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TRANSFORMATION FORMULAE FOR MODIFIED MULTI-VARIABLE I-FUNCTION OF PRASAD

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ABSTRACT. Three transformations of double infinite series involving the modified multivariable I-function(MMIF) examined in this article. These transformations further used to obtain double summation formulae for the said function. Our results are quite general in character and a number of transformation formulae and summation formulae deducted as particular cases.

1. INTRODUCTION

We establish four summation formulae for the multiple series involving MMIF defined here. In our results, on specialization of parameters, we can deduce some more results as particular cases. Here, we assume $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ be the sets of positive integers, real numbers and complex numbers respectively. Also $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Prasad and Singh [7] defined MMHF. Later Prasad [6] studied I-function with multivariables. These are the extensions of H-functin with multivariable. [9, 10]. first, we define MMIF and we note it as I.

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(1.1)

$$I(z_1,...,z_r) = I^{0,n_2;0,n_3;....;0,n_r:\big|R':m',n';....;m^{(r)},n^{(r)}}_{p_2,q_2;p_3,q_3;....;p_r,n_r:\big|R:p',q';....;p^{(r)},q^{(r)}}$$

$$\left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix}\left|\begin{matrix} (a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};(\alpha_{3j};\alpha'_{3j},\alpha''_{3j},\alpha'''_{3j})_{1,p_3};....;\left(a_{rj};\alpha^{1}_{rj},....,\alpha^{(r)}_{rj}\right)_{1,p_r}: \\ . \\ \left(b_{2j};\beta'_{2j},\beta''_{2j}\right)_{1,q_2};\left(b_{3j};\beta'_{3j},\beta''_{3j},\beta'''_{3j}\right)_{1,q_3};....;\left(b_{rj};\beta'_{rj},....,\beta^{(r)}_{rj}\right)_{1,q_r}: \end{matrix}\right.\right.$$

$$\left.\begin{matrix} \left(a_{rj};\alpha^{1}_{rj},....,\alpha^{(r)}_{rj}\right)_{1,p_r}:\left(e_j;u'_jg'_j,....,u^{(r)}_jg^{(r)}_j\right)_{1,R'}:(a'_j;\alpha'_j)_{1,p^{(1)}},\left(a^{(r)}_j;\alpha^{(r)}_j\right)_{1,p^{(r)}} \\ : \\ \left(b_{rj};\beta'_{rj},....,\beta^{(r)}_{rj}\right)_{1,q_r}:\left(l_j;U'_jf'_j,....,U^{(r)}_jf^{(r)}_j\right)_{1,R}:\left(b'_j,\beta'_j\right)_{1,q^{(1)}},\left(b^{(r)}_j,\beta^{(r)}_j\right)_{1,q^{(r)}} \end{matrix}\right]$$

$$= \frac{1}{(2\pi\, w)^r}\int_{L_1}....\int_{L_r}\xi(s_1,....,s_r)\prod_{i=1}^{r}\phi(s_i)\,z_i^{\,s_i}ds_1....ds_r\,,$$

where

$$\xi(s_1,....,s_r) = \frac{\prod_{j=1}^{n_2}\Gamma\left(1-a_{2j}+\sum_{i=1}^{2}\alpha^{(i)}_{2j}s_i\right)\prod_{j=1}^{n_3}\Gamma\left(1-a_{3j}+\sum_{i=1}^{3}\alpha^{(i)}_{3j}s_i\right)}{\prod_{j=1+n_2}^{p_2}\Gamma\left(a_{2j}-\sum_{i=1}^{2}\alpha^{(i)}_{2j}s_i\right)\prod_{j=n_3+1}^{p_3}\Gamma\left(a_{3j}-\sum_{i=1}^{3}\alpha^{(i)}_{3j}s_i\right)}$$

$$\times\frac{....\prod_{j=1}^{n_r}\Gamma\left(1-a_{rj}+\sum_{i=1}^{r}\alpha^{(i)}_{rj}s_i\right)}{....\prod_{j=1+n_r}^{p_r}\Gamma\left(a_{rj}-\sum_{i=1}^{r}a^{(i)}_{rj}s_i\right)\prod_{j=1}^{q_2}\Gamma\left(1-b_{2j}+\sum_{i=1}^{2}\beta^{(i)}_{2j}s_i\right)}$$

$$\times\frac{1}{\prod_{j=1}^{q_3}\Gamma\left(1-b_{3j}+\sum_{i=1}^{3}\beta^{(i)}_{3j}s_i\right)....\prod_{j=1}^{q_r}\Gamma\left(1-b_{rj}+\sum_{i=1}^{r}\beta^{(i)}_{rj}s_i\right)}$$

$$\times\frac{\prod_{j=1}^{R'}\Gamma\left(e_j+\sum_{i=1}^{r}u^{(i)}_jg^{(i)}_js_i\right)}{\prod_{j=1}^{R}\Gamma^{L_j}\left(l_j+\sum_{i=1}^{r}U^{(i)}_jf^{(i)}_js_i\right)},$$

$$\phi(s_i) = \frac{\prod_{j=1}^{n^{(i)}}\Gamma\left(1-a^{(i)}_j-\alpha^{(i)}_js_i\right)\prod_{j=1}^{m^{(i)}}\Gamma\left(b^{(i)}_j-\beta^{(i)}_js_i\right)}{\prod_{j=1+n^{(i)}}^{p^{(i)}}\Gamma\left(a^{(i)}_j-\alpha^{(i)}_js_i\right)\prod_{j=1+m^{(i)}}^{q^{(i)}}\Gamma\left(1-b^{(i)}_j-\beta^{(i)}_js_i\right)},$$

where $\alpha_j^{(i)}, \beta_j^{(i)}, \alpha_{kj}^{(i)}, \beta_{kj}^{(i)} \, \forall i = 1, ..., r; k = 1, ..., r,\; g_j^{(i)} \, \forall i = 1, ..., r; j = 1, ..., R$ and $f_j^{(i)} \, \forall i = 1, ..., r; j = 1, ..., R'$ all are positive reals. Further,

$$e_j\,(j = 1, ..., R'),\; l_j(j = 1, ..., R),\; a_j^{(i)}, b_j^{(i)}(i = 1, ...r); \alpha_{kj}, \beta_{kj}(k = 2, ..., r)$$

are complex and here $p^{(i)},\, n^{(i)},\, q^{(i)}, m^{(i)} \, \forall\, i = 1, ..., r$ and $q_k\,,\, p_k,\, n_k \, \forall\, k = 2, ..., r$ are non-negative integers, where $0 \le q_k; 0 \le n_k \le p_k$ and $0 \le m^{(i)} \le q^{(i)}\,;\; 0 \le n^{(i)} \le p^{(i)} \, \forall\, i = 1, ..., r.$

Here "i" represents the number of dashes. The contour L_k, is in $s_k -$ plane, where $k = 1, ..., r$ which lies from $\sigma - i\infty\, to\, \sigma + i\infty$; σ is real with the loop. It is necessary to ensure that the poles of $\Gamma\left(1 - a_{2j} + \sum_{k=1}^{2} \alpha_{2j}^{(k)} s_k\right)$, for all $j = 1, ..., n_2$; $\Gamma\left(1 - a_{3j} + \sum_{k=1}^{3} \alpha_{3j}^{(k)} s_k\right)$; for all $j = 1, ..., n_3$. $\Gamma\left(1 - a_{rj} + \sum_{k=1}^{r} \alpha_{rj}^{(k)} s_k\right)$, for all $j = 1, ..., n_r$, $\Gamma\left(1 - a_j^{(k)} - \alpha_j^{(k)} s_k\right) : j = 1, ..., n^{(k)},\, k = 1, ..., r$ are in left of 'L_k' and their poles of $\Gamma\left(b_j^{(k)} - \beta_j^{(k)} s_k\right) : j = 1, ..., m^{(k)},\, k = 1, ..., r$ are in right of 'L_k'. Refer [6], for further details and asymptotic expansion of I-function. Assume the poles of the integrand are simple. Also assume that the pole of $\Gamma\left(e_j + \sum_{i=1}^{r} u_j^{(i)} g_j^{(i)} s_i\right)$ lies to the left or right of $u_j^{(i)}$is positive or negative. The point $z_i = 0$ for all $i = 1, ..., r$, is excluded. The multiple contour integral converges absolutely if $|\arg z_i| < \frac{1}{2}\Omega_i \pi$, where

(1.2)

$$\begin{aligned}\Omega_i \;=\; & \sum_{k=1}^{n^{(i)}} \alpha_k^{(i)} - \sum_{k=n^{(i)}+1}^{p^{(i)}} \alpha_k^{(i)} + \sum_{k=1}^{m^{(i)}} \beta_k^{(i)} - \sum_{k=m^{(i)}+1}^{q^{(i)}} \beta_k^{(i)} + \sum_{k=1}^{n_2} \alpha_{2k}^{(i)} \\ & - \sum_{k=n_2+1}^{p_2} \alpha_{2k}^{(i)} + \sum_{k=1}^{n_3} \alpha_{3k}^{(i)} \\ & - \sum_{k=n_3+1}^{p_3} \alpha_{3k}^{(i)} + \cdots \\ & + \sum_{k=1}^{n_r} \alpha_{rk}^{(i)} - \sum_{k=n_r+1}^{p_r} \alpha_{rk}^{(i)} - \sum_{k=1}^{q_2} \beta_{2k}^{(i)} - \sum_{k=1}^{q_3} \beta_{3k}^{(i)} \cdots - \sum_{k=1}^{q_r} \beta_{rk}^{(i)} \\ & + \sum_{j=1}^{R'} g_j^{(i)} - \sum_{j=1}^{R} f_j^{(i)} > 0\end{aligned}$$

$\forall i = 1, ..., r$. We note

$$\begin{aligned}
A &= \left(a_{2j};\alpha'_{2j},\alpha''_{2j}\right)_{1,p_2};...;\left(a_{(r-1)j};\alpha'_{(r-1)j},...,\alpha^{r-1}_{(r-1)j}\right)_{1,p_{r-1}}\\
B &= \left(b_{2j};\beta'_{2j},\beta''_{2j}\right)_{1,q_2};....;\left(b_{(r-1)j};\beta'_{(r-1)j},...,\beta^{r-1}_{(r-1)j}\right)_{1,q_{r-1}}\\
\Im &= \left(a'_j,\alpha'_j\right)_{1,p'};....;\left(a^{(r)}_j,\alpha^{(r)}_j\right)_{1,p^{(r)}}\\
A &= \left(a_{rj};\alpha'_{rj},....,\alpha^{(r)}_{rj}\right)_{1,p_r};E=\left(e_j;u'_jg'_j,....,u^{(r)}_jg^{(r)}_j\right)_{1,R'}\\
B &= \left(b_{rj};\beta'_{rj},....,\beta^{(r)}_{rj}\right)_{1,q_r};\ \mathrm{L}=\left(l_j;U'_jf'_j,....,U^{(r)}_jf^{(r)}_j\right)_{1,R}\\
B &= \left(b'{}_j,\beta'{}_j\right)_{1,q'};....;\left(b^{(r)}_j,\beta^{(r)}_j\right)_{1,q^{(r)}}\\
U &= p_2,q_2;p_3,q_3;\ldots;p_{r-1},q_{r-1};\ V\ =\ 0,n_2;0,n_3;\ldots;0,n_{r-1}\\
Y &= (p',q');...;(p^{(r)},q^{(r)});\ X=(m',n');...;(m^{(r)},n^{(r)})\,.
\end{aligned}$$

2. Main Results

In this part, we examine the following three transformation iformulae for the imultivariable Gimel-function:

Theorem 2.1.

(2.1)

$$\sum_{m,n=0}^{\infty}\frac{x^my^n}{m!\,n!}I^{V;0,n_r+4:|R':X}_{U;p_r+4,q_r+1:|R:Y}\left(\begin{array}{c|c} z_1 & A;(1-a-m;a_1,...,a_r;1),(1-b-m;b_1,...,b_r),\\ \cdot & \cdot\\ \cdot & \cdot\\ z_r & B;B,(1-c-m-n;c_1,...,c_r)\end{array}\right.$$

$$\left.\begin{array}{c}(1-c+a-n;c_1-a_1,...,c_r-a_r),(1-c+b-n;c_1-b_1,...,c_r-b_r),A:E:\Im\\ \cdot\\ \cdot\\ :L{:}\mathbf{B}\end{array}\right)$$

$$=\sum_{s=0}^{\infty}\frac{(1-y)^{a+b-c}(x+y-xy)^s}{s!}I^{V;0,n_r+4:|R':X}_{U;p_r+4,q_r+1:|R:Y}$$

$$\left(\begin{array}{c|c} z_1(1-y)^{u_1+v_1-w_1} & A;(1-a-s;a_1,...,a_r;1),(1-b-s;v_1,...,v_r), \\ . & . \\ . & . \\ z_r(1-y)^{u_r+v_r-w_r} & B;B,(1-c-s;c_1,...,c_r) \end{array}\right.$$

$$\left.\begin{array}{c} (1-c+a;c_1-a_1,...,c_r-a_r),(1-c+b;b_1,...,b_r),\mathbf{A}:E:\Im \\ . \\ . \\ :L:\mathbf{B} \end{array}\right)$$

provided, $a_i, b_i, c_i > 0$, $c_i - a_i > 0$, $c_i - b_i > 0\, \forall\, i = 1, ..., r$. $|arg(z_i)| < \frac{1}{2}(\Omega_i - c_i)\pi$ *, where* Ω_i *is defined by (1.2),* $max\{|x|\,,\,|y|\,\} < 1$*, either* $|x + y - xy| < 1$ *or* $x = 1$ *with* $Re(c - a - b) > 0$.

Proof. Expressing the MMIF [6] by using (1.1) and changing the order of iintegration and summation, we arrive at (say I):

$$\begin{aligned} I \;=\; & \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,...,s_r)\prod_{k=1}^{r}\theta_k(s_k)\,z_k^{s_k} \\ & \times\frac{\Gamma(a+\sum_{i=1}^{r}a_i\,s_i)\,\Gamma(b+\sum_{i=1}^{r}b_is_i)}{\Gamma(c+\sum_{i=1}^{r}c_is_i)} \\ & \times\Gamma(c-a+\sum_{i=1}^{r}(c_i-a_i)s_i)\,\Gamma(c-b+\sum_{i=1}^{r}(c_i-b_i)s_i \\ & \times F_2\left[a+\sum_{i=1}^{r}a_is_i, c-a+\sum_{i=1}^{r}(c_i-a_i)s_i, b+\sum_{i=1}^{r}b_is_i,\right. \\ & \left. c-b+\sum_{i=1}^{r}(c_i-b_i)s_i; c+\sum_{i=1}^{r}b_is_i\right] ds_1...ds_2\,. \end{aligned}$$

By applying the result([4], p.238, eq.(14)) in the above, we get

$$\begin{aligned} I &= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\phi(s_1,...,s_r)\prod_{k=1}^{r}\theta_k(s_k)\,z_k^{s_k} \\ &\quad\times\frac{\Gamma(a+\sum_{i=1}^r a_i\,s_i)\,\Gamma(b+\sum_{i=1}^r b_is_i)}{\Gamma(c+\sum_{i=1}^r c_is_i)} \\ &\quad\times\Gamma(c-a+\sum_{i=1}^{r}(c_i-a_i)s_i)\,\Gamma(c-b+\sum_{i=1}^{r}(c_i-b_i)s_i)\,(1-y)^{a+b+c+\sum_{i=1}^r(a_i+b_i-c_i)s_1} \\ &\quad\times{}_2F_1\left[a+\sum_{i=1}^{r}a_is_i,1;b+\sum_{i=1}^{r}b_is_i;c+\sum_{i=1}^{r}c_i\,s_i;x+y-xy\right]ds_1...ds_2\,. \end{aligned}$$

Now, expressing ${}_2F_1$ in terms of its series and changing the order of integration and summation iand then iinterpreting the resulting expression by using(1.1), we get desired theorem. □

Theorem 2.2.

$$\sum_{m,n=0}^{\infty}\frac{x^my^n}{m!\,n!}I_{U;p_r+2,q_r+1:|R:Y}^{V;0,n_r+2:|R':X}\left(\begin{array}{c|c} z_1 & A;(1-a-m;a_1,...,a_r), \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;B,(1-c-m-n;b_1,...,b_r) \end{array}\right. \left.\begin{array}{c} (1-c+a-n;b_1-a_1,...,b_r-a_r),A:E:\Im \\ \cdot \\ \cdot \\ :L:\mathbf{B} \end{array}\right) \tag{2.2}$$

$$= e^y\sum_{s=0}^{\infty}\frac{(x-y)^{-a}}{s!}I_{U;p_r+2,q_r+1:|R:Y}^{V;0,n_r+2:|R':X}\left(\begin{array}{c|c} & A;(1-a-s;a_1,...,a_r),(1-c+a;b_1-a_1,...,b_r-a_r),A::E:\Im \\ z_1 & \cdot \\ z_r & \cdot \\ & B;B,(1-c-s;b_1,...,b_r):L:\mathbf{B} \end{array}\right),$$

provided, $a_i,b_i>0,\ b_i-a_i>0\,\forall\,i=1,...,r.\ |arg(z_i)|<\frac{1}{2}(\Omega_i-b_i)\pi$*, where* Ω_i *is defined by* (1.2).

Theorem 2.3.

$$\sum_{m,n=0}^{\infty}\frac{(16x)^m(1-x)^n}{m!\,n!}\,I^{V;0,n_r+5:|R':X}_{U;p_r+5,q_r+2:|R:Y}\left(\begin{matrix}z_1\\ \cdot\\ \cdot\\ z_r\end{matrix}\middle|\begin{matrix}A;(1-a-m-n;a_1,..,a_r),(1-b-m-n;b_1,..,b_r),\\ \cdot\\ \cdot\\ B;B,(\tfrac{1}{2}-a-b-2m;a_1+b_1,...,a_r+b_r),\end{matrix}\right.$$

$$\left.\begin{matrix}(\tfrac{1}{2}-a-m;a_1,...,a_r),(\tfrac{1}{2}-b-m;b_1,...,b_r),(\tfrac{3}{2}-c-2m;c_1,...,c_r),A:E:\Im\\ \cdot\\ \cdot\\ (2-2c-2m;2c_1,...,2c_r):L:\mathbf{B}\end{matrix}\right)$$

$$=\pi 4^{1-c}\sum_{s=0}^{\infty}\frac{(x)^s}{s!}I^{V;0,n_r+2:|R':X}_{U;p_r+2,q_r+1:|R:Y}\left(\begin{matrix}z_1 4^{-c_1}\\ \cdot\\ \cdot\\ z_r 4^{-c_r}\end{matrix}\middle|\begin{matrix}A;(1-a-s;a_1,...,a_r),\\ \\ \\ B;B,\end{matrix}\right.$$

$$\left.\begin{matrix}(1-b-s;b_1,...,b_r),\mathbf{A}:E:\Im\\ \cdot\\ \cdot\\ (1-c-s;\omega_1,...,\omega_r):L:\mathbf{B}\end{matrix}\right),$$

provided, $a_i, b_i, c_i > 0\,\forall\, i = 1,...,r.$ *and* $|\,arg(z_i)| < \frac{1}{2}(\Omega_i - a_i - b_i - c_i)\pi$, $|x| < 1$ *or* $x = 1$ *with* $Re(c-a-b) > 0$, *where* Ω_i *is defined by* (1.2).

We can make proof of similar steps to the formula (2.1) by using ([3],p.124, Eq.64), see also([8],p.322, Eq.181) and ([2],p.339, Eq.12) respectively, instead of the result ([5],p.238, Eq.(14)) to prove the above two theorems.

3. Summation Formulae

If we take x = 1 in (2.1) and using Gauss's summation formula, we obtain:

Corollary 3.1. $\sum_{m,n=0}^{\infty}\frac{y^n}{m!\,n!}\,I^{V;0,n_r+4:|R':X}_{U;p_r+4,q_r+1:|R:Y}$

$$\left(\begin{array}{c|c} z_1 & A; (1-a-m; a_1, ..., a_r), (1-c+a-n; c_1-a_1, ..., c_r-a_r), \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B; B, (1-c-m-n; c_1, ..., c_r) : L : \mathbf{B} \end{array}\right.$$

$$\left.\begin{array}{c} (1-b-n; b_1, ..., b_r), (1-c-b-n; c_1-b_1, ..., c_r-b_r), A{:}E : \Im \\ \cdot \\ \cdot \\ -- \end{array}\right)$$

$$= \sum_{s=0}^{\infty} \frac{(1-y)^{a+b-c}(x+y-xy)^s}{s!} I^{V;0,n_r+2:|R':X}_{U;p_r+2,q_r+1:|R:Y}$$

$$\left(\begin{array}{c|c} z_1(1-y)^{u_1+v_1-\omega_1} & A; (1-a-s; a_1, ..., a_r), (1-b-s; v_1, ...v_r), \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r(1-y)^{u_r+v_r-\omega_r} & B; B, \end{array}\right.$$

$$\left.\begin{array}{c} (1-c+a; c_1-a_1, ..., c_r-a_r), (1-c+b; b_1, ..., b_r), A : E : \Im \\ \cdot \\ \cdot \\ (1-c-s; c_1, ..., c_r) : L{:}\mathbf{B} \end{array}\right)$$

provided, $a_i, b_i, c_i > 0$, $c_i - a_i > 0$, $c_i - b_i > 0 \, \forall \, i = 1, ..., r$. *and* $|arg(z_i)| < \frac{1}{2}(\Omega_i - c_i)\pi$, $|y| < 1$.

Consider the above formula, by taking $y \to 0$, we get

Corollary 3.2. $\sum_{m=0}^{\infty} \frac{1}{m!} I^{V;0,n_r+2:|R':X}_{U;p_r+2,q_r+1:|R:Y}$

$$\left(\begin{array}{c|c} z_1 & A; (1-a-m; a_1, ..., a_r), (1-b-m; b_1, ..., b_r), \mathbf{A} : E : \Im \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B; B, (1-c-m; c_1, ..., c_r) : L : \mathbf{B} \end{array}\right)$$

$$= e^x I^{V;0,n_r+3:|R':X}_{U;p_r+3,q_r+2:|R:Y}\left(\begin{matrix} z_1(1-y)^{u_1+v_1-\omega_1} \\ \cdot \\ \cdot \\ z_r(1-y)^{u_r+v_r-\omega_r} \end{matrix}\middle| \begin{matrix} \mathbf{A};(1-a;a_1,...,a_r), \\ \\ \\ B;\mathbf{B},(1-c+a;c_1-a_1,...,c_r-a_r): \end{matrix}\right.$$

$$\left.\begin{matrix} (1-b;v_1,...v_r),(1-c+a+b;c_1-a_1-b_1,...,c_r-a_r-b_r),A:E:\Im \\ \cdot \\ \cdot \\ L:\mathbf{B} \end{matrix}\right)$$

Taking x = y in (2.2), we obtain the following formula:

Corollary 3.3.

$$\sum_{m=0}^{\infty}\frac{x^{m+n}}{m!\,n!}\,I^{V;0,n_r+2:|R':X}_{U;p_r+2,q_r+1:|R:Y}\left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_r \end{matrix}\middle| \begin{matrix} A;(1-a-m;a_1,...,a_r), \\ \cdot \\ \cdot \\ B;B,(1-c-m-n;b_1,...,b_r): \end{matrix}\right.$$

$$\left.\begin{matrix} (1-c+a-n;b_1-a_1,...,b_r-a_r),\mathbf{A}:E:\Im \\ \cdot \\ \cdot \\ L:\mathbf{B} \end{matrix}\right)$$

$$= e^x I^{V;0,n_r+2:|R':X}_{U;p_r+2,q_r+1:|R:Y}\left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_r \end{matrix}\middle| \begin{matrix} \mathbf{A};(1-a;a_1,...,a_r), \\ \\ \\ B;\mathbf{B},(1-c;b_1,...,b_r): \end{matrix}\right.$$

$$\left.\begin{matrix} (1-c+a;b_1-a_1,...,b_r-a_r,\mathbf{A}:E:\Im \\ \cdot \\ \cdot \\ L:\mathbf{B} \end{matrix}\right)$$

Taking x = 1 in (2.2), we obtain the following formula:

Corollary 3.4.

$$\sum_{m,n=0}^{\infty}\frac{1}{(2m)!}\, I^{V;0,n_r+3:|R':X}_{U;p_r+3,q_r+2:|R:Y}\left(\begin{array}{c|c} z_1 & A;(1-2a-2m;2a_1,...,2a_r), \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;B,(\tfrac{1}{2}-a-b+2m;a_1+b_1,...,a_r+b_r), \end{array}\right.$$

$$\left.\begin{array}{c} (1-2b-2m;2b_1,...,2b_r),(\tfrac{3}{2}-c-2m;c_1,...,c_r),A{:}E;\Im \\ \cdot \\ \cdot \\ (2+2c-2m;2c_1,...,2c_r:L:\mathbf{B} \end{array}\right)$$

$$= 4^{a+b-c} I^{V;0,n_r+3:|R':X}_{U;p_r+3,q_r+2:|R:Y}\left(\begin{array}{c|} z_1 4^{a_1+b_1-c_1} \\ \cdot \\ \cdot \\ z_r 4^{a_r+b_r-c_r} \end{array}\right.$$

$$\left.\begin{array}{c} A;(1-2a;2a_1,...,2a_r),(1-2b;2b_1,...,2b_r),(\tfrac{3}{2}-c;c_1,...c_r),\mathbf{A}:E:\Im \\ \\ B;B,(\tfrac{1}{2}-a-b;a_1+b_1,...,a_r+b_r),(2+2c;2c_1,...2c_r:L:\mathbf{B} \end{array}\right)$$

4. REMARKS

By using analogue technique, we can obtain the similar type of relations if MMIF by Prasad [6] reduces into modified generalized H-function [2, 7]. If the above three conditions are satisfied at a time, then the MMIF by Prasad [6] reduces in to the generalized multivariable i H-function by [9, 10] and then we can obtain the same summation iformulae by Audich [1]. We can also obtain different transformations, derivative formulae [12, 13]and we can apply the I-function for solving the application problems [11, 14].

5. CONCLUSION

The significance of all these results lies in their manifold generality. First, in the sense of general arguments utilized in these itransformation of double infinite series formulae, we can obtain a large number of single, double or multiple summations. Later, by specializing the various parameters as well as variables in the MMIF, we get a several formulae involving a wide variety of useful relations.

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