \wedge (v])^*$) = lub($(u]^*$, $(v]^*$).*
(iv) *Any two minimal fuzzy-foremost Ideals A and B of $\mathbb{S}$ are co-maximal.*

Proof. To prove that $(i) \Rightarrow (ii)$: Consider a fuzzy-semilattice $\mathbb{S}$ with 0 , and u, $v \in \mathbb{S}$, such that $u \wedge v = 0$. But lub($(u]^*$,$(v]^*$) $\neq \mathbb{S}$. Then the lub($(u]^*$, $(v]^*$) is contained in some fuzzy-foremost ideal T of $\mathbb{S}$. Then $(u]^* \subseteq T$ or $(v]^* \subseteq T$. Let us, suppose that $u \notin K(T)$, otherwise $u \wedge t = 0$ for some $t \in \mathbb{S} \setminus T$. This implies $t \in \mathbb{S}$ but $t \notin T$. As $u \wedge t$ =0, $t \in (u]^* \subseteq T$. This leads to $t \in T$, a contradiction. Similarly $v \notin K(T)$, as $K(T)$ is a fuzzy-foremost Ideal. Thus $\mathbb{S} \setminus K(T)$ is a fuzzy-dual Ideal. Thus for $u \in K(T)$ and $v \in K(T)$, there exists d in $\mathbb{S} \setminus K(T)$ such that $d \leq u$ and $d \leq v$. This implies, $d \leq u \wedge v = 0$, which is contradiction to the fact that $0 \in \mathbb{S}$. Hence lub($(u]^*$, $(v]^*$) = $\mathbb{S}$.
To prove that $(ii) \Rightarrow (iii)$: If $u \wedge v = 0$, then lub($(u]^*$, $(v]^*$) = $\mathbb{S}$, where $(u]^* = \{t \mid t \wedge u = 0\}$. To prove that ($((u] \wedge (v])^*$ = lub($(u]^*$, $(v]^*$). Let $t \in (u]^* \vee (v]^*$. Then $t = a \vee b$ for $a \in (u]^*$ and $b \in (v]^*$. This implies $(t] = (u] \vee (v]$ also $a \wedge u = 0$ and $b \wedge v = 0$. This leads to $(a] \wedge (u] = (0]$ and $(b] \wedge (v] = (0]$. Now $(a] \wedge (u] \wedge (v]$ $= (0] \wedge (v] = (0]$ and $(b] \wedge (v] \wedge (u] = (0] \wedge (u] = (0]$. This implies $(a] \wedge ($ $(u] \wedge$

$(v]) = (b] \wedge ((u] \wedge (v]) = (0]$. Hence, $((a] \vee (b]) \wedge ((u] \wedge (v]) = (0]$. Therefore, $(t] \wedge ((u] \wedge (v]) = (0]$. Which implies that $t \in ((u] \wedge (v])^*$. Therefore, $(u]^* \vee (v]^* \subseteq ((u] \wedge (v])^* \longrightarrow (1)$. If $t \in ((u] \wedge (v])^*$, then which implies $(t] \wedge ((u] \wedge (v])=(0]$. So we have that $((t] \wedge ((u]) \wedge (v] = (0]$, which is in the form of $u \wedge v = 0$. Then by (ii) we have that $(u]^* \vee (v]^* = \mathbb{S}$. Then $((t] \wedge ((u]) \vee (v]^* = \mathbb{S}$. Hence, $t = p \vee q$ for $p \in ((t] \wedge ((u])^*$ and $q \in (v]^*$. This implies $(p] \wedge (t] \wedge (u] = (0]$; $(q] \wedge (v] = (0]$. Thus, $(p] \wedge (t] \subseteq (u]^*$ and $(q] \subseteq (v]$ and since $t = p \vee q$, we have $(t] = (p] \vee (q]$. As $(t] = (t] \wedge (t] = (t] \wedge ((p] \vee (q]) = ((t] \wedge (p]) \vee ((t] \wedge (q])$, where $(t] \wedge (p] \subseteq (u]^*$ and $(t] \wedge (q] \subseteq (v]^*$. Thus, it implies that $(t] \subseteq (u]^* \vee (v]^* \longrightarrow (2)$. Therefore from (1) and (2) we determine that $((u] \wedge (v])^* = \text{lub}((u]^* , (v]^*)$.
To prove that $(iii) \Rightarrow (ii)$: Let us suppose that $((u] \wedge (v])^* = \text{lub}((u]^* , (v]^*)$ for $u , v \in \mathbb{S}$. Now to prove that If $u \wedge v = 0$. Then $\text{lub}((u]^* , (v]^*) = \mathbb{S}$, where $(u]^* = \{ t \mid t \wedge u = 0\}$. Since for $u, v \in \mathbb{S}$, we have that $(u] \subseteq \mathbb{S}$ and $(v] \subseteq \mathbb{S}$, Clearly $(u]^* \vee (v]^* \subseteq \mathbb{S} \longrightarrow (1)$. Let $t \in \mathbb{S}$ such that $t \in ((u] \wedge (v])^* \subseteq \mathbb{S}$. This implies $(t] \wedge ((u] \wedge (v]) = (0]$. Therefore, it gives that $t \wedge u =0$ and $t \wedge v =0$. Thus $t \in (u]^*$ and $t \in (v]^*$. Therefore, $t \in (u]^* \vee (v]^*$, to each t in $\mathbb{S}$. Hence $\mathbb{S} \subseteq (u]^* \vee (v]^* \longrightarrow (2)$. Therefore, from (1) and (2) we have that the $\text{lub}((u]^* , (v]^*) = \mathbb{S}$.
To prove that $(ii) \Rightarrow (iv)$: Let us suppose that if $u \wedge v =0$, then $\text{lub}((u]^*,(v]^*) = \mathbb{S}$, where $(u]^* = \{a \mid a \wedge u =0\}$. To prove that any two minimal fuzzy-foremost Ideals A and B of S are co- maximal. Suppose A and B are any two distinct minimal fuzzy-foremost Ideals of $\mathbb{S}$. Let $u\ in\ A \setminus B$. Then $u \in A$ and $u \notin B$. This implies $(u]^* \subseteq B$. Now for $v \in (u]^* \subseteq B$, we have $v \wedge u =0$. Since $\text{lub}((u]^* , (v]^*) = \mathbb{S}$ and also $v \notin A$, where A is a fuzzy-foremost Ideal. This leads to $(v]^* \subseteq A$. Therefore, for $\text{lub}((u]^*, (v]^*) = \mathbb{S}$, we have $A \vee B = \mathbb{S}$. Hence A and B are co-maximal. □

4. Conclusion

This manuscript is able to produce the following results: It is proved that, an fuzzy- Ideal of an distributive fuzzy- semilattice with 1, is an fuzzy -dual ideal. The concepts of fuzzy- minimal foremost Ideal and a Conventional fuzzy- semilattice are initiated. An equivalent condition for an fuzzy- Ideal of an distributive fuzzy- semilattice with 1 is maximal is obtained. A result that an distributive fuzzy- semilattice is conventional if and only if any two different fuzzy- foremost

Ideals are co-maximal is obtained. It is identified the Equivalent Conditions for the Distributive fuzzy- semilattice to be conventional.

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