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ON INTUITIONISTIC FUZZY NANO SEMI GENERALIZED CONTINUOUS MAPPINGS

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ABSTRACT. In this article, we define a new class of continuous functions called intuitionistic fuzzy nano semi generalized continuous functions and discuss some of its properties in terms of *IFNsg*-closed sets, *IFNsg*-open sets, *IFNsg*-closure and *IFNsg*-interior.

1. INTRODUCTION

Nano set theory proposed by Lellis Thivagar and Carmel Richard in [3] is a new concept that supports uncertainty. It may be seen as an extension of classical set theory and has been in applied decision analysis. Its basic structure is an approximation space. Bhattacharyya et.al in [1] investigated semi generalized closed sets. Noiri in [4] gave the remarks on semi-open mappings. Devi et.al in [2] defined semi generalized closed maps. Stephan Antony Raj et.al in [5-7], introduced and studied the properties of intuitionistic fuzzy nano generalized and nano semi generalized closed sets in intuitionistic fuzzy nano topological spaces. In this article, we investigate further on the notion of intuitionistic fuzzy nano semi generalized closure and intuitionistic fuzzy nano semi generalized interior and study a new class of continuous functions called intuitionistic fuzzy nano semi generalized continuous functions and discuss some of its properties in terms of *IFNsg*-closed sets, *IFNsg*-open sets, *IFNsg*-closure and *IFNsg*-interior.

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2. PRELIMINARIES

Definition 2.1. *[7] A subset A of an intuitionistic fuzzy nano topological space U is said to be intuitionistic fuzzy nano semi-generalized closed (briefly, IFNsg-closed), if IFNscl(A) $\subseteq$ G whenever G is intuitionistic fuzzy nano semi-open (In short IFNSO) and A $\subseteq$ G. The set A is said to be intuitionistic fuzzy nano semi-generalized open (IFNsg-open) if A^c is intuitionistic fuzzy nano semi-generalized closed(IFNsg-closed).*

Definition 2.2. *[7] If A $\subseteq$ U ,then the intuitionistic fuzzy nano semi generalized closure denoted by IFNsg-Cl(A) is defined as the smallest intuitionistic fuzzy nano sg-closed set containing A. The intuitionistic fuzzy nano semi generalized interior of A , denoted by IFNsg-Int(A) is defined as the largest intuitionistic fuzzy nano sg-open subset of A.*

Definition 2.3. *[6] A mapping f* : $(\mathrm{U},\tau_{\mathrm{R}}(\mathrm{X})) \rightarrow (\mathrm{V},\tau_{\mathrm{R'}}(\mathrm{Y}))$ *is said to be intuitionistic fuzzy nano generalized continuous (In short IFNGC) if the inverse image of every intuitionistic fuzzy nano closed set of V is intuitionistic fuzzy nano g-closed(briefly IFNg-open) in U.*

Definition 2.4. *[8] Let X and Y be IFT's. A mapping f : X $\rightarrow$ Y is called intuitionistic fuzzy semi-generalized continuous (intuitionistic fuzzy sg-continuous), if f^{-1}(B) is an IFSGCS in X for every IFCS B in Y .*

3. INTUITIONISTIC FUZZY SEMI-GENERALIZED CONTINUOUS MAPPING AND ITS PROPERTIES

This area examines into *IFNsg*-continuous and the characterizations of *IFNsg*-continuous mappings in terms of *IFNsg*-open sets, image of *IFNsg*-closure and converse image of *IFNsg*-closure are likewise inferred. Thus, proportional characterizations of *IFNsg*-open mappings are likewise decided.

Definition 3.1. *Let* $(S,\, \tau_R(Y)\,)$ *and* $(\mathrm{T},\tau_{\mathrm{R'}}(\mathrm{Z}))$ *be two IFNTS. Then a function* $f\ :\ (\mathrm{S},\tau_{\mathrm{R}}(\mathrm{Y}))\ \rightarrow\ (\mathrm{T},\tau_{\mathrm{R'}}(\mathrm{Z}))$ *is said to be intuitionistic fuzzy nano semi generalized continuous (IFNsg-continuous), if the inverse image of every IFNCS in T is IFNsg-closed in S.*

Example 1. *Let (S, R) be an intuitionistic fuzzy approximation space (In short IFAS) where* $S = \{a_1, a_2, a_3\}$ *with* $R = \{\langle(a_1, a_1), 1, 0\rangle, \langle(a_1, a_2), 0.5, 0.5\rangle, \langle(a_2, a_1), 0.5, 0.5\rangle, \langle(a_2, a_2), 1, 0\rangle, \langle(a_2, a_3), 0.3, 0.7\rangle, \langle(a_3, a_2), 0.3, 0.7\rangle, \langle(a_3, a_3), 1, 0\rangle, \langle(a_1, a_3), 0.4, 0.6\rangle, \langle(a_3, a_1), 0.4, 0.6\rangle\}$.
Let $B = \{\langle a_1, 0.7, 0.3\rangle, \langle a_2, 0.5, 0.5\rangle, \langle a_3, 0.61, 0.31\rangle\}$ *be an intuitionistic fuzzy set (In short IFS) on S then*
$\tau_R(Y) = \{1\sim, 0\sim, \{\langle a_1, 0.7, 0.3\rangle, \langle a_2, 0.5, 0.5\rangle, \langle a_3, 0.6, 0.31\rangle\}, \{\langle a_1, 0.5, 0.5\rangle, \langle a_2, 0.5, 0.5\rangle, \langle a_3, 0.61, 0.31\rangle\}, \{\langle a_1, 0.5, 0.5\rangle, \langle a_2, 0.5, 0.5\rangle, \langle a_3, 0.31, 0.61\rangle\}\}$.
Let (T, R) be an IFAS where $T = \{u, v, w\}$ *with* $R = \{\langle(u, u), 1, 0\rangle, \langle(u, v), 0.21, 0.21\rangle, \langle(v, u), 0.21, 0.21\rangle, \langle(v, v), 1, 0\rangle, \langle(v, w), 0.31, 0.31\rangle, \langle(w, v), 0.31, 0.31\rangle, \langle(w, w), 1, 0\rangle, \langle(u, w), 0.11, 0.21\rangle, \langle(w, u), 0.11, 0.21\rangle\}$.
Let $D = \{\langle u, 0.51, 0.31\rangle, \langle v, 0.51, 0.41\rangle, \langle w, 0.41, 0.31\rangle\}$ *be an intuitionistic fuzzy set (In short IFS) on T then*
$\tau_R(Z) = \{1\sim, 0\sim, \{\langle u, 0.51, 0.31\rangle, \langle v, 0.51, 0.31\rangle, \langle w, 0.41, 0.31\rangle\}, \{\langle u, 0.41, 0.31\rangle, \langle v, 0.41, 0.41\rangle, \langle w, 0.41, 0.31\rangle\}, \{\langle u, 0.31, 0.41\rangle, \langle v, 0.41, 0.41\rangle, \langle w, 0.31, 0.41\rangle\}\}$.
Let $C = \{\langle u,0.31,0.51\rangle, \langle v,0.31,0.51\rangle, \langle w,0.31,0.41\rangle\}$ *be an IFCS in T is IFNsg-closed in S. Hence, it is intuitionistic fuzzy nano semi generalized continuous (IFNsg-continuous), since the inverse image of every IFNCS in T is IFNsg-closed in S.*

Remark 3.1. *Since the complement of an IFNCS is IFNOS and the complement of an IFNsg-closed (IFNg-closed) set is IFNsg-open (IFNg-open), f is IFNg-continuous if and only if the inverse image of every IFNOS in T is IFNg-open in S. Also, f is IFNsg-continuous, if and only if the inverse image of every IFNOS in T is IFNsg-open in S.*

Theorem 3.1. *Every IFN-continuous function is IFNsg-continuous.*

Proof. If $f : (\mathrm{S}, \tau_{\mathrm{R}}(\mathrm{Y})) \rightarrow (\mathrm{T}, \tau_{\mathrm{R}'}(\mathrm{Z}))$ is *IFN*-continuous on *S* and if *H* is IFNOS in *T*, then $f^{-1}(H)$ is IFNOS in *S*. Therefore, $f^{-1}(H)$ is *IFNsg*-open, since any IFNOS is IFNSO and any IFNSO set is *IFNsg*-open. Therefore, *f* is *IFNsg*-continuous. □

Remark 3.2. *An IFNsg-continuous function on an IFNTS is not IFN-continuous. For example, that is, the inverse image of every IFNCS in T is IFNsg-closed in S. Therefore, f is IFNsg-continuous, but f is not IFN-continuous.*

Theorem 3.2. *The following are equivalent for any function* $f : (\mathrm{S}, \tau_{\mathrm{R}}(\mathrm{Y})) \rightarrow (\mathrm{T}, \tau_{\mathrm{R}'}(\mathrm{Z}))$

(i) *f is nano IFNsg-continuous.*
(ii) *for every* $x \in S$ *and for every IFNOS G in T such that* $f(x) \in G$*, there exists a IFNsg-open set H in S such that* $x \in H$ *and* $f(H) \subseteq G$.
(iii) *for every* $B \subseteq S$, f *(IFNsg-Cl(B))* $\subseteq$ *IFNCl(f (B))*
(iv) *for every* $D \subseteq T$, *IFNsg-Cl*$(f^{-1}(D)) \subseteq f^{-1}$*(IFNCl(D)).*

Proof. Assume (i). Let $x \in S$ and G be a IFNOS in T such that $f(x) \in G$. Since f is *IFNsg*-continuous, $f^{-1}(G)$ is *IFNsg*-open in S. Let $K = f^{-1}(G)$. Then $f(K) = G$ where K is *IFNsg*-open in S and $x \in K$. That is (i) $\Rightarrow$ (ii).

Now, assume (ii). Let G be IFNOS in T. We know that *IFNsg-Int*$(f^{-1}(G)) \subseteq f^{-1}(G)$. Let $x \in f^{-1}(G)$. Then $f(x) \in G$ and G is IFNOS in T. Therefore, there exists an *IFNsg*-open set F such that $x \in F$ and $f(K) \subseteq G$. If $x \notin$ *IFNsg-Int*$(f^{-1}(G))$, then $K \not\subseteq f^{-1}(G)$ for every *IFNsg*-open set K in S containing x and hence, in particular, $K \not\subseteq f^{-1}(G)$. Therefore, $f(K) \not\subseteq G$, which is a contradiction. Therefore, $x \in$ *IFNsg-Int*$(f^{-1}(G))$. Thus, $f^{-1}(G) \subseteq$ *IFNsg-Int*$(f^{-1}(G))$ and therefore, $f^{-1}(G)$ = *IFNsg-Int*$(f^{-1}(G))$. Hence $f^{-1}(G)$ is *IFNsg*-open in S. Thus, the inverse image of every IFNOS in T is *IFNsg*-open in S. Hence, f is *IFNsg*-continuous. Thus (ii) $\Rightarrow$ (i). Therefore (i) $\Leftrightarrow$ (ii).

Now, let $B \subseteq S$ and let $y \in f$ (*IFNsg-Cl*(B)). Then $y = f(x)$ for some $x \in$ *IFNsg-Cl*(B). Let G be an IFNOS in T such that $y \in G$. Then there exists an *IFNsg*-open set K such that $x \in K$ and $f(K) \subseteq G$, by (ii). Since $x \in$ *IFNsg-Cl*(B), $K \cap B = 0\sim$. Therefore, $f(K \cap B) = 0\sim$. Since $f(K) \subseteq G$, $G \cap f(B) = 0\sim$. Thus, whenever G is IFNOS in T and $x \in G$, $G \cap f(B) = 0\sim$. Therefore, $y \in$ *IFNCl*$(f(B))$. Therefore, f (*IFNsg-Cl*$(B)) \subseteq$ *IFNCl*$(f(B))$. That is (ii) $\Rightarrow$ (iii).

Next, assume (iii). Let $x \in S$ and G be an IFNOS in T such that $f(\mathrm{x}) \in G$. Let $B = f^{-1}(G^c)$. Then $x \notin B$ and G^c is IFNCS. Also by (iii), f (*IFNsg-Cl*$(B)) \subseteq$ *IFNCl*$(f(B)) \subseteq G^c$, since *IFNClf*(B) = *IFNCl*$(f(f^{-1}(G^c))) \subseteq$ *IFNCl*$(G^c) = G^c$, as G^c is IFNCS. Therefore, f (*IFNsg-Cl*$(B)) \subseteq G^c$. That is, *IFNsg-Cl*$(B) \subseteq f^{-1}(G^c) = B$. Therefore, B = *IFNsg-Cl*(B). Since $x \notin B$, $x \notin$ *IFNsg-Cl*$(B.)$ Therefore, there exists a *IFNsg*-open set K containing x such that $K \cap B = 0\sim$. That is, $K \subseteq B^c$. Thus, $f(K) \subseteq f(B^c) \subseteq G$. Thus for every $x \in S$ and for every IFNOS G in T containing $f(x)$, there exists a *IFNsg*-open set K containing x such that $f(K) \subseteq G$. Therefore, (iii) $\Rightarrow$ (ii). Thus (ii) $\Leftrightarrow$ (iii).

Now assume (iii). Let $D \subseteq T$. Then $f^{-1}(D) \subseteq S$. Let $B = f^{-1}(D)$ in (iii). Then f (*IFNsg-Cl*$(f^{-1}(D))) \subseteq$ *IFNCl* $f(f^{-1}(D)) \subseteq$ *IFNCl*(D). Therefore, *IFNsg-Cl*$(f^{-1}(D)) \subseteq f^{-1}($*IFNCl*$(D))$. That is, (iii) $\Rightarrow$ (iv). If (iv) holds and B $\subseteq S$, then $f(B) \subseteq T$. Let $D = f(B)$, in (iv). Then *IFNsg-Cl*$(f^{-1}(f(B))) \subseteq f^{-1}($*IFNCl*$(f(B)))$. That is, *IFNsg-Cl*$(B) \subseteq f^{-1}($*IFNCl*$(f(B)))$. Therefore, f (*IFNsg-Cl*$(B)) \subseteq$ *IFNCl*$(f(B))$. That is, (iv) $\Rightarrow$ (iii). Thus, (iii) $\Leftrightarrow$ (iv). □

Theorem 3.3. *The following are equivalent for any function* $f : (\mathrm{S}, \tau_{\mathrm{R}}(\mathrm{Y})) \rightarrow (\mathrm{T}, \tau_{\mathrm{R}'}(\mathrm{Z}))$

(i) *f is nano IFNsg-open.*
(ii) *f (IFNInt(B))* $\subseteq$ *IFNsg-Int(f (B)), for every B* $\subseteq$ *S.*
(iii) *for every x* $\in$ *S and for each IFNOS H such that x* $\in$ *H, there exists a IFNsg-open set G such that f(x)* $\in$ *G, and G* $\subseteq$ *f (H).*
(iv) *for every D* $\subseteq$ *T,* f^{-1}*(IFNsg-Cl(D))* $\subseteq$ *IFNCl(*f^{-1}*(D)).*

Proof. Assume (i). Let $B \subseteq S$ and $y \in f$ (*IFNInt*(B)). Then $y = f(x)$ for some $x \in$ *IFNInt*(B). Since $x \in$ *IFNInt*(B), $x \in H$ for some IFNOS $H \subseteq B$. Therefore, $y = f(x)$ for some $x \in H$ where H is IFNOS and $H \subseteq B$. Since f is *IFNsg*-open, $f(H)$ is *IFNsg*-open. Also, $f(x) \in f(H)$ and $f(H) \subseteq f(B)$. Therefore, $y = f(x) \in$ *IFNsg-Int*$(f(B))$. Thus, f (*IFNInt*$(B)) \subseteq$ *IFNsg-Int*$(f(B))$. Thus, (i) $\Rightarrow$ (ii).

Next, assume (ii). Let H be an IFNOS in S. Since $H \subseteq B$, by (ii), f (*IFNInt*$(H)) \subseteq$ *IFNsg-Int*$(f(H))$. That is, $f(H) \subseteq$ *IFNsg-Int*$(f(H))$, since H is IFNOS. But *IFNsg-Int*$(f(H)) \subseteq f(H)$. Therefore, $f(G) =$ *IFNsg-Int*$(f(H))$. Thus, $f(H)$ is *IFNsg*-open whenever H is IFNOS in S. Therefore, f is *IFNsg*-open. Thus, (ii) $\Rightarrow$ (i). Hence, (i) $\Leftrightarrow$ (ii).

Now, assume (ii). Let $x \in S$ and H be IFNOS in S such that $x \in H$. By (ii), f (*IFNInt*$(H)) \subseteq$ *IFNsg-Int*$(f(H))$. That is, $f(H) \subseteq$ *IFNsg-Int*$(f(H))$. Therefore, $f(H) =$ *IFNsg-Int*$(f(H))$. Therefore, $f(H)$ is *IFNsg*-open. Thus there exists a *IFNsg*-open set K, namely $f(H)$ such that $f(x) \in K$. That is, (ii) $\Rightarrow$ (iii).

Assume (iii). Let $B \subseteq S$ and Let $y \in f$ (*IFNInt*(B)). Then $y = f(x)$ where $x \in$ *IFNInt*(B). Then $x \in H$ for some IFNOS $H \subseteq B$. Therefore, by (iii), there exists an *IFNsg*-open set K containing y such that $K \subseteq f(H) \subseteq f(B)$. Thus, there exists an *IFNsg*-open set K containing y such that $K \subseteq f(B)$. Therefore, $y \in$ *IFNsg-Int*(f(A)). Thus, f (*IFNInt*$(B)) \subseteq$ *IFNsg-Int*$(f(B))$. Thus (ii) $\Leftrightarrow$ (iii).

Now, assume (iii). Let $D \subseteq T$ and $x \in f^{-1}($*IFNsg-Cl*$(D))$. If $x \notin$ IFNCl$(f^{-1}(D))$, then $x \in S -$ *IFNCl*$(f^{-1}(D))$, which is an IFNOS in S. Therefore, there exists an *IFNsg*-open set K such that $f(x) \in K$ and $K \subseteq f(S - IFNCl(f^{-1}(D))) \subseteq f(S - f^{-1}(D))$, since $f^{-1}(D) \subseteq IFNCl(f^{-1}(D))$. Thus, $K \subseteq f(S - f^{-1}(D)) \subseteq T - D$. Therefore, $D \subseteq K^c$, where K^c is *IFNsg*-closed. Therefore, $IFNsg\text{-}Cl(D) \subseteq K^c$, where $f(x) \in IFNsg\text{-}Cl(D)$ but $f(x) \notin K^c$ which is a contradiction. Therefore, $x \in IFNCl(f^{-1}(D))$. Thus, $f^{-1}(IFNsg\text{-}Cl(D)) \subseteq IFNCl(f^{-1}(D))$, for every $D \subseteq T$. Hence, (iii) $\Rightarrow$ (iv).

Now assume (iv). Let $x \in S$ and H be IFNOS in S such that $x \in H$. Let $D = f(H^c)$ where H^c is an IFNCS. Since $x \in H$, $f(x) \notin D$. Since $D \subseteq T$, $f^{-1}(IFNsg\text{-}Cl(D)) \subseteq IFNCl(f^{-1}(D)) = IFNCl(H^c) = H^c$.
That is, $f^{-1}(IFNsg\text{-}Cl(D)) \subseteq H^c$. Then $IFNsg\text{-}Cl(D) \subseteq f(H^c) = D$. Therefore, $D = IFNsg\text{-}Cl(D)$. There exists an *IFNsg*-open set K in T containing $f(x)$ such that $D \cap K = 0\sim$. Therefore, $K \subseteq D^c$. Then $f^{-1}(K) \subseteq f^{-1}(D^c) \subseteq H$. Therefore, $K \subseteq f(H)$. Thus, K is an *IFNsg*-open set in T such that $f(x) \in K$ and $K \subseteq f(H)$. Thus (iv) $\Rightarrow$ (iii). Hence (iii) $\Leftrightarrow$ (iv). □

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