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COMPUTATION OF EXPECTED LIVE BIRTHS AS PER PREFERENCE OF COUPLES STOPPING RULE

VASAM NAGARAJU[1], V. VASANTA KUMAR, AND K. VISWESWARA RAO

ABSTRACT. Owing to socio economic reasons, in some families couples may have preference over the number of male or female births they wish to have. Since, an important characteristics which distinguish births with respect to their sex ratio are age of mother, order of birth, sex and biological factors the number of male births or female births the couple wants to have may not be exactly as per their preference but will be more than that number. In this paper using a Probabilistic model we calculate the expected number of live births to the couple when their preference number is specified. Also, the sex ratio at birth is presented. The values have been calculated with the help of MATLAB and MS-Excel.

1. INTRODUCTION

The birth of a male or a female to a couple is a random phenomenon. Some couples may have preferences over the number of only male births or only female births or both to give birth. In such situations, the number of actual births will be more than their expectation. So the point of interest is what will be the expected number of live births?

[1]*corresponding author*

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Biggar et al. (1999) [1] found that the declining male: female ratio in Denmark from 1960-1964 and probably other European populations is mainly attributable to three effects: declining family size, biologic heterogeneity, and child sex preference. Wang and Gray (2009) investigated [2] changes in family sizes and sex ratios using the survey data obtained from students enrolled at three universities located in Shenyang, China. They reported that Binomial distribution and correlation analyses for the present generation demonstrated highly significant differences (P, 0.01) between observed and expected combination of sex in two-child families. The response to China's One Child Family Planning Policy has resulted in an unmatched one generation reduction in family size from 4.5 to 1.6 children. These results indicated that there may be a waning of the historically strong son preference.

Using the best estimate of the sex ratio from the [3] data (NHIS, 1998 to 2002),on families with two biological children (10 years of age and younger) Stanfield and Carlton (2006) found that there were significantly more families with [4] opposite-sex siblings than families with same-sex siblings.

Jayachandran (2014) quantified [5] the relationship between desired fertility and the sex ratio, focusing on India. She found that the desired sex ratio increases sharply as the fertility rate falls, and that fertility decline can explain 30-50% of India's sex ratio increase over the past thirty years. She also showed that female education could counter intuitively worsen the sex ratio because while it reduces the desired sex ratio at any given family size, it also reduces desired family size. Chai Bin Park 1983, has been investigated [6] the effects of son preferences on Sex Ratio and fertility at the family level, utilizing World fertility survey data for Korea, whose population is known to have a strong preference for sons and fairly high level of contraceptive use.

Peit Hein Jongbloet (2001). The Sex Ratio of all newborn [7] babies from 1751 to 1997 in order evaluate whether Finnish long-term data are compatible with the hypothesis that the decrease in the ratio of male to female births after World war I and World war II in industrial countries is caused by environmental factors.Reiko Mizuno MA (2000), analysed [8] about the decline in the male/female ratio of births has not yet been explained. In Japan, the male/female ratio of foetal deaths has been increasing since the 1970's reaching over 2.0 in 1996. FabioParrazzini (1998). The proportion [9] male: female ratio among the 29 countries, the proportion of males declined in 16, increased

in sex and remained stable in seven. Lillian Belmont, [10]it was found that birth order and family size had independent effects on intellectual performance. Effects of family size were not present in all social classes, but effects of birth order were consistent across social class.

Elmer gray and Jehud Bertolozzi (1977) have investigated [11] to extend the studies of the human sex ratio and of factors influencing family size to a subset of the population that has not experienced a high level of technological development.Gray E Duckworth D. Nakajima Y (1980). They have investigated to extend the studies of the human sex ratio and of the factors that influence family size to the Japanese population which has experienced a rapidly developed, high-level technology.

In this paper we build probabilistic model for the problem and used to calculate the expected number of live births to the couple when they want to have specified number of male births and female births or only male births or only female births. The rest of the paper is organized as follows: Mathematical model is presented in section-2. In section-3, expected numbers of births for specified values of the model parameters are presented in tabular form. Expected number of live births in the family and Sex Ratio at birth are computed and presented in section-4, Conclusion in section-5 and References in section-6.

2. Mathematical Model

Let α and β be the number of sons and daughters a couple wants to have. Since every couple is not fortunate to have exactly α boys and β girls, in the process N unwanted live births will born. N is a random variable whose range is $0, 1, 2, 3, ...$

Let X is the number of live births in the family. Then $X = \alpha + \beta + N$, is a random variable whose range is $\{\alpha + \beta, \alpha + \beta + 1, \alpha + \beta + 2, \alpha + \beta + 3, ...\}$.

Let p be the probability that the new born is a boy and $q = (1 - p)$ be the probability that the new born is a girl.

$$P(X = \alpha + \beta + N) = \binom{\alpha + \beta + N - 1}{\alpha - 1}.p^{\alpha}.q^{\beta+N} + \binom{\alpha + \beta + N - 1}{\beta - 1}.p^{\alpha+N}.q^{\beta}$$

$N = 0, 1, 2, 3, \ldots$

Expected number of live births in the family is given by

$$E(X) = \alpha + \beta + E(N)\,.$$

Since it is difficult to find the analytical expression for the E(X), we compute expected number of live births in the family for specific values of α, β, p and q numerically.

3. Particular Cases

Case(i) Let the couple wants to have only α boys. Then $\beta = 0$, Let Y= $\alpha + N$, where N is a random variable whose range is $\{0, 1, 2, 3, ...\}$ The probability distribution of Y is given by

$$P(X = \alpha + N) = \binom{\alpha + N - 1}{\alpha - 1} .p^{\alpha}.q^{N}, N = 0, 1, 2, 3, \ldots$$

$$E(Y) = \alpha + E(N)\,.$$

In particular if the couple wants to have only one boy,
Y = N+1; N = 0,1,2,3...
In this case, the probability mass function of Y is

$$P(X = N + 1) = p.q^{N}\,,\ N = 0, 1, 2, 3, ... \tag{3.1}$$

In equation (3.1) is the mass function of the geometric distribution with parameter 'p' Expected number of children in the family is given by $E(Y) = \frac{1}{p}$. When p = 0.55, the expected family size is 1.82.

Case (ii) Let the couple wants to have only β girls. Then $\alpha = 0$.
Let $Z = \beta + N$, where N is a random variable whose range is $0, 1, 2, 3, \ldots$
The probability distribution of Z is given by,

$$E(Z) = \beta + E(N)\,.$$

3.1. **SEX RATIO.** Sex ration is also an index of fertility. It is defined as,

$$\text{Sex Ratio} = \frac{(Numberoffemales)}{(Numberofmales)}\text{X } 1000\,.$$

This sex ratio at birth is a better index of fertility than the overall sex ratio. It is defined as:

$$\text{Sex ratio at birth} = \frac{Numberoffemalelivebirths}{Numberofmalelivebirths}\text{ X } 1000\,.$$

As per the 2011 Census the sex ratio was 943 in India. But when there is preference for the number of boys and girls the sex ratio calculated in Table-4 is almost 1000 which is a healthy number. Example of a definition.

4. Numerical Computation

In this section we present the numerical results for specified values of α,β , p and q.

Table 1: Expected number of live births (Total) and for different values of α,β, p and q

α	β	p	q	Total	α	β	p	q	Total
1	1	0.48	0.52	2.8606	2	2	0.51	0.49	5.0618
1	1	0.49	0.51	2.8597	2	2	0.52	0.48	5.0596
1	1	0.50	0.50	2.8594	2	3	0.48	0.52	6.0788
1	1	0.51	0.49	2.8597	2	3	0.49	0.51	6.0737
1	1	0.52	0.48	2.8606	2	3	0.50	0.50	6.0645
1	2	0.48	0.52	4.0799	2	3	0.51	0.49	6.0507
1	2	0.49	0.51	4.0912	2	3	0.52	0.48	6.0320
1	2	0.50	0.50	4.1016	3	1	0.48	0.52	5.1563
1	2	0.51	0.49	4.1108	3	1	0.49	0.51	5.1795
1	2	0.52	0.48	4.1187	3	1	0.50	0.50	5.1973
1	3	0.48	0.52	5.2177	3	1	0.51	0.49	5.2099
1	3	0.49	0.51	5.2099	3	1	0.52	0.48	5.2177
1	3	0.50	0.50	5.1973	3	2	0.48	0.52	6.0320
1	3	0.51	0.49	5.1795	3	2	0.49	0.51	6.0507
1	3	0.52	0.48	5.1763	3	2	0.50	0.50	6.0645
2	1	0.48	0.52	4.118	3	2	0.51	0.49	6.0737
2	1	0.49	0.51	4.1108	3	2	0.52	0.48	6.0788
2	1	0.50	0.50	4.1016	3	3	0.48	0.52	6.9867
2	1	0.51	0.49	4.0912	3	3	0.49	0.51	6.9959
2	1	0.52	0.48	4.0799	3	3	0.50	0.50	6.9990
2	2	0.48	0.52	5.0596	3	3	0.51	0.49	6.9959

2	2	0.49	0.51	5.0618	3	3	0.52	0.48	6.9867
2	2	0.50	0.50	5.0625					

TABLE 2. Expected number of live births when the couple wants to have only of α boys.

α	p	q	Expected no. of live male births
1	0.48	0.52	2.07
1	0.49	0.51	2.03
1	0.50	0.50	1.99
1	0.51	0.49	1.96
1	0.52	0.48	1.92
2	0.48	0.52	3.07
2	0.49	0.51	3.03
2	0.50	0.50	2.99
2	0.51	0.49	2.96
2	0.52	0.48	2.92
3	0.48	0.52	4.07
3	0.49	0.51	4.03
3	0.50	0.50	3.99
3	0.51	0.49	3.95
3	0.52	0.48	3.92
4	0.48	0.52	5.07
4	0.49	0.51	5.03
4	0.50	0.50	4.99
4	0.51	0.49	4.95
4	0.52	0.48	4.92

From table-1 it can be observed that expected number of child births shows inverse trend as p increases (q decreases) for different combinations of the values of α and β. It is noticed from table-2 that expected number of live births decreases as p increases (q decreases) and from table-3 this number decreases as q increases (p decreases).

TABLE 3. Expected number of livebirths when the couple wants to have only β girls

β	p	q	Expected no. of live male births
1	0.48	0.52	1.9191
1	0.49	0.51	1.9557
1	0.50	0.50	1.9937
1	0.49	0.51	2.0329
1	0.48	0.52	2.0735
2	0.52	0.48	2.9187
2	0.51	0.49	2.9553
2	0.50	0.50	2.9932
2	0.51	0.49	3.0323
2	0.52	0.48	3.0727
3	0.48	0.52	3.9184
3	0.49	0.51	3.9549
3	0.50	0.50	3.9927
3	0.51	0.49	4.0317
3	0.52	0.48	4.0720
4	0.48	0.52	4.9181
4	0.49	0.51	4.9545
4	0.50	0.50	4.9922
4	0.51	0.49	5.0311
4	0.52	0.48	5.0712

Table 4: Expected number of live births and births by sex for different values of α, β, p and q.

α	β	p	p	Live births	Male live births	Female live births
1	1	0.48	0.52	2.8606	1.3731	1.4875
1	1	0.49	0.51	2.8597	1.4013	1.4584
1	1	0.50	0.50	2.8594	1.4297	1.4297
1	1	0.51	0.49	2.8597	1.4584	1.4013
1	1	0.52	0.48	2.8606	1.4875	1.3731

1	2	0.48	0.52	4.0799	1.9584	2.1215
1	2	0.49	0.51	4.0912	2.0047	2.0865
1	2	0.50	0.50	4.1016	2.0508	2.0508
1	2	0.51	0.49	4.1108	2.0965	2.0143
1	2	0.52	0.48	4.1187	2.1417	1.9770
1	3	0.48	0.52	5.2177	2.5045	2.7132
1	3	0.49	0.51	5.2099	2.5529	2.6570
1	3	0.50	0.50	5.1973	2.5987	2.5987
1	3	0.51	0.49	5.1795	2.6415	2.5380
1	3	0.52	0.48	5.1763	2.6917	2.4846
2	1	0.48	0.52	4.1187	1.9770	2.1417
2	1	0.49	0.51	4.1108	2.0143	2.0965
2	1	0.50	0.50	4.1016	2.0508	2.0508
2	1	0.51	0.49	4.0912	2.0865	2.0047
2	1	0.52	0.48	4.0799	2.1215	1.9584
2	2	0.48	0.52	5.0596	2.4286	2.6310
2	2	0.49	0.51	5.0618	2.4803	2.5815
2	2	0.50	0.50	5.0625	2.5313	2.5313
2	2	0.51	0.49	5.0618	2.5815	2.4803
2	2	0.52	0.48	5.0596	2.6310	2.4286
2	3	0.48	0.52	6.0788	2.9178	3.1610
2	3	0.49	0.51	6.0737	2.9761	3.0976
2	3	0.50	0.50	6.0645	3.0323	3.0323
2	3	0.51	0.49	6.0507	3.0859	2.9648
2	3	0.52	0.48	6.0320	3.1366	2.8954
3	1	0.48	0.52	5.1563	2.4750	2.6813
3	1	0.49	0.51	5.1795	2.5380	2.6415
3	1	0.50	0.50	5.1973	2.5987	2.5987
3	1	0.51	0.49	5.2099	2.6570	2.5529
3	1	0.52	0.48	5.2177	2.7132	2.5045
3	2	0.48	0.52	6.0320	2.8954	3.1366
3	2	0.49	0.51	6.0507	2.9648	3.0859
3	2	0.50	0.50	6.0645	3.0323	3.0323

3	2	0.51	0.49	6.0737	3.0976	2.9761
3	2	0.52	0.48	6.0788	3.1610	2.9178
3	3	0.48	0.52	6.9867	3.3536	3.6331
3	3	0.49	0.51	6.9959	3.4280	3.5679
3	3	0.50	0.50	6.9990	3.4995	3.4995
3	3	0.51	0.49	6.9959	3.5679	3.4280
3	3	0.52	0.48	6.9867	3.6331	3.3536
			Total	228.1147	114.0578	114.057
					Sex-Ratio	999.993

5. CONCLUSION

The couples wish to have according to their preference of sons or daughters; then they must have to maintain certain limit of births (Stopping Rule). The population also can be under controlled, mean while the Sex Ratio followed by a healthy figure (1000), preference of son and daughter is nearly same probability. This has been computed and presented in above table-4.

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DEPARTMENT OF MATHEMATICS
RESEARCH SCHOLAR IN KONERU LAKSHMAIAH EDUCATION FOUNDATION
ANDHRA PRADESH, INDIA
E-mail address: nagaraju.stats@gmail.com

DEPARTMENT OF MATHEMATICS
KONERU LAKSHMAIAH EDUCATION FOUNDATION
ANDHRA PRADESH, INDIA
E-mail address: vvkumar@kluniversity.in

DEPARTMENT OF BIO-STATISTICS
(RETD.) NIN
HYDERABAD, INDIA
E-mail address: visweswararao.stats@gmail.com