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A JOINT ACTIVITY OF SINGLE RETAILER AND SINGLE SUPPLIER WITH PRICE DEPENDENT DEMAND IN CHANGEABLE DECISION OF LEAD TIME WITH SHORTAGES

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ABSTRACT. To develop the model of Joint activity of single Retailer and Supplier model of deteriorating items in supply chain with Price dependent demand in time linked Holding Cost of Changeable decision in lead time with shortages. To arrive at an EOQ with the following assumption of price dependent demand for Retailer and quadratic time function of supplier demand has considered. The retailer may meet the shortages, so shortages considered with the supplier salvage cost has taken. In Retailer and Supplier holding cost considered as time function. Using this ideas to formulate a mathematical model and to find optimal solution in Lead time with Retailers total cost how much affect the Supplier total cost, finally all the parameters compares with integrated(Retailer and Supplier) total cost. To show the sensitivity analysis for all the parameter, using this value graphs are shown.

1. INTRODUCTION

In the real life any business the coordination between Retailer and Supplier in necessary and Supplier supply the items whenever needed for Retailer and cannot expect all the time same. Some time both Retailer and supplier have the same demand and this situation is not unique and some situation different demand for Retailer and Supplier, it is depend on the situation of assumed.

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So here we considered two types of demand namely for Retailer price dependent demand, for supplier quadratic time dependent of higher powers of time is negative. By usual way two types of holding cost considered and changeable deterioration is taken.

Detreating things like frozen cream, woollen materials, dairy - items, green vegetables, and so on , and at the same time which also gets diminished without a trace in course of time. These things are considered. The decaying cannot be avoid for any business , to reduce this decaying cost is the one of the main purpose.

Sreelekha Biswas., C. Giri. [1] discussed A Single - vendor Single - buyer Model for Partial Backlogging and Price - dependent Market Demand. This paper developed the vendor supplies the buyer's order quantity in a number of unequal sized shipments. Hesham et al [2] Studied EOQ and EPQ Variable Holding Cost. This paper discussed more time dependent holding cost. Savitha., K. K. Achary. [3], has developed An Integrated Vendor - buyer Production System for Deteriorating Time Shortages for Buyer. It gave the consequence of single - seller single - purchaser incorporated blemished creation stock model with learning underway and speculation for process quality improvement. The lead - time is thought to be parcel size ward, and the security stock factor is thought to be diverse for the main clump and the remainder of the groups. The yearly expected all out expense of the coordinated framework is determined. Nejah Ben Mabrouk. [4] has established an Interpretive structural modeling of critical factors for buyer - supplier partnerships in supply chain management, Because of this work, it very well may be proposed that purchaser and provider ought to be given unique consideration to defeat the key elements like development and innovation and data trade. Furthermore, the created considered as basic components to support connections among purchaser and provider.

Dr Parmjot Singh, [5] proposed a work of An Investigation of Vendor Buyer. Here between close to home and between hierarchical associations between Indian car associations are the key drivers of for rising and supporting trust, duty and building up joint effort between ccomplices. Diaz - Mateus, [6] has formulated a paper of Pricing and lot sizing optimization in a two - echelon supply chain with a constrained logic demand function. Pavan Kumar. [7] has developed an inventory planning problem for time - varying linear demand and parabolic holding cost with salvage value.The parabolic holding cost give dealt.Urmila Chaudhari, [8] has investigated an Inventory Modelling of Deteriorating Item and Preservation Technology with Advance Payment Scheme Under Quadratic Demand.

Maragatham, M. and Palani, R. [9] An inventory model for deteriorating items with lead time. In this work a different type of demand of price sensitivity demand has considered, using this type of demand the demand will increase with low price items. Mohit Rastogi1 S. R. Singh. [10] have studied a model of A Pharmaceutical Inventory Model for Varying Deteriorating Items with Price Sensitive Demand and Partial Backlogging Under the Effect of Learning.Magfura Pervin, (et. al) [11] has described An integrated vendor - buyer model under inspection policy and preservation technology. The framework is considered for both seller and purchaser, The merchant applies the conservation innovation to decrease the misfortune because of crumbling. Be that as it may, it is noticeable from the model that while the underlying weakening rate is extremely low, burning through cash on conservation innovation won't be advantageous. It is additionally worth referencing that in the event that there exists a spending plan on the speculation capital, at that point there will be a likelihood to get more benefit for that association. Srabani Shee and Tripti Chakrabarti. [12] have presented a model of A Fuzzy two echelon Supply Chain Model for Time as a Decision Variable, to built up the ideal request procedure of a provider retailer's stock model for falling apart things under fluffy condition, it is seen that provider's normal all out cost diminishes and the retailer's normal all out cost increments.

In this manuscript, to develop a Joint activity of single Retailer and Supplier model of Deteriorating items in supply chain with price dependent demand in Time linked Holding Cost of Changeable decision in lead time with shortages. Deterioration rate is not fixed changing with time. Our target is to find the least overall average cost with lead time, leftover paper is construct To arrange bellow as quoted. In the section 2, symbols and Inferences is listed. then section 3, we formulated a Math Optimizing model of the recommended inventory problem. By next section 3.1 Retailor model, 3.2 Supplier model, in Section 4, Retailor and Supplier joint level numerical problems is carried out and the section 5, Using table values sensitivity test is performed, graphical depictions are showed in section 6, Observations are submitted in section 7, Conclusion and provide a few future investigate chance.

2. Symbols and Assumptions

2.1. **Symbols.**

(i) The Retailer Demand function is selling price dependent $(\alpha_1 - \alpha_2 P) > 0$.
(ii) The Retailer Holding Cost $H_R(t) = b_1 t^3$.
(iii) The Supplier Demand function is $\psi(1 + t\psi_2 - \psi_3 t^3)$.
(iv) The Supplier Deterioration is $\theta_S t$.
(v) The Retailer Deterioration is $\theta_R t$.
(vi) The Retailer Inventory is $I_{R_1}(t)$.
(vii) The Supplier Inventory is $I_S(t)$.
(viii) C_{J_2} Retailer stock out cost.
(ix) C_{J_3} Retailer setup cost.
(x) C_{J_4} Supplier setup cost.
(xi) C_{J_5} Retailer deterioration cost.
(xii) C_{J_6} Supplier deterioration cost.
(xiii) C_{J_7} Lot sale cost of Retailer.
(xiv) λ Supplier Salvage cost.
(xv) AT_{JC} Average Total Cost.
(xvi) C_{JS} Cost of shortage for cycle.
(xvii) C_{JD} cost of deterioration for cycle.
(xviii) C_{JH} cost of holding for cycle.
(xix) C_{JL} cost of lost sale for a cycle.

2.2. **Assumptions.**

(i) The demand for product is price dependent and time dependent.
(ii) The inventory system involves production of single item.
(iii) The constant rate of deterioration is consider.
(iv) The model is developed for finite time horizon.
(v) The holding cost considered as changing with time.

3. The equation using mathematical model can be form as follows

3.1. **Retailer's Model.** Q_j is the retailer's beginning inventory level is at time $t = 0$. The inventory level gradually reduces to zero at time $t = T_J - L_J$. The joined operations in deterioration and demand. The differential equation can be formed as follows

$$\begin{aligned}\frac{d\,I_{R_1}(t)}{dt} + \theta_R\, t\, I_{R_1}(t) &= -(\alpha_1 - \alpha_2 P),\ 0 < t < T_J - L_J \\ \frac{d\,I_{R_2}(t)}{dt} &= -\frac{(\alpha_1 - \alpha_2 P)}{1 + \delta(T_J - t)},\ if\, T_J - L_J < t < T_J\end{aligned}$$

Using the boundary $Q_{J_1} = I_{R_1}(0),\ I_{R_2}(T_J - L_J) = 0$, we can write

$$\begin{aligned}I_{R_1}(t) = (\alpha_1 - \alpha_2 P)\Bigg[(T_J - L_J) + \left(\frac{(T_J - L_J)^3}{6}\right)\theta_R - t \\ - \frac{(T_J - L_J)\theta_R\, t^2}{2} + \left(\frac{\theta_R\, t^2}{6}\right)\Bigg]\end{aligned}$$

$$I_{R_2}(t) = (\alpha_1 - \alpha_2 P)\log\left|\frac{1 + \delta(T_J - t)}{1 + \delta L_j}\right|$$

Using $Q_{J_1} = I_{R_1}(0)$, which implies

$$Q_{J_1} = (\alpha_1 - \alpha_2 P)\left[T_J - L_J) + \left(\frac{(T_J - L_J)^3}{6}\right)\theta_R\right]$$

Using $Q_2 = I_{R_2}(T)$, implies that

$$Q_2 = -\frac{(\alpha_1 - \alpha_2 P)}{\delta}\log|1 + \delta L|$$

$$\begin{aligned}Q_R &= Q_{J_1} + Q_{J_2} \\ &= (\alpha_1 - \alpha_2 P)\left[T_J - L_J) + \left(\frac{(T_J - L_J)^3}{6}\right)\theta_R - \frac{1}{\delta}\log|1 + \delta L_J|\right]\end{aligned}$$

The shortage cost of the retailer is

$$\begin{aligned}\text{Shortage Cost}\, C_{J_S} &= -C_{J_2}\int_{T_J - L_J}^{T_J} T_{R_2}(t)\, dt \\ &= C_{J_2}(\alpha_1 - \alpha_2 P)\left[\frac{L_J}{\delta} - \frac{1}{\delta^2}\log|1 + \delta L_J|\right]\end{aligned}$$

The lost sale quantity of the Retailer is

$$\begin{aligned}\text{Lost Sale Quantity Cost}\, C_{J_L} &= C_{J_7}\int_{T_J - L_J}^{T_J}\left[1 - \frac{1}{1 + \delta(T_J - t)}\right] D(P)\, dt \\ &= C_{J_7}(\alpha_1 - \alpha_2 P)\left[L_J - \frac{1}{\delta}\log|1 + \delta L_J|\right]\end{aligned}$$

The deterioration cost of the Retailer is

$$\text{Deterioration Cost} = \frac{C_{J_5}(\alpha_1 - \alpha_2 P)\theta_R(T_J - L_J)^3}{6}$$

The Holding cost of the Retailer is

$$\begin{aligned}\text{Holding Cost } C_{J_H} &= \int_0^{T_J} h_r(t)\, I_{R_1}(t)\, dt \\ &= b_1\,(\alpha_1 - \alpha_2 P)\left[\frac{(T_J - L_J)^5}{20} + \frac{1}{168}\,(T_J - L_J)\,\theta_R\right]\end{aligned}$$

The setup cost of the Retailer C_{J_3}. Therefore the Retailer average overall cost is

$$\begin{aligned}C_{J_R}(T_J,\, L_J) &= \frac{1}{T_J}\left[\begin{array}{l} Retailer\ Deterioratio\, Cost + Retailer\ Holding\, Cost \\ +Retailer\ Setup\, Cost + Retailor\ Lost\ SaleCost \\ +Retailor\ Shortage\, Cost, \end{array}\right] \\ &= \frac{(\alpha_1 - \alpha_2 P)}{T_J}\left\{C_{J_3} + C_{J_2}\left(\frac{L_J}{\delta} - \frac{1}{\delta^2}\log|1 + \delta L_J|\right)\right. \\ &\quad + C_{J_7}\left(L_J - \frac{1}{\delta}\log|1 + \delta + L_J|\right) + \frac{C_{J_5}(\alpha_1 - \alpha_2 P)\theta_R(T_J - L_J)^3}{6} \\ &\quad \left. + b_1\left(\frac{(T_J - L_J)^5}{20} - \frac{1}{168}\,(T_J - L_J)\,\theta_R\right)\right\}\end{aligned}$$

3.2. **Supplier's Model.** Q_S is the Supplier beginning inventory level is at time $t = 0$. The inventory level gradually reduces to zero at time $t = T_J - L_J$. The joined operations in deterioration and demand. The differential equation can be formed as follows

$$\frac{d\,I_S(t)}{dt} + \theta_S t I_S(t) = -\psi_1(1 + \psi_2 t - \psi t^3),\ if\ 0 \le t \le T_J - L_J$$

Using $I_S(0) = Q_S$, $I_S(T_J - L_J) = 0$

$$\begin{aligned}I_S(t) &= \psi_1\left\{(T_J - L_J) - t - \frac{(T_J - L_J)\theta_S\, t^2}{2} + \frac{\theta_S\, t^3}{2} + \frac{\psi_2}{2}\left[(T_J - L_J)^2 - t^2\right]\right. \\ &\quad + \left(\frac{\theta_S - 2\psi_3}{6}\right)\left[(T_J - L_J)^3 - t^3\right] + \frac{\psi_2\theta_S}{8}\left[(T_J - L_J)^4 - t^4\right] \\ &\quad + \frac{\psi_3\theta_S}{10}\left[(T_J - L_J)^5 - t^5\right] - \frac{\psi_2\theta_S}{4}\left[(T_J - L_J)^2\, t^2 - t^4\right] \\ &\quad \left. + \frac{\psi_3\theta_S}{6}\left[(T_J - L_J)^3\, t^2 - t^5\right]\right\}\end{aligned}$$

after simplificaiton

$$I_S(t) = \psi_1 \left\{ T_J - L_J + \frac{\psi_2}{2}(T_J - L_J)^2 + \left(\frac{\theta_S - 2\psi_3}{6}\right)(T_J - L_J)^3 + \frac{\psi_2\theta_S}{8}(T_J - L_J)^4 + \frac{\psi_3\theta_S}{10}(T_J - L_J)^5 \right\}$$

The deterioration cost of the Supplier is

$$Deteriration\,Cost = C_{J_6}\,\theta_S\,\psi_1 \left[\frac{(T_J - L_J)^3}{6} + \frac{\psi_2(T_J - L_J)^4}{8} + \frac{\psi_3(T_J - L_J)^5}{10}\right]$$

$$\begin{aligned} Salvage\,Cost &= \lambda[Deterioration\,Cost] \\ &= \lambda\,C_{J_6}\,\theta_S\,\psi_1 \left[\frac{(T_J - L_J)^3}{6} + \frac{\psi_2(T_J - L_J)^4}{8} + \frac{\psi_3(T_J - L_J)^5}{10}\right] \end{aligned}$$

The holding cost of the supplier is

$$\begin{aligned} Holding\,Cost = \psi_1 \Bigg\{ & f\left[\frac{(T_J - L_J)^2}{2} + \left(\frac{\theta_S - 3\psi_3}{12}\right)(T_J - L_J)^4 + \frac{\psi_2(T_J - L_J)^3}{3} + \frac{\psi_2\theta_S(T_J - L_J)^5}{15} - \frac{\psi_3\theta_S(T_J - L_J)^6}{18}\right] \\ & + g\left[+\frac{(T_J - L_J)^4}{12} + + \frac{\psi_3(T_J - L_J)^5}{15}\left(\frac{\theta_S - 5\psi_3}{90}\right)(T_J - L_J)^6 + \frac{\psi_2\theta_S(T_J - L_J)^7}{105} - \frac{\psi_3\theta_S(T_J - L_J)^8}{120}\right]\Bigg\} \end{aligned}$$

The Setup cost of supplier $=C_{J_4}$. Therefore, the suppliers average total cost in the cycle T_J is

$$C_S(T_J, L_J) = \frac{1}{T_J}\left[\begin{array}{l} Deterioration\,Cost + Holding\,Cost \\ +Setup\,Cost - Salvage\,Cost \end{array}\right]$$

$$
\begin{aligned}
= \; & \frac{1}{T_J}\Bigg[\Bigg[\frac{(T_J-L_J)^3}{6}+\frac{\psi_2(T_J-L_J)^4}{8}+\frac{\psi_3(T_J-L_J)^5}{10}\Bigg] \\
& \times(1-\lambda)C_{J_6}\,\theta_S\,\psi_1 \;+C_{J_4}\psi_1\Bigg\{f\;\Bigg[\frac{(T_J-L_J)^2}{2}+\left(\frac{\theta_S-3\psi_3}{12}\right) \\
& \times(T_J-L_J)^4+\frac{\psi_2(T_J-L_J)^3}{3}+\frac{\psi_2\theta_S(T_J-L_J)^5}{15} \\
& -\frac{\psi_3\theta_S(T_J-L_J)^6}{18}\Bigg]+g\;\Bigg[+\frac{(T_J-L_J)^4}{12}+\frac{\psi_3(T_J-L_J)^5}{15} \\
& +\left(\frac{\theta_S-5\psi_3}{90}\right)(T_J-L_J)^6+\frac{\psi_2\theta_S(T_J-L_J)^7}{105} \\
& -\frac{\psi_3\theta_S(T_J-L_J)^8}{120}\Bigg]\Bigg\}\Bigg]
\end{aligned}
$$

Therefore the total average cost in cycle T_J is

$$
\begin{aligned}
AT_JC(T_J,L_J) &= C_R(T_J,L_J)+C_S(T_J,L_J) \\
T_C(T_J,L_J) &= \frac{1}{T_J}\Bigg\{(\alpha_1-\alpha_2P)\Bigg[C_{J_3}+C_{J_2}\left(\frac{L_J}{\delta}-\frac{1}{\delta^2}\log|1+\delta L_J|\right) \\
&\quad +C_{J_7}\left(L_J-\frac{1}{\delta}\log|1+\delta L_J|\right)+\frac{C_{J_5}\theta_R(T_J-L_J)^3}{6} \\
&\quad +b_1\left(\frac{(T_J-L_J)^5}{20}-\frac{1}{168}(T_J-L_J)\,\theta_R\right)\Bigg]+C_{J_4}+(1-\lambda) \\
&\quad \times\; C_{J_6}\,\theta_S\,\psi_1\Bigg[\frac{(T_J-L_J)^3}{6}+\frac{\psi_2(T_J-L_J)^4}{8}+\frac{\psi_3(T_J-L_J)^5}{10}\Bigg] \\
&\quad +C_{J_4}\psi_1\Bigg\{f\;\Bigg[\frac{(T_J-L_J)^2}{2}+\left(\frac{\theta_S-3\psi_3}{12}\right)(T_J-L_J)^4 \\
&\quad +\frac{\psi_2(T_J-L_J)^3}{3}+\frac{\psi_2\theta_S(T_J-L_J)^5}{15}-\frac{\psi_3\theta_S(T_J-L_J)^6}{18}\Bigg] \\
&\quad +g\;\Bigg[+\frac{(T_J-L_J)^4}{12}+\frac{\psi_3(T_J-L_J)^5}{15}+\left(\frac{\theta_S-5\psi_3}{90}\right)(T_J-L_J)^6 \\
&\quad +\frac{\psi_2\theta_S(T_J-L_J)^7}{105}-\frac{\psi_3\theta_S(T_J-L_J)^8}{120}\Bigg]\Bigg\}\Bigg\}
\end{aligned}
\tag{3.1}
$$

The necessary conditions for least value of $AT_JC(T_J,L_J)$ are

$$\frac{\partial(AT_JC(T_J,L_J))}{\partial L_J}=0\;and\;\frac{\partial(AT_JC(T_J,L_J))}{\partial T_J}=0.$$

The sufficient condition for least of $AT_JC(T_J, L_J), L_J > 0,\ T_J > 0$ is

$$\begin{vmatrix} \frac{\partial^2(AT_JC)}{\partial L_J^2} & \frac{\partial^2(AT_JC)}{\partial L_J \partial T_J} \\ \frac{\partial^2(AT_JC)}{\partial T_J \partial L_J} & \frac{\partial^2(AT_JC)}{\partial T_J^2} \end{vmatrix} > 0$$

The equations (3.1) are non linear so, to solve using the any technique of computer based software and obtain optimal order cycle time $L_J,\ T_J$ is $(L_J^*,\ T_J^*)$, the calculation of Second derivatives of the function $AT_JC(L_J,\ T_J)$, is complicated. That is also verified using computer based software and with the help of a graph the progress can be identified and tabulated.

4. Numerical Examples (Retailor and Supplier)

Example 1. *Let us consider the input data: (Smaller data)*

Let $\alpha_1 = 140, \alpha_2 = 0.18, P = 22, \theta_R = 0.04, b_1 = 9, C_{J_2} = 0.5, C_{J_3} = 90, C_{J_5} = 5, C_{J_7} = 4, \lambda = 0.4, \psi_1 = 86, \psi_2 = 20, \psi_3 = 24, \theta_S = 0.04, f = 5, g = 0.5, C_{J_4} = 150, C_{J_6} = 6, \delta = 0.4$.

The lead time $L_J^* = 1.0318,\ T_J^* = 1.7080,\ RTC^* = 351.6234,\ STC^* = 352.9889,\ AT_JT^* = 704.6127$.

Example 2. *Let us consider the input data: (Medium data) Let* $\alpha_1 = 150, \alpha_2 = 0.2, P = 20, \theta_R = 0.05, b_1 = 10, C_{J_2} = 0.6, C_{J_3} = 100, C_{J_5} = 6, C_{J_7} = 5, \lambda = 0.625, \psi_1 = 60, \psi_2 = 20, \psi_3 = 10, \theta_S = 0.03, f = 6, g = 0.6, C_{J_4} = 175, C_{J_6} = 7, \delta = 0.6$.

The lead time $L_J^* = 1.1331, T_J^* = 1.8013, RTC^* = 430.5565, STC^* = 444.577, AT_JT^* = 875.1320$.

Example 3. *Let us consider the input data: (Larger data)*

Let $\alpha_1 = 160, \alpha_2 = 0.25, P = 22, \theta_R = 0.06, b_1 = 12, C_{J_2} = 0.7, C_{J_3} = 120, C_{J_5} = 7, C_{J_7} = 6, \lambda = 0.7, \psi_1 = 55, \psi_2 = 18, \psi_3 = 8, \theta_S = 0.01, f = 4, g = 0.5, C_{J_4} = 160, C_{J_6} = 5, \delta = 0.5$.

The lead time $L_J^* = 0.8848, T_J^* = 1.6413, RTC^* = 484.6065, STC^* = 400.906, AT_JT^* = 885.512$.

5. Sensitivity Analysis

Table 1: Sensitivity Analysis

	Parameter	% Change	Value	J_J^*	T_J^*	AT_JC^*	RTC^*	STC^*
150	α_1	$+20\%$	180	1.0542	1.7389	975.521	491.476	484.046
		$+10\%$	165	1.0911	1.768	926.361	461.438	464.922
		-10%	135	1.1814	1.8402	821.537	398.715	422.822
		-20%	120	1.2378	1.8862	765.196	365.78	399.416
0.2	α_2	$+20\%$	0.24	1.1355	1.8033	872.336	428.882	443.454
		$+10\%$	0.22	1.1343	1.8023	873.735	429.719	444.016
		-10%	0.18	1.1319	1.8004	876.527	431.39	445.137
		-20%	0.16	1.1307	1.7995	877.921	432.225	445.696
20	P	$+20\%$	24	1.1355	1.8033	872.336	428.882	443.454
		$+10\%$	22	1.1343	1.8023	873.735	429.719	444.016
		-10%	18	1.1319	1.8004	876.527	431.39	445.137
		-20%	16	1.1307	1.7995	877.921	432.225	445.696
0.05	θ_R	$+20\%$	0.06	1.1333	1.8015	875.279	430.83	444.449
		$+10\%$	0.055	1.1332	1.8014	875.206	430.693	444.513
		-10%	0.045	1.133	1.8013	875.058	430.417	444.641
		-20%	0.04	1.1329	1.8013	874.985	430.28	444.705
10	b_1	$+20\%$	12	1.134	1.8021	875.906	431.782	444.124
		$+10\%$	11	1.1336	1.8017	875.519	431.169	444.35
		-10%	9	1.1326	1.801	874.745	429.94	444.805
		-20%	8	1.1322	1.8006	874.356	429.323	445.033
0.6	C_{J_2}	$+20\%$	0.72	1.1187	1.7899	891.838	440.415	451.422
		$+10\%$	0.66	1.1258	1.7955	883.516	435.499	448.017
		-10%	0.54	1.1406	1.8073	866.683	425.582	441.101
		-20%	0.48	1.1482	1.8134	858.169	420.579	437.589
100	C_{J_3}	$+20\%$	120	1.1464	1.8166	886.267	442.798	443.469
		$+10\%$	110	1.1398	1.809	880.722	436.703	444.019
		-10%	90	1.1264	1.7936	869.495	424.352	445.143
		-20%	80	1.1196	1.7858	863.809	418.091	445.718
6	C_{J_5}	$+20\%$	7.2	1.1333	1.8015	875.282	430.833	444.449
		$+10\%$	6.6	1.1332	1.8014	875.207	430.694	444.513
		-10%	5.4	1.133	1.8013	875.057	430.416	444.641
		-20%	4.8	1.1329	1.8013	874.982	430.277	444.706
5	C_{J_7}	$+20\%$	6	1.0667	1.7488	956.286	478.778	477.508
		$+10\%$	5.5	1.0982	1.7736	916.436	454.997	461.438
		-10%	4.5	1.1722	1.8327	832.213	405.389	426.824
		-20%	4	1.2163	1.8685	787.481	379.427	408.054
0.6	λ	$+20\%$	0.72	1.1329	1.8012	874.956	430.514	444.442
		$+10\%$	0.66	1.133	1.8013	875.067	430.54	444.527
		-10%	0.54	1.1333	1.8015	875.29	430.592	444.698
		-20%	0.48	1.1334	1.8016	875.401	430.618	444.783
60	ψ_1	$+20\%$	72	1.1802	1.8279	914.642	440.221	474.421
		$+10\%$	66	1.1575	1.8149	895.532	435.6	459.932

Continued on next page

Table 1 – *Continued from previous page*

	Parameter	% Change	Value	J_J^*	T_J^*	AT_JC^*	RTC^*	STC^*
		−10%	54	1.1069	1.7873	853.214	425.006	428.208
		−20%	48	1.0785	1.7728	829.48	418.85	410.63
20	ψ_2	+20%	24	1.1881	1.8344	921.226	441.396	479.83
		+10%	22	1.1617	1.8182	899.104	436.282	462.822
		−10%	18	1.1017	1.7838	848.864	424.042	424.822
		−20%	16	1.0667	1.7657	819.638	416.476	403.162
10	ψ_3	+20%	12	1.1166	1.7904	861.327	427.461	433.865
		+10%	11	1.125	1.7959	868.324	429.04	439.283
		−10%	9	1.141	1.8067	881.766	432.011	449.755
		−20%	8	1.1488	1.812	888.237	433.413	454.824
0.03	θ_S	+20%	0.036	1.1334	1.8016	875.392	430.613	444.78
		+10%	0.033	1.1333	1.8015	875.262	430.584	444.678
		−10%	0.027	1.133	1.8013	875.002	430.526	444.476
		−20%	0.024	1.1328	1.8012	874.872	430.497	444.374
6	f	+20%	7.2	1.1797	1.8275	914.176	440.129	474.047
		+10%	6.6	1.1572	1.8147	895.282	435.549	459.733
		−10%	5.4	1.1073	1.7876	853.509	425.072	428.437
		−20%	4.8	1.0792	1.7732	830.13	419.004	411.126
0.6	g	+20%	0.72	1.1336	1.8017	875.534	430.638	444.896
		+10%	0.66	1.1334	1.8015	875.333	430.597	444.736
		−10%	0.54	1.1329	1.8012	874.931	430.513	444.418
		−20%	0.48	1.1326	1.801	874.73	430.472	444.258
175	C_{J_4}	+20%	210	1.1562	1.8279	894.499	432.694	461.806
		+10%	192.5	1.1448	1.8147	884.885	431.636	453.249
		−10%	157.5	1.1213	1.7878	865.235	429.45	435.785
		−20%	140	1.1093	1.774	855.187	428.321	426.866
7	C_{J_6}	+20%	8.4	1.1333	1.8015	875.271	430.587	444.684
		+10%	7.7	1.1332	1.8014	875.202	430.571	444.63
		−10%	6.3	1.133	1.8013	875.063	430.539	444.524
		−20%	5.6	1.1329	1.8013	874.993	430.523	444.471
0.625	δ	+20%	0.75	1.1686	1.8353	866.202	431.926	434.276
		+10%	0.6875	1.1552	1.8224	869.157	431.116	438.041
		−10%	0.5625	1.1222	1.7911	878.579	430.62	447.96
		−20%	0.5	1.1016	1.7718	885.96	431.342	454.618

5.1. Graphs using tabular value. Parametric Changes with integrated average total cost(Retailer and Supplier).

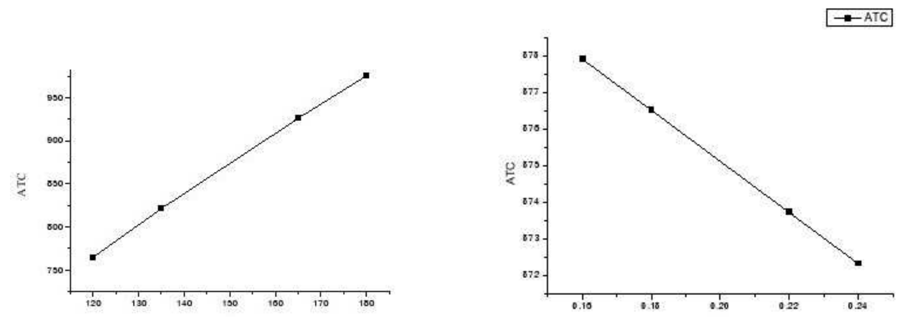

FIGURE 1. The Impact of α_1 with AT_JC and of α_2 with AT_JC

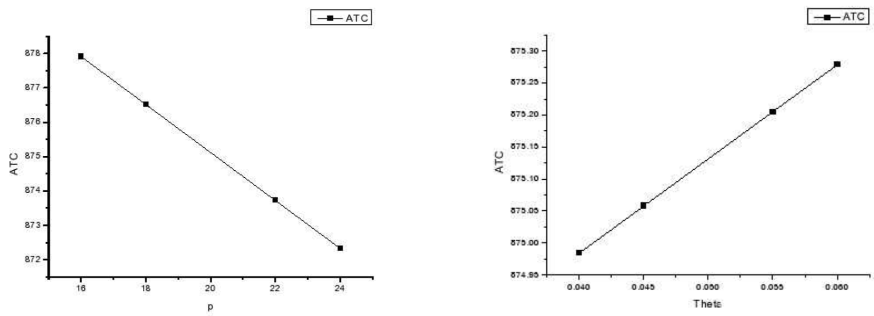

FIGURE 2. The Impact of P with AT_JC and of θ_R with AT_JC

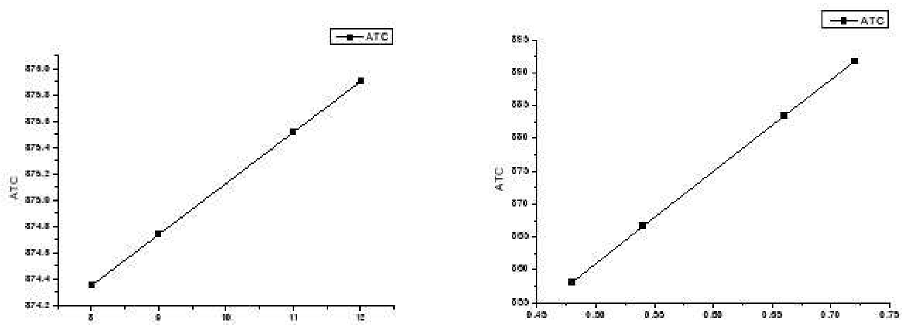

FIGURE 3. The Impact of b_1 with AT_JC and of C_J with AT_JC

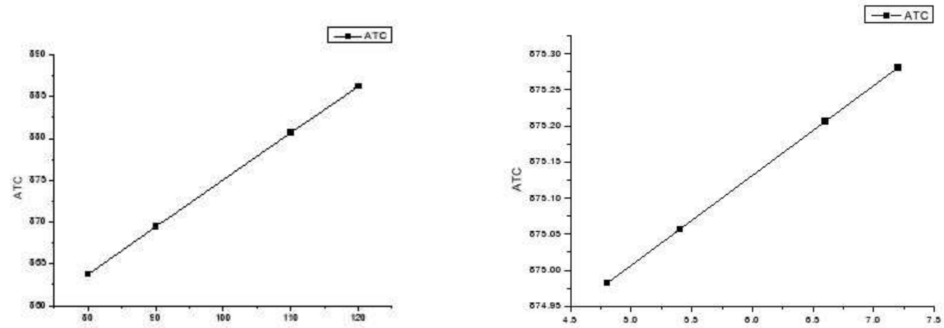

FIGURE 4. The Impact of C_{J_3} with AT_JC and C_{J_5} with AT_JC

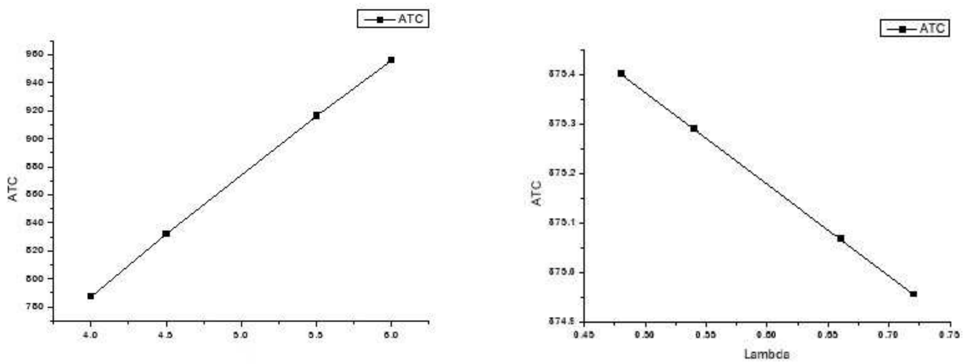

FIGURE 5. The Impact of AT_JC and of λ with AT_JC

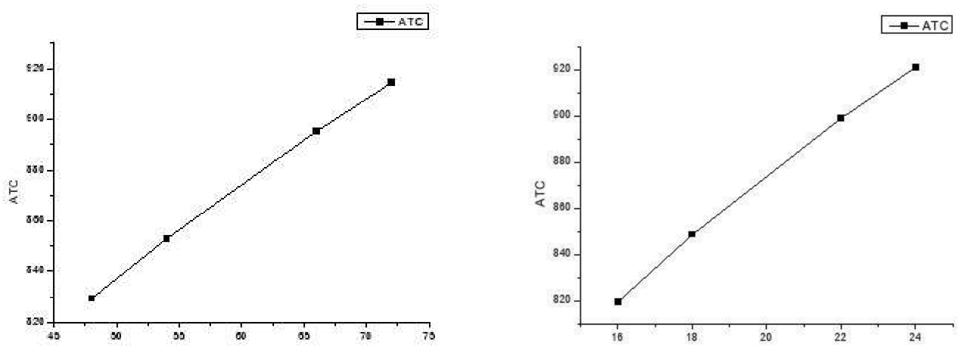

FIGURE 6. The Impact of ψ_1 with AT_JC and of ψ_2 with AT_JC

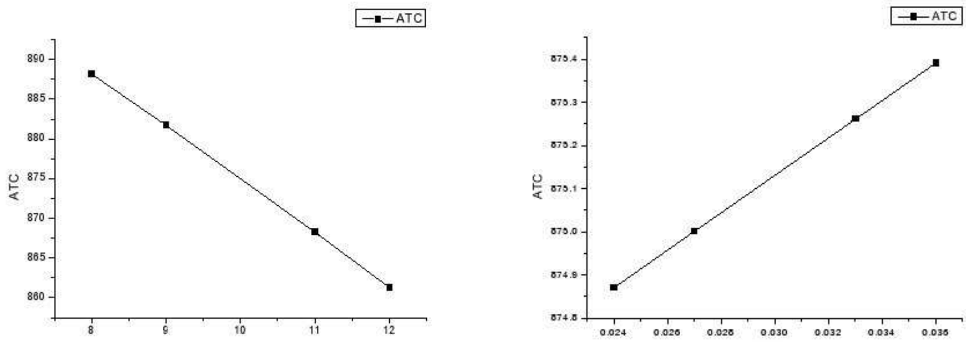

FIGURE 7. The Impact of ψ_3 with AT_JC and of θ_S with AT_JC

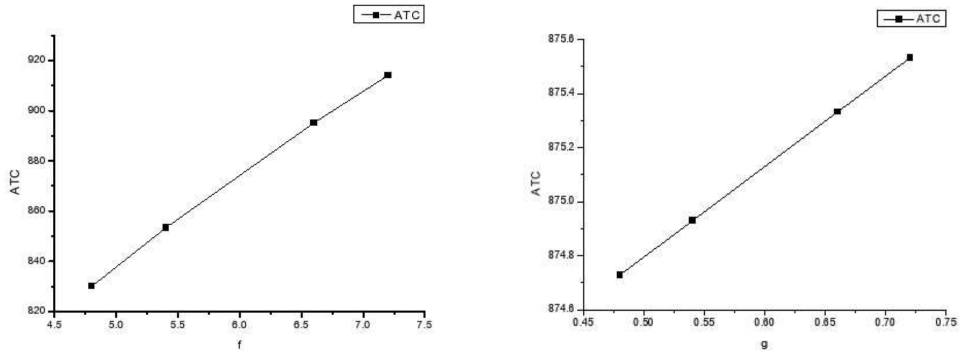

FIGURE 8. The Impact of f with AT_JC and of g with AT_JC

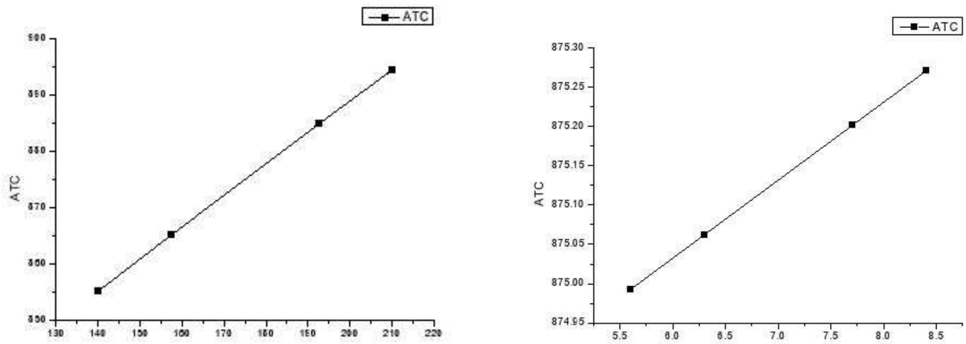

FIGURE 9. The Impact of C_{J_4} with AT_JC and of C_{J_6} with AT_JC

6. OBSERVATIONS USING YOUR TABLE VALUE

Here the investigation are using tabular values we can observe the following progress.

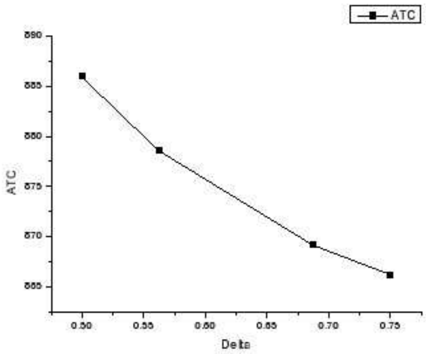

FIGURE 10. The Impact of δ with AT_JC

(1) The raising in α_1 results in time L_J decline and Time T_J also decline, there by Retailor total cost (RTC) has raising, Supplier total cost(STC) is also raising and over all Avg cost AT_JC has also been raising.
(2) The growing in α_2 results in time L_J raising and Time T_J also raising, there by Retailor total cost (RTC) has diminishing, Supplier total cost(STC) is also diminishing and over all Avg cost AT_JC has also been diminishing.
(3) The growing in P results in time L_J raising and Time T_J also raising, there by Retailor total cost (RTC) has diminishing, Supplier total cost(STC) is also diminishing and over all Avg cost AT_JC has also been diminishing.
(4) The growing in Retailor deterioration results in time L_J raising and Time T_J also raising, there by Retailor total cost (RTC) has increasing, Supplier total cost(STC) is also diminishing and over all Avg cost AT_JC has also been diminishing.
(5) The augmentation of holding Coefficient b_1 leads to the time raising in L_J and T_J, there by Retailor total cost (RTC), Supplier total cost(STC) and over all Avg cost AT_JC has oscillating.
(6) The growing in C_{J_2} results in time L_J and Time T_J is diminishing, there by Retailor total cost (RTC), Supplier total cost(STC) and over all Avg cost AT_JC has also been raising.
(7) The growing in C_{J_3} results in time L_J raising and Time T_J also raising, there by Retailor total cost (RTC) has raising, Supplier total cost(STC) is also diminishing and over all Avg cost AT_JC has also been raising.

(8) The growing in C_{J_5} results in time L_J diminishing and Time T_J is increasing, there by Retailor total cost (RTC) increasing, Supplier total cost(STC) diminishing and over all Avg cost AT_JC has also been raising.

(9) The growing in C_{J_7} results in time L_J Time T_J diminishing, there by Retailor total cost (RTC) increasing, Supplier total cost(STC) and over all Avg cost AT_JC has also been raising.

(10) The growing in λ results in time L_J and Time T_J is diminishing, there by Retailor total cost (RTC), Supplier total cost(STC) and over all Avg cost AT_JC has also been diminishing.

(11) The growing in ψ_1, ψ_1 results in time L_J and Time T_J, Retailor total cost (RTC) , Supplier total cost(STC) and over all Avg cost AT_JC has also been raising.

(12) The growing in ψ_3 results in time L_J and Time T_J, Retailor total cost (RTC) , Supplier total cost(STC) and over all Avg cost AT_JC has also been diminishing.

(13) The growing in θ_S, f results in time L_J and Time T_J, Retailor total cost (RTC) , Supplier total cost(STC) and over all Avg cost AT_JC has also been raising.

(14) The growing in g results in time L_J and Time T_J, Retailor total cost (RTC) , Supplier total cost(STC) has raising and over all Avg cost AT_JC is oscillating.

(15) The growing in C_{J_4}, C_{J_6} results in time L_J and Time T_J, Retailor total cost (RTC) , Supplier total cost(STC) and over all Avg cost AT_JC has also been raising.

(16) The growing in δ results in time L_J and Time T_J are raising , Retailor total cost (RTC) is oscillating , Supplier total cost(STC) has diminishing and over all Avg cost AT_JC is mounting.

7. Conclusion

The model of Joint activity of single Retailer and Supplier model of deteriorating items in supply chain with Price dependent demand in time linked Holding Cost of Changeable decision in lead time with shortages has arrived. To arrive this model we considered price dependent demand for Retailer and quadratic time function of supplier demand, The variable deterioration is considered. The holding cost of Retailer and Supplier were time dependent function. All computation process in these models is carried out by using the suitable software programming. Three level data checked for numerical problems. Using graphs the impact of overall average cost versus all parameters are shown. In some sit uation Retailer overall average is raising and few changes of parameters Supplier overall average cost is raising. Finally the progress of all parameters with joint (Retailer and Supplier) average overall cost is verified. All the changes we can observe using observa tion and computation table. This model can be further developed using the following assumptions namely Stochastic demand and two parameter of Weibull distribution taken as deterioration and by allowing shortages in Supplier and to reduce the deterioration using preservation technology etc.

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