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OPTIMAL THREE STAGE SPECIALLY STRUCTURED FLOWSHOP SCHEDULING TO MINIMIZE THE TOTAL WAITING TIME OF JOBS, PROCESSING TIME ASSOCIATED WITH PROBABILITIES

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ABSTRACT. This paper is an attempt to minimize the total waiting time of jobs for the nx3 Specially Structured Flow shop Scheduling in which processing time are associated with their respective probabilities. The problem discussed has wider applicability and has better practical usage when it comes to reducing the overall waiting time of scheduled jobs. The objective of the observation is to bring forth the arrangement/sequence of jobs associated with the least amount of total waiting time using iterative algorithm. The application of the algorithm is universal under given constraints discussed here. Furthermore, a practical example is discussed step by step to clarify the application of the algorithm.

1. INTRODUCTION

Scheduling might be defined as the problem of taking decision when to execute a given set of actions, subject to chronological constraints and possessions capacities, in order to optimize some function. A Flow shop problem survives when all the jobs share the same dealing out order on all the machines. Technological constraints insist that the jobs get ahead between the machines in the same order. Hence there is ordinary sequence of the machines characterized by the technological constraints for each and every job in flow shop. The flow shop contains n different machines set in series on which a set of m jobs are to be

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executed. Each of the m jobs requires n operations and each function is to be performed on a separate machine. The flow of the occupation is unidirectional; thus each job must be processed through each machine in a prearranged order. The general m jobs, n machine flow shop scheduling is quite terrifying. Consider an arbitrary sequence of jobs on each machine, there are (m!)n probable schedules which poses computational difficulties. With the aim to trim down the counting of probable schedules it is logical to assume that all machines practice jobs in the alike order. Efforts in the past have been made by researchers to reduce this number of possible schedules as much as achievable without compromising on optimality condition. Today's large-scale markets and immediate interactions mean that clients expect high-class goods and services at what time they require them, anywhere they require them. Organizations, whether public or private, have to make available these products and services as effectively and efficiently as possible.

The principle of optimality in the given flow shop scheduling problem is precised as minimization of waiting time of jobs. The waiting time of a job is defined as the subtraction of the completion time of job on the first machine from the starting time of job on second machine and the subtraction of the completion time of job on the second machine from the starting time of job on third machine. There are some papers in the literature of scheduling theory which put stress on the waiting time for scheduling the jobs on the machines.

2. LITERATURE REVIEW

The Johnson's method is exclusively famous amidt scientific access that are used to clarify n- jobs, two- machines order complication to optimize the total lapsed time. Schrage and Ignall popularized the flow shop complication with branch and bound method to make span minimization. Muhlemann A.P. and Lockett A.G., Crowin and Esogbue, Maggu and Dass made experiment to broad the study by popularized different specification. Yoshida and Hitomi solved norigination program, the setting time of machine being different from working time. Resulting methods for flow shop program using heurist made by Singh T.P. [6], Rajendran and Chaudhuri. Singh T.P., Gupta D. studied the complication related with get together job condition in a flow shop which involves different setting time of machines and transport time. Singh V. [7] put his achievement to solve 3 machine flow shop program complication with total renowned cost. Gupta D. studied optimization of Renowned Cost in n Flow Shop roster with job block concept and setting Time of machines was different from working Time of machines and each coupled with possibilities. Gupta D. [1], [2] considered optimize 2 and 3 grade open shop specially framed roster in which the working times of machines are connected along possibilities along transport time to optimize the renowned cost,. Recently Gupta D. and Goyal B. [4], [3] examined the theory of compressing holding time of jobs by examing transforming times connected along probabilities [5], reducing in the three machine without setup time. Also examined theory of Pradeep Bishnoi [8] on optimization of Useful Time for Specially framed n $\times$ three roster exemplary along Jobs in a cord of non linked Job section. The complication negotiated here has evident use of imaginary solution in process factory or in the conditions where the goal is to optimize the total holding time of jobs. This paper considered here wider area of Gupta D. and Goyal B. [5] That mean we have take three machine alienated from processing time. Extending the study three machine specially structured flow shop scheduling with the aim of optimize total waiting period of jobs.

3. PRACTICAL SITUATION

Manufacturing units/industries play a important role in the economic progress of a country. Flow shop scheduling occurs in various offices, service stations, banks, airports, etc. In our routine working in industrial and manufacturing units diverse jobs are practiced on a variety of machines. In cars industry different types of works done on machines. For example Body shop, Paint shop and assemble shop. Here, the different equivalent processing time taken in body shop on first machine. is always less than or equal to the minimum equivalent time taken in paint shop on the second machine and the maximum equivalent time taken in paint shop of on second machine is always less than or equal to the minimum equivalent time taken in assemble shop on the third machine.

4. NOTATIONS

a_k: Order made by the algorithm.
A_i: Processing time of i^{th} job on machine A.
A_i': Equivalent time for processing of j^{th} job on machine A.

TABLE 1

Jobs	Machine A		Machine B		Machine C	
I	A_i	a_i	B_i	b_i	C_i	c_i
1	A_1	a_1	B_1	b_1	C_1	c_1
2	A_2	a_2	B_2	b_2	C_2	c_2
3	A_3	a_3	B_3	b_3	C_3	c_3
⋮	⋮	⋮	⋮	⋮	⋮	⋮
n	A_n	a_n	B_n	b_n	C_n	c_n

B_i: Processing time of i^{th}job on machine B.
B_i': Equivalent time for processing of j^{th} job on machine B.
C_i: Processing time of i^{th} job on machine C.
C_i': Equivalent time for processing of j^{th} job on machine C.
P_i :Processing time for i^{th}job on first fictitious machine
Q_i: Processing time for i^{th} job on second fictitious machine
W_T: Sum of holding time of all the jobs.

5. PROBLEM FORMULATION

Assume that three machines A,B and C are transforming n jobs in the form ABC, A_i, B_i and C_i are the respective processing times Our aim is to find an optimize oder a_k of jobs optimization the total holding time of all jobs.

6. ASSUMPTIONS

1) There are n figure of jobs (I) and 3 machines (A, B and C).
2) The arrangements of procedure in all machines act by identical.
3) Jobs are not dependent on one another.
4) Once a job is begin on a machine, the processing can't be put to an end as long as the job is finished.

7. ALGORITHM

Step 1: Equivalent processing times of i^{th} job on machine A,B and C are defined as $A_i'= A_i * a_i$, $B_i'= B_i * b_i$, $C_i',= C_i * c_i$.

TABLE 2

A'_i	B'_i	C'_i
A'_1	B'_1	C'_1
A'_2	B'_1	C'_2
A'_3	B'_3	C'_3
$\vdots$	$\vdots$	$\vdots$
A'_n	B'_n	C'_n

TABLE 3

Jobs	Machine P	Machine Q
I	P_i	Q_i
1	P_1	Q_1
2	P_2	Q_2
3	P_3	Q_3
4	P_4	Q_4
5	P_5	Q_5

Step 2: Check the condition either $maxB_i \leq minA_i$ or $maxB_i \leq minC_i$ if higher terms are fulfill that time we substitute the 3 machine by two imaginary machine P and Q along with uniform transaction time defined as

$$P_i = A_i + B_i \quad \text{and} \quad Q_i = B_i + C_i,$$

where P_i and Q_i are the transaction times for i^{th} job on machine P and Q independently .

By Computing the new transaction times, We consider the preferable flow of the jobs for the machines P and Q in the normal way. Transaction times satisfies structural relationship

$$MaxP_i \leq MinQ_i$$

Step 3: Equivalent processing times P_i and Q_i on machine P and Q respectively be calculated is defined in steps

Step 4: Calculate the entries for the following table.

Step 5: Assemble the jobs in enhancing series of x_j

Assuming the sequence found be $(\sigma_1, \sigma_2, \sigma_3, \dots\dots\dots \sigma_m)$

Step 6: Locate Min P_i

TABLE 4

Jobs (J)	Machine P (P_i)	Machine Q (Q_i)	$z_{a_r} = (n-r)x_k$				
			$x_i = Q_i - P_i$	$r = 1$	$r = 2$	$\ldots$	$r = (n-1)$
1	P_1	Q_1	x_1	z_{11}	z_{12}	...	$z_{1(n-1)}$
2	P_2	Q_2	x_2	z_{21}	z_{22}	...	$z_{2(n-1)}$
3	P_3	Q_3	x_3	z_{31}	z_{32}	...	$z_{3(n-1)}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	P_n	Q_n	x_n	z_{n1}	z_{n2}	...	$z_{n(n-1)}$

For the following two possibilities
P_{α_1}=min P_i arranged aligning to step 4 is the needed preferable flow
$P_{\alpha_1} \neq minP_i$ shift at step 6
Step 7: Examine the distinct order of jobs $a_1, a_2, a_3, \ldots\ldots a_m$ .Where a_1 is the flow accomplished in step 4, Sequence a_k (k= 2,3,... m) can be accomplished by shifting k^{th} job in the flow a_1 to the 1^{st} spot and stub of the flow staying identical.
Step 8: Figure out the total holding time W_T for all the order $a_1, a_2, a_3, \ldots\ldots a_m$. Using the following formula:

$$W_T = nX_i + \sum_{r=1}^{n-1} Z_{a_r} - \sum_{k=1}^{n} X_i$$

$P_i =$ Correspond transforming time of the 1st job on machine P in order j

$$Z_{ar} = (n-r)x_i; \qquad a = \sigma_1, \sigma_2, \sigma_3, \ldots\ldots\ldots \sigma_n.$$

The flow along lesser total holding time is the needed preferable flow.

8. NUMERICAL ILLUSTRATION

Considered five jobs 1^{st},2^{nd}, 3_{rd}, 4^{th}, 5^{th} along transforming times A_i and B_i and B_i to be transacted on the machines A ,B and C.

Our purpose leads to attain preferable flow which lesser the total holding time of the jobs.

Solution-

Acc. to step 1- After satisfying structured conditions reducing the three machine into two fictitious machine P and Q.

TABLE 5

Jobs	A_i	a_i	B_i	b_i	C_i	c_i
1	20	0.1	4	0.1	26	0.1
2	7	0.2	2	0.2	11	0.4
3	5	0.3	3	0.1	15	0.2
4	10	0.2	6	0.1	31	0.1
5	8	0.2	1	0.5	16	0.2

TABLE 6

Jobs	Machine A	Machine B	Machine C
I	A_i'	B_i'	C_i'
1	2.0	0.4	2.6
2	1.4	0.4	4.4
3	1.5	0.3	3.0
4	2.0	0.6	3.1
5	1.6	0.5	3.2

TABLE 7

Jobs	Machine P	Machine Q
I	P_i	Q_i
1	2.4	3.0
2	1.8	4.8
3	1.8	3.3
4	2.6	3.7
5	2.1	3.7

Acc. to step 2- After satisfying structured conditions reducing the three machine into two fictitious machine P and Q satisfying max B_i'= 4≤ min A_i'=4 two imaginary machine G and H along with uniform transaction time given by $P_i = A_i' + B_i'$ and $Q_i = B_i' + C_i'$.

Max P_i=2.6≤ Min $Q_i = 3.0$

Acc. to step 5-The order so established be 1^{st}, 4^{th}, 3^{rd} , 5^{th} , 2^{nd}.

Acc. to step 6- MinP_i=1.8≠ P_1

TABLE 8

Jobs (J)	Machine X (X_k)	Machine Y (Y_k)	$z_{kr} = (n-r)x_k$				
			$x_k = Y_k - X_k$	$r=1$	$r=2$	$r=3$	$r=4$
1	2.4	3.0	0.6	2.4	1.8	1.2	0.6
2	1.8	4.8	3.0	12.0	9.0	6.0	3.0
3	1.8	3.3	1.5	6.0	4.5	3.0	1.5
4	2.6	3.7	1.1	4.4	3.3	2.2	1.1
5	2.1	3.7	1.6	6.4	4.8	3.2	1.6

TABLE 9

a_i	Sequence	Total Waiting Time W_T	
a_1	1-4-3-5-2	11.6	
a_2	4-1-3-5-2	13.1	
a_3	3-1-4-5-2	9.9	Optimal
a_4	5-1-4-3-2	11.7	
a_5	2-1-4-3-5	15.8	

Acc. to step 7- Different sequence of jobs can be considered as:
a_1: 1,4,3,5,2;
a_2 : 4,1,3,5,2;
a_3 : 3,1,4,5,2;
a_4 :5,1,4,3,2;
a_5 : 2,1,4,3,5

Acc. to step 8- The total holding time for the order accomplished in step five can be computed
Present sum of $P_i = 10.7$
Hence arrangement is a_3: 3,1,4,5,2;the needed arrangement with lesser total holding time.

9. REMARKS

The present study deals with the flow shop scheduling problem with the main idea to reduce the total waiting time of jobs. However it may add to the other costs lime machine idle cost or penalty cost of the jobs, yet the idea of minimizing the waiting time may be an economical aspect from Factory /Industry manager's observation point when he has minimum time contract with a commercial party to complete the jobs. Extension to the present paper can be made by introducing various parameters like transportation time, break down interval etc.

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