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# A DECOMPOSITION OF NANO SEMI-$\mathcal{I}$-CONTINUITY

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ABSTRACT. In this paper, we introduce the notions of nano $\mathcal{A}^*_{\mathcal{R}}$-$\mathcal{I}$-sets, nano $\mathcal{A}^*_{\delta}$-$\mathcal{I}$-sets and nano $\mathcal{A}^*_{\mathcal{I}\delta}$-sets and investigate some of their basic properties. Also, we introduce the notions of their respective continuous functions and we obtain the decomposition of nano semi-$\mathcal{I}$-continuity and nano $\alpha$-$\mathcal{I}$-continuity in nano ideal topological spaces.

## 1. INTRODUCTION AND PRELIMINARIES

M. L. Thivagar [3] introduced the notion of nano topology which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. Let $U$ be a non-empty, finite universe of objects and $R$ be an equivalence relation on $U$ and $X \subseteq U$. Let $\tau_R(X) = \{U, \phi, U_R(X), L_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on $U$, called as the nano topology with respect to $X$. An Ideal $\mathcal{I}$ [7] on a topological space $(X, \tau)$ is a non-empty collection of subsets of $X$ which satisfies the following conditions: (1) $A \in \mathcal{I}$ and $B \subseteq A$ imply $B \in \mathcal{I}$ and (2) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$. Given a topological space $(X, \tau)$ with ideal $\mathcal{I}$ on $X$. If $P(X)$ is the family of all subsets of $X$, a set operator $(.)^* : P(X) \to P(X)$, called a local function, see [6], of $A$ with respect to $\tau$ and $\mathcal{I}$ is defined as follows: For $A \subseteq X$, $A^*(\mathcal{I}, \tau)$ = $\{x \in X : U \cap A \notin \mathcal{I}$, for all $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [3]. The

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topological space together with ideal on $X$ is called an ideal topological space and it is denoted by $(X, \tau, \mathcal{I})$. A nano topological space $(U, \tau_R(X))$ with an ideal $\mathcal{I}$ on $U$ is called a nano ideal topological space [4] or nano ideal space and is denoted by $(U, \tau_R(X), \mathcal{I})$.

Let $(U, \tau_R(X), \mathcal{I})$ be a nano ideal topological space. A set operator $(A)^{*N}$ : $P(U) \to P(U)$, is called the nano local function, see [5], of $\mathcal{I}$ on $U$ with respect to $\mathcal{I}$ on $\tau_R(X)$, is defined as $(A)^{*N} = \{x \in: U \cap A \notin \mathcal{I}$; for all $U \in \tau_R(X)\}$ and is denoted by $(A)^{*N}$, where nano closure operator is defined as $NCl^*(A) = A \cup (A)^{*N}$. Let $A$ be a subset of a nano topological space $(U, \tau_R(X))$, then $A$ is said to be regular open [3] (resp.nano regular closed [3]), if $A = NInt(NCl(A))$(resp.$A = NCl(NInt(A))$. Let $A$ be a subset of a nano ideal topological space $(U, \tau_R(X), \mathcal{I})$, then $A$ is said to be nano semi-$\mathcal{I}$-open [4] (resp. nano $\alpha$-$\mathcal{I}$-open [4] , nano regular-$\mathcal{I}$-open (nano $R$-$\mathcal{I}$-open) [4], nano pre-$\mathcal{I}$-open [2], nano $\beta$-$\mathcal{I}$-open [1], nano strong $\beta$-$\mathcal{I}$-open [1], nano $\delta$-$\mathcal{I}$-open, nano $\mathcal{I}_\delta$ set), if $A \subseteq NCl^*(NInt(A))$, (resp. $A \subseteq NInt(NCl^*(NInt(A)))$, $A = NInt(NCl^*(A))$, $A \subseteq NInt(NCl^*(A))$, $A \subseteq NCl(NInt(NCl^*(A)))$, $A \subseteq NCl^*(NInt(NCl^*(A)))$, $NInt(NCl^*(A)) \subseteq NCl^*(NInt(A))$, $NInt(A^*) \subseteq (NInt(A))^*$).

A mapping $f : \mathcal{U}_\mathcal{I} \to \mathcal{U}_1$ is said to be nano semi-$\mathcal{I}$-continuous [4] (resp. nano $\delta$-$\mathcal{I}$-continuous [1], nano strong $\beta$-$\mathcal{I}$-continuous [1], nano $\alpha$-$\mathcal{I}$-continuous [4], nano pre-$\mathcal{I}$-continuous [2]) if for every $V \in \tau_{R_1}(X_1)$, $f^{-1}$(V) is a nano semi-$\mathcal{I}$-open set (resp. nano $\delta$-$\mathcal{I}$-open set, nano strong $\beta$-$\mathcal{I}$-open set, nano $\alpha$-$\mathcal{I}$-open set, nano pre-$\mathcal{I}$-open set).

## 2. Nano $\mathcal{A}^*_\mathcal{R}$-$\mathcal{I}$-sets, nano $\mathcal{A}^*_\delta$-$\mathcal{I}$-sets and nano $\mathcal{A}^*_{\mathcal{I}\delta}$-sets

**Definition 2.1.** *Let $A$ be a subset of a nano ideal topological space $\mathcal{U}_\mathcal{I}$, then $A$ is said to be a:*

(i) *nano $\mathcal{A}^*_\mathcal{R}$-$\mathcal{I}$-set (briefly $\mathcal{N}\mathcal{A}^*_\mathcal{R}$-$\mathcal{I}$-set), if $A = S \cap V$, where $S$ is a nano closed set and $V$ is a nano regular-$\mathcal{I}$-open set.*

(ii) *nano $\mathcal{A}^*_\delta$-$\mathcal{I}$-set (briefly $\mathcal{N}\mathcal{A}^*_\delta$-$\mathcal{I}$-set), if $A = S \cap V$, where $S$ is a nano closed set and $V$ is a nano $\delta$ -$\mathcal{I}$-open set.*

(iii) *nano $A^*_{I\delta}$-set (briefly $NA^*_{I\delta}$-set), if $A = S \cap V$, where $S$ is a nano closed set and $V$ is a nano $\mathcal{I}_\delta$-set.*

**Proposition 2.1.** *Let $A$ be a subset of a nano ideal topological space $\mathcal{U}_{\mathcal{I}}$. Then the following hold:*

(1) *If $A$ is a nano closed set in $\mathcal{U}_{\mathcal{I}}$, then $A$ is a $\mathcal{N}\mathcal{A}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*
(2) *If $A$ is a nano regular-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$, then $A$ is a $\mathcal{N}\mathcal{A}^*_{\mathcal{R}}$ -$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*

*Proof.* (1). Let $A$ be a nano closed set in $\mathcal{U}_{\mathcal{I}}$. Then $A = U \cap A$, where $U$ is a nano regular-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$, which implies $A$ is a $\mathcal{N}\mathcal{A}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.
(2). Let $A$ be a nano regular-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. Then $A = U \cap A$, where $U$ is a nano closed set in $\mathcal{U}_{\mathcal{I}}$. Therefore $A$ is a $\mathcal{N}\mathcal{A}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.

□

**Remark 2.1.** *The converse of the Proposition 2.1 need not be true as shown in the following example.*

**Example 1.** *Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$, $X = \{b, d\}$, $\tau_R(X) = \{\phi, U, \{a, b, d\}, \{a\}, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. Then $\{d\}$ is a $\mathcal{N}\mathcal{A}^*_{\mathcal{R}}$-$\mathcal{I}$-set but not a nano closed set in $\mathcal{U}_{\mathcal{I}}$. The set {a,b,c} is a $\mathcal{N}\mathcal{A}^*_{\mathcal{R}}$-$\mathcal{I}$-set but not a nano regular-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$.*

**Theorem 2.1.** *Let $A$ and $B$ be subsets of a nano ideal topological space $\mathcal{U}_{\mathcal{I}}$. Then the following hold:*

(1) *If $A$ and $B$ are nano regular-$\mathcal{I}$-open, then $A \cap B$ is nano regular-$\mathcal{I}$-open.*
(2) *If $A$ is nano regular open, then it is nano regular-$\mathcal{I}$-open.*
(3) *If $A$ is nano regular-$\mathcal{I}$-open, then it is nano $\delta$-$\mathcal{I}$-open.*

*Proof.* (1). Let $A$ and $B$ be nano regular open sets. Then, we have $A \cap B = NInt(NCl^*(A) \cap NInt(NCl^*(B)) = NInt(NCl^*(A) \cap NCl^*(B)) \supseteq NInt(NCl^*(A \cap B)) \supseteq NInt(A \cap B) = A \cap B$. Thus, $A \cap B = NInt(NCl^*(A \cap B))$ which implies $A \cap B$ is a nano regular-$\mathcal{I}$ set.
(2). Let $A$ be a nano regular open. Since $\tau^*$ is finer than $\tau$ we have $A = NInt(A) \subseteq NInt(NCl^*(A)) \subseteq NInt(NCl(A)) = A$. Hence $A = NInt(NCl^*(A))$. Therefore $A$ is nano regular-$\mathcal{I}$-open.
(3). Let $A$ be nano regular-$\mathcal{I}$-open, then we have $A = NInt(NCl^*(A))$, which implies $NIntA = NInt(NCl^*(A)) \Rightarrow NInt(NCl^*(A)) \subseteq NInt(A) \subseteq NCl^*(NInt(A))$. Therefore $NInt(NCl^*(A)) \subseteq NCl^*(NInt(A))$. Hence $A$ is nano $\delta$ -$\mathcal{I}$-open. □

**Proposition 2.2.** *Every nano $\delta$-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$ is a nano $\mathcal{I}_\delta$-set in $\mathcal{U}_{\mathcal{I}}$.*

*Proof.* Let $A$ be a nano $\delta$-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. Then we have $NInt(NCl^*(A)) \subseteq NCl^*(NInt(A)) \Rightarrow NInt(A \cup A^*) \subseteq ((NInt(A)^* \cup (NInt(A))$. Now, $NInt(A) \cup NInt(A^*) \subseteq ((NInt(A)^* \cup (NInt(A))$ which shows that $(NInt(A^*) \subseteq (NInt(A))^*$. Thus $A$ is a nano $\mathcal{I}_\delta$-set in $\mathcal{U}_{\mathcal{I}}$. □

**Proposition 2.3.** *Let $A$ be a subset of a nano ideal topological space $\mathcal{U}_{\mathcal{I}}$. Then the following hold:*

(1) *If $A$ is a nano regular open in $\mathcal{U}_{\mathcal{I}}$, then $A$ is nano $\delta$-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$.*
(2) *If $A$ is a nano regular open in $\mathcal{U}_{\mathcal{I}}$, then $A$ is a nano $\mathcal{I}_\delta$-set in $\mathcal{U}_{\mathcal{I}}$.*

*Proof.* (1). Let $A$ be a nano regular-open set in $\mathcal{U}_{\mathcal{I}}$. By Proposition 2.1 (2), $A$ is nano regular-$\mathcal{I}$-open in $\mathcal{U}_{\mathcal{I}}$. By Proposition 2.1.(3), $A$ is a nano $\delta$-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$.
(2). Let $A$ be a nano regular-open set in $\mathcal{U}_{\mathcal{I}}$. By (1) $A$ is nano $\delta$-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. Thus, by Proposition 2.2, $A$ is nano $\mathcal{I}_\delta$-set in $\mathcal{U}_{\mathcal{I}}$. □

**Proposition 2.4.** *Intersection of two $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$ is a $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*

*Proof.* Since the intersection of two nano closed sets is a nano closed and intersection of two nano regular-$\mathcal{I}$-open set is nano regular-$\mathcal{I}$-open, The proof follows immediately. □

**Remark 2.2.** *The following examples shows that*

(1) *The converse of the Proposition 2.3 need not be true.*
(2) *The union of two nano $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$-set need not be a $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*

**Example 2.** *Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$, $X = \{b, d\}$ , $\tau_R(X) = \{\phi, U, \{a, b, d\}, \{d\}, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. Then,*

(1) *The set $\{a\}$ is a nano $\delta$-$\mathcal{I}$-open set and nano $\mathcal{I}_\delta$-set but not a nano regular-open set in $\mathcal{U}_{\mathcal{I}}$.*
(2) *The sets $A = \{d\}$ and $B = \{a, b\}$ are $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$ sets but the set $A \cup B = \{a, b, d\}$ is not a $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$ set in $\mathcal{U}_{\mathcal{I}}$.*

**Proposition 2.5.** *Let $\mathcal{U}_{\mathcal{I}}$ be a nano ideal topological space. Then the following hold:*

(1) *Every nano regular-$\mathcal{I}$-open set is nano open.*
(2) *Every nano $\delta$-$\mathcal{I}$-open set is a $\mathcal{NA}^*_{\delta}$-$\mathcal{I}$-set.*
(3) *Every nano $\mathcal{I}_\delta$-set ia a $\mathcal{NA}^*_{\delta}$-$\mathcal{I}$-set.*

*Proof.* (1). Let $A$ be a nano regular-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. Then $A = NInt(NCl^*(A))$, which shows that $NInt(A) = NInt(Cl^*(A))$. Now, $A = NInt(Cl^*(A))$ which implies $A \subseteq NInt(Cl^*(A)) = NInt(A)$. Also, we know that $NInt(A) \subseteq A$. Therefore $A = NInt(A)$ and hence $A$ is a nano open set in $\mathcal{U}_{\mathcal{I}}$.
(2). Let $A$ be a $\delta$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$. Then $A = A \cap U$, where $U$ is nano closed in $\mathcal{U}_{\mathcal{I}}$. Thus, $A$ is a $\mathcal{NA}^*_\delta$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.
(3). Let $A$ be a nano $\mathcal{I}_\delta$-set in $\mathcal{U}_{\mathcal{I}}$. Then $A = A \cap U$, where $U$ is nano closed in $\mathcal{U}_{\mathcal{I}}$. Thus, $A$ is a $\mathcal{NA}^*_\delta$ -$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$. □

**Proposition 2.6.** *For a subset $A$ of a nano ideal topological space $\mathcal{U}_{\mathcal{I}}$, the following hold:*

(1) *Every $\mathcal{NA}^*$-set is a $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*
(2) *Every $\mathcal{NA}^*_{\mathcal{R}}$ -$\mathcal{I}$-set is a $\mathcal{NA}^*_\delta$ -$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*
(3) *Every $\mathcal{NA}^*_\delta$-$\mathcal{I}$-set is a $\mathcal{NA}^*_{\mathcal{I}\delta}$-set in $\mathcal{U}_{\mathcal{I}}$.*

*Proof.* (1). Let $A$ be an $\mathcal{NA}^*$-set in $\mathcal{U}_{\mathcal{I}}$. Then $A = S \cap V$, where S is a nano closed set and V is a nano regular open set in $\mathcal{U}_{\mathcal{I}}$. By Proposition 2.1(2), $V$ is a nano regular-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$, which implies $A$ is a $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.
(2). Let $A$ be a $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$. Then $A = S \cap V$, where $S$ is a nano closed set and V is a nano regular-$\mathcal{I}$- open set in $\mathcal{U}_{\mathcal{I}}$. By Proposition 2.1 (3), $V$ is a nano $\delta$-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$, which implies $A$ is a $\mathcal{NA}^*_\delta$ -$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.
(3). Let $A$ be a $\mathcal{NA}^*_\delta$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$. Then $A = S \cap V$, where $S$ is a nano closed set and $V$ is a nano $\delta$-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. By Proposition 2.2, $V$ is a nano $\mathcal{I}_\delta$-set in $\mathcal{U}_{\mathcal{I}}$, which implies $A$ is a $\mathcal{NA}^*_{\mathcal{I}\delta}$-set in $\mathcal{U}_{\mathcal{I}}$. □

**Remark 2.3.** *The following examples shows that the converses of the Proposition 2.6 need not be true.*

**Example 3.** *Let $U = \{a, b, c, d\}$, $U/R = \{\{a, c\}, \{b\}, \{d\}\}$, $X = \{a, b\}$, $\tau_R(X) = \{\phi, U, \{a, b, c\}, \{b\}, \{a, c\}\}$ and $\mathcal{I} = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$. Then $\{a, b, c\}$ is a $\mathcal{NA}^*_{\mathcal{R}}$ -$\mathcal{I}$-set and but not a $\mathcal{NA}^*$-set in $\mathcal{U}_{\mathcal{I}}$.*

**Example 4.** *Let $U = \{a, b, c\}$, $U/R = \{\{a, b\}, \{c\}\}$, $X = \{c\}$, $\tau_R(X) = \{\phi, U, \{c\}\}$ and $\mathcal{I} = \{\phi, \{c\}\}$. Then*

(1) *The set $\{a\}$ is a $\mathcal{NA}^*_\delta$-$\mathcal{I}$-set but not a $\mathcal{NA}^*_{\mathcal{R}}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*
(2) *The set $\{b, c\}$ is a $\mathcal{NA}^*_{\mathcal{I}\delta}$-set but not a $\mathcal{NA}^*_\delta$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*

## 3. Decomposition of nano semi-$\mathcal{I}$-continuity

**Definition 3.1.** *A mapping $f : \mathcal{U}_{\mathcal{I}} \to \mathcal{U}_1$ is said to be $\mathcal{NA}_\delta^*$-$\mathcal{I}$-continuous if for every $V \in \tau_{R_1}(X_1)$, $f^{-1}$(V) is $\mathcal{NA}_\delta^*$-$\mathcal{I}$-set.*

**Proposition 3.1.** *Every nano $\alpha$-$\mathcal{I}$-open set is a nano pre-$\mathcal{I}$-open.*

*Proof.* Let $A$ be a nano $\alpha$-$\mathcal{I}$-open set. Then we have, $A \subseteq NInt(NCl^*(NInt(A))) \subseteq NInt(NCl^*(A))$. Therefore $A$ is nano pre-$\mathcal{I}$-open.

□

**Proposition 3.2.** *Let $\mathcal{U}_{\mathcal{I}}$ be a nano ideal topological spaces. Then a subset $A$ of $\mathcal{U}_{\mathcal{I}}$ is nano semi-$\mathcal{I}$-open in $\mathcal{U}_{\mathcal{I}}$ if and only if $A$ is nano $\delta$-$\mathcal{I}$-open and nano strong $\beta$-$\mathcal{I}$-open in $\mathcal{U}_{\mathcal{I}}$.*

*Proof.* **Necessity.** Let $A$ be a nano semi-$\mathcal{I}$-open set. Then we have $A \subseteq NCl^*(NInt(A))$, which implies, $A \subseteq NCl^*(NInt(A)) \subseteq NCl^*(NInt(NCl^*(A)))$. Thus, $A \subseteq NCl^*(NInt(NCl^*(A)))$ which shows that $A$ is nano strong $\beta$-$\mathcal{I}$-open. Now since $A$ is nano semi-$\mathcal{I}$-open, we have $A \subseteq NCl^*(NInt(A)) \Rightarrow NCl^*(A) \subseteq NCl^*(NCl^*(NInt(A))) \Rightarrow NCl^*(A) \subseteq NCl^*(NInt(A)) \Rightarrow NInt(NCl^*(A)) \subseteq NCl^*(A) \subseteq NCl^*(NInt(A))$. Thus, $NInt(NCl^*(A)) \subseteq NCl^*(NInt(A))$. Hence $A$ is nano $\delta$-$\mathcal{I}$-open.
**Sufficiency.** Let $A$ be a nano $\delta$-$\mathcal{I}$-open and nano strong $\beta$-$\mathcal{I}$-open in $(U, \tau_R(X), \mathcal{I})$, then we have $NInt(NCl^*(A)) \subseteq NCl^*(NInt(A))$ and $A \subseteq NCl^*(NInt(NCl^*(A)))$. Thus we get, $A \subseteq NCl^*(NInt(NCl^*(A))) \subseteq NCl^*(NCl^*(NInt(A)))$ which implies $A \subseteq NCl^*(NInt(A))$. Thus $A$ is nano semi-$\mathcal{I}$-open. □

**Remark 3.1.** *The following examples shows that the concept of nano $\delta$-$\mathcal{I}$-open sets and nano strong $\beta$-$\mathcal{I}$-open sets are independent.*

**Example 5.** *Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$, $X = \{b, d\}$, $\tau_R(X) = \{\phi, U, \{a, b, d\}, \{d\}, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. Then,*

1. *The set $\{c\}$ is a nano $\delta$-$\mathcal{I}$-open set but not a nano strong $\beta$-$\mathcal{I}$-open set.*
2. *The set $\{b, c\}$ is a nano strong $\beta$-$\mathcal{I}$-open set but not a nano $\delta$-$\mathcal{I}$-open set.*

**Theorem 3.1.** *Let $\mathcal{U}_{\mathcal{I}}$ be a nano ideal topological space. A subset $A$ of $\mathcal{U}_{\mathcal{I}}$ is nano $\alpha$-$\mathcal{I}$-open in $\mathcal{U}_{\mathcal{I}}$ if and only if $A$ is nano pre-$\mathcal{I}$-open and $\mathcal{NA}_\delta^*$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$.*

*Proof.* **Necessity:** Let $A$ be a nano $\alpha$-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. By Proposition 3.1(2), $A$ is a nano pre-$\mathcal{I}$-open set. By Proposition 3.1 (3), $A$ is a nano semi-$\mathcal{I}$-open set in

$\mathcal{U}_{\mathcal{I}}$. Since $A$ is a nano semi-$\mathcal{I}$-open in $\mathcal{U}_{\mathcal{I}}$. $A$ is nano $\delta$ -$\mathcal{I}$-open in $\mathcal{U}_{\mathcal{I}}$, by Theorem 3.2. Since $A$ is $\delta$-$\mathcal{I}$-open in $\mathcal{U}_{\mathcal{I}}$, it is an $\mathcal{NA}_{\delta}^{*}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$, by Proposition 2.5 (2). Therefore, $A$ is both nano pre-$\mathcal{I}$-open and $NA_{\delta}^{*}\mathcal{I}$ set in $\mathcal{U}_{\mathcal{I}}$.
**Sufficiency:** Let $A$ be a nano pre-$\mathcal{I}$-open and $\mathcal{NA}_{\delta}^{*}$-$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$. Since $A$ is nano pre-$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$, $A \subseteq NInt(NCl^{*}(A))$. Since $A$ is a $\mathcal{NA}_{\delta}^{*}$ -$\mathcal{I}$-set in $\mathcal{U}_{\mathcal{I}}$, $A = S \cap V$, where $S$ is nano closed and $V$ is nano $\delta$ -$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. Now, $A \subseteq NInt(NCl^{*}(A)) = NInt(NCl^{*}(S \cap V)) \subseteq NInt((NCl^{*}(S) \cap NCl^{*}(V))$ $= NInt((NCl^{*}(S)) \cap NInt(NCl^{*}(V)) \subseteq NInt(S) \cap NCl^{*}(NInt(V))$, (since $V$ is a nano $\delta$ -$\mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. ) $\subseteq NCl^{*}(NInt(S) \cap NInt(V)) = NCl^{*}(NInt(S \cap V))$ $= NCl^{*}(NInt(A))$. Thus $A \subseteq NCl^{*}(NInt(A))$. But $A \subseteq NInt(NCl^{*}(A)) \subseteq NInt(NCl^{*}(NCl^{*}(NInt(A)))$. Therefore $A \subseteq NInt((NCl^{*}(NInt(A))$. Hence $A$ is nano $\alpha - \mathcal{I}$-open set in $\mathcal{U}_{\mathcal{I}}$. □

**Remark 3.2.** *The concept of nano pre-$\mathcal{I}$-open and $\mathcal{NA}_{\delta}^{*}$-$\mathcal{I}$-set are independent as shown in the following examples.*

**Example 6.** *Let* $U = \{a, b, c, d\}$, $U/R = \{\{a, c\}, \{b\}, \{d\}\}$, $X = \{a, b\}$, $\tau_R(X) = \{\phi, U, \{a, b, c\}, \{b\}, \{a, c\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. *Then ,*

(1) *The set* $\{c\}$ *is a nano pre-$\mathcal{I}$-open set but not $\mathcal{NA}_{\delta}^{*}\mathcal{I}$-set.*
(2) *The set* $\{a, b, d\}$ *is $\mathcal{NA}_{\delta}^{*}\mathcal{I}$-set but not nano pre-$\mathcal{I}$-open set.*

**Remark 3.3.** *The notion of nano $\delta$-$\mathcal{I}$-continuity is independent of notion of strong $\beta$-$\mathcal{I}$-continuity, as seen from the following examples.*

**Example 7.** *Let* $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$, $X = \{b, d\}$, $\tau_R(X) = \{\phi, U, \{a, b, d\}, \{d\}, \{a, b\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}\}$. *Let* $U_1 = \{x, y, z, w\}$, *Let* $U_1/R_1 = \{\{x, y\}, \{z\}, \{w\}\}$, $X_1 = \{z\}$, $\tau_{R_1}(X_1) = \{\phi, U_1, \{z\}\}$. *Now define* $f : \mathcal{U}_{\mathcal{I}} \to \mathcal{U}_1$ *as* $f(a) = x$, $f(b) = y$, $f(c) = z$, $f(d) = w$. *Then $f$ is nano $\delta$-$\mathcal{I}$-continuous but not nano strong $\beta$-$\mathcal{I}$-continuous.*

**Example 8.** *Let* $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$, $X = \{b, d\}$, $\tau_R(X) = \{\phi, U, \{a, b, d\}, \{d\}, \{a, b\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}\}$. *Let* $U_1 = \{x, y, z, w\}$, $U_1/R_1 = \{\{x, y\}, \{z\}, \{w\}\}$, $X_1 = \{x, y\}$, $\tau_{R_1}(X_1) = \{\phi, U_1, \{x, y\}\}$. *Now define* $f : \mathcal{U}_{\mathcal{I}} \to \mathcal{U}_1$ *as* $f(a) = w$, $f(b) = x$, $f(c) = y$, $f(d) = w$. *Then $f$ is nano strong $\beta$-$\mathcal{I}$-continuous but not nano $\delta$-$\mathcal{I}$-continuous.*

**Remark 3.4.** *The notion of nano pre-$\mathcal{I}$-continuity is independent of notion of $\mathcal{NA}_{\delta}^{*}$-$\mathcal{I}$-continuity, as seen from the following examples.*

**Example 9.** *Let* $U = \{a, b, c, d\}$, $U/R = \{\{a, c\}, \{b\}, \{d\}\}$*,* $X = \{a, b\}$, $\tau_R(X) = \{\phi, U, \{a, b, c\}, \{b\}, \{a, c\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}, \{b\}, \{a, b\}\}$. *Let* $U_1 = \{x, y, z, w\}$, $U_1/R_1 = \{\{x, y\}, \{z\}, \{w\}\}$, $X_1 = \{z\}$, $\tau_{R_1}(X_1) = \{\phi, U_1, \{z\}\}$. *Now define* $f : \mathcal{U}_{\mathcal{I}} \to \mathcal{U}_1$ *as* $f(a) = x$*,* $f(b_1) = y$, $f(c) = z$*,* $f(d) = w$. *Then* $f$ *is nano pre-*$\mathcal{I}$*-continuous but not* $\mathcal{NA}^*_\delta$ *-*$\mathcal{I}$*-continuous.*

**Example 10.** *Let* $U = \{a, b, c, d\}$, $U/R = \{\{a, c\}, \{b\}, \{d\}\}$, $X = \{a, b\}$, $\tau_R(X) = \{\phi, U, \{a, b, c\}, \{b\}, \{a, c\}\}$*,* $\mathcal{I} = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, $U_1 = \{x, y, z, w\}$, $U_1/R_1 = \{\{x, y\}, \{z\}, \{w\}\}$, $X_1 = \{x, y, z\}$, $\tau_{R_1}(X_1) = \{\phi, U_1, \{x, y, z\}\}$. *Now define* $f : \mathcal{U}_{\mathcal{I}} \to \mathcal{U}_1$ *as* $f(a) = x$*,* $f(b) = y$, $f(c) = w$, $f(d) = z$. *Then* $f$ *is* $\mathcal{NA}^*_\delta$*-*$\mathcal{I}$*-continuous but not nano pre-*$\mathcal{I}$*-continuous.*

**Theorem 3.2.** *For a mapping* $f : \mathcal{U}_{\mathcal{I}} \to \mathcal{U}_1$, *then, f is nano semi-*$\mathcal{I}$*-continuous if and only if f is nano* $\delta$*-*$\mathcal{I}$*-continuous and nano strong* $\beta$*-*$\mathcal{I}$*-continuous.*

**Theorem 3.3.** *For a mapping* $f : \mathcal{U}_{\mathcal{I}} \to \mathcal{U}_1$, *then, f is nano* $\alpha$*-*$\mathcal{I}$*-continuous if and only if f is nano pre-*$\mathcal{I}$*-continuous and* $\mathcal{NA}^*_\delta$*-*$\mathcal{I}$*-continuous.*

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