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ON WEAKLY G''-CONTINUOUS MAPPING AND WEAKLY G''-IRRESOLUTE MAPPING IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

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ABSTRACT. In this paper,I introduce the notions of intuitionistic fuzzy weakly g''-continuous mappings, intuitionistic fuzzy weakly g''- irresolute mappings, separation Axioms and study some of its properties in intuitionistic fuzzy topological space.

1. INTRODUCTION

The intuitionistic fuzzy set was introduced by Atanassov [1]. In 1997, Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. Continuity is a property of transformation one space to another. It is a natural curiosity to study how does the fuzziness of continuity passes the information of spatial characteristic under transformation. In 1997, Gurcay, Coker and Haydar [4] have introduced the Continuous mapping . Here the intuitionistic fuzzy weakly g''-continuous mapping and intuitionistic fuzzy weakly g''-irresolute mapping and its properties are introduced.

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2. PRELIMINARIES

Here (W,τ) or W denoted as the IF-topological space. The closure,the interior and the complement of a subset Pof Ware denoted by c(P), i(P) and P^c respectively.

Definition 2.1. *[1] Consider W is a non empty set and IF-set Pin W is of the form*$P = \{< w, \mu_P(w), \nu_P(w) > /w \in W\}$ *where the functions* $\mu_P(w) : W \to [0,1]$ *and* $\nu_P(w) : W \to [0,1]$ *denotes the degree of membership and the degree of non membership of the element w∈ W respectively and* $0 \leq \mu_P(w)+\nu_P(w) \leq 1, w \in W$.

Definition 2.2. *[1] Consider P and Q are the IFSets of the form* $P = \{< w, \mu_P(w), \nu_P(w) > /w \in W\}$ *and* $Q = \{< w, \mu Q(w), \nu_Q(w) > /w \in W\}$. *Then:*

(i) $P \subseteq Q$ *if and only if* $\mu_P(w) \leq \mu Q(w)$*and* $\nu_P(w) \geq \nu_Q(w)$ *for all* $w \in W$,
(ii) *P=Q if and only if* $P \subseteq Q$ *and*$Q \subseteq P$,
(iii) $P^c = \{< w, \nu_P(w), \mu_P(w) > /w \in W\}$,
(iv) $P \cap Q = \{< w, \mu_P(w) \wedge \mu Q(w), \nu_P(w) \vee \nu_Q(w) > /w \in W\}$,
(v) $P \cup Q = \{< w, \mu_P(w) \vee \mu Q(w), \nu_P(w) \wedge \nu_Q(w) > /w \in W\}$.

Definition 2.3. *[2] Consider IF-topology Wis a family of IFSets satisfying:*

(i) $0, 1 \in \tau$,
(ii) $Z_1 \cap Z_2 \in \tau$ *,for any* $Z_1, Z_2 \in \tau$,
(iii) $\cup Z_i \in \tau$, *the family* $\{Z_i/i \in J\} \subseteq \tau$.

The pair (W,τ) *is said to be IF-topological space and every IFSet of* τ *is called as IF-open set in W.*

Definition 2.4. *[3] Consider* (W,τ) *is IF-topological space and* $P =< w, \mu_P, \nu_P >$ *be an IFS in W. Then*

(i) $i(P) = \cup\{H/HisIFOSinWandH \subseteq P\}$,
(ii) $c(P) = \cap\{B/BisIFCSinWandP \subseteq B\}$.

Note that for any IFS P in (W,τ), $c(P^c) = (i(P))^c$ and $i(P^c) = (c(P))^c$.

Definition 2.5. *[3] Consider the IFS* $P = \{< w, \mu_P(w), \nu_P(w) > /w \in W\}$ *in W is*

(i) *intuitionistic fuzzy alpha-open set if* $P \subseteq i(c(i(P)))$,

(ii) *intuitionistic fuzzy regular-open set if* $P = i(c(P))$,

(iii) *intuitionistic fuzzy semi-open set if* $P \subseteq c(i(P))$.

Definition 2.6. *[4] An IFS* $C = \{< w, \mu c(w), \nu c(w) > /w \in W\}$ *inan IFTS* (W, τ) *is an*

(i) *IF*g''*CS if* $c(C) \subseteq Y$ *whenever* $C \subseteq Y$ *and Y is IFGSOS in W,*

(ii) *IFGSCS if* $sc(C) \subseteq Y$ *whenever* $C \subseteq Y$ *and Y is IFOS in W,*

(iii) *IFRWGCS if* $c(i(C)) \subseteq Y$ *whenever* $C \subseteq Y$ *and Y is IFROS in W,*

(iv) *IFWGCS if* $c(i(C)) \subseteq Y$ *whenever* $C \subseteq Y$ *and Y is IFOS in W.*

Definition 2.7. *The IFS P in* (W, τ) *is said to be IFW*g''*CS if* $c(i(P)) \subseteq Y$ *whenever* $P \subseteq Y$ *and Y is an IFGSOS in* (W, τ).

Definition 2.8. *Consider*(W, τ) *and* (D, σ) *are intuitionistic fuzzy topological spaces and*$h : W \to D$ *is a function. Then h is said to be IF-continuity if the pre image of each intuitionistic fuzzy open set of D isintuitionistic open set in W.*

Definition 2.9. *[5] A mapping* $h : (W, \tau) \to (D, \sigma)$*is said to be IF*$\alpha$ *-continuity if* $h^{-1}(V) \in IF\alpha O(W)$ *for every* $V \in \sigma$.

Definition 2.10. *A mapping* $h : (W, \tau) \to (D, \sigma)$ *is said to be IF*g''*-continuity if* $h^{-1}(V)$ *is IF*g''*CS in* (W, τ) *for every IFCS V of* (D, σ).

3. INTUITIONISTIC FUZZY WEAKLY g''-CONTINUOUS MAPPING

In this chapter intuitionistic fuzzy weakly g''-continuous mapping and its properties are introduced.

Definition 3.1. *A mapping* $h : (W, \tau) \to (D, \sigma)$ *is said to be intuitionistic fuzzy weakly* g''*-continuous mapping if* $h^{-1}(V)$ *is IFW*g''*CS in* (W, τ) *for every IFCS V of* (D, σ).

Theorem 3.1. *Every IF-continuity is IFW*g''*-continuity, but the converse is not true.*

Proof. Consider $h : (W, \tau) \to (D, \sigma)$ is IF-continuous mapping and the set V is IF-closed in D. Hence $h^{-1}(V)$ isIF-closed in W. We know that every IF-closed set is IFWg''-closed set, Therefore $h^{-1}(V)$ is IFWg''-closed set of W. Hence h is IFWg''-continuous mapping. □

Example 1. *Consider $W = \{e, f\}, D = \{g, h\}$ and $I =< w, (.2, .2), (.4, .5) >$ $J =< d, (.5, .6), (.1, .1) >$. Then $\tau = \{0, I, 1\}, \sigma = \{0, J, 1\}$ are intuitionistic fuzzy topologies of W and D respectively. Define a transformation $h : (W, \tau) \to (D, \sigma)$ by h(e)=g and h(f)=h. Then IF-set $V =< d, (.1, .1), (.5, .6) >$is IF-closed in D. Therefore$h^{-1}(V)$ is IFWg''CS but not an IFCS in W. Therefore h is IFWg''-continuous mapping but not IF-continuous mapping.*

Theorem 3.2. *Every IFα -continuity is IFWg''-continuity,but the converse is not true.*

Proof. Consider $h : (W, \tau) \to (D, \sigma)$ is IFα -continuity and the set V is IF-closed in D. Hence $h^{-1}(V)$ is IFα -closed of W. We know that every IFα -closed set is IFWg''-closed set. Therefore $h^{-1}(V)$ is IFWg''-closed in W. Hence h is IFWg''-continuous mapping. □

Example 2. *Consider $W = \{e, f\}, D = \{g, h\}$ and $I =< w, (.2, .2), (.4, .5) >$, $J =< d, (.5, .6), (.1, .1) >$. Then $\tau = \{0, I, 1\}, \sigma = \{0, J, 1\}$ are intuitionistic fuzzy topologies of W and D respectively. Define a transformation $h : (W, \tau) \to (D, \sigma)$ by h(e)=g and h(f)=h. Then IF-set $V =< d, (.1, .1), (.5, .6) >$ is IF-closed in D. Hence $h^{-1}(V)$ is IFWg''CS but not an IFα CS in W. Therefore h is IFWg''-continuous mapping but not IFα-continuous mapping.*

Theorem 3.3. *Every IFg''-continuity is IFWg''-continuity, but the converse is not true.*

Proof. Consider $h : (W, \tau) \to (D, \sigma)$ is IFg''-continuous mapping and V is IF-closed in D. Hence $h^{-1}(V)$ is IFg''-closed in W. We know that every IFg''-closed set is IFWg''-closed set, Therefore $h^{-1}(V)$ is IFWg''-closed set of W. Hence h is IFWg''-continuous mapping. □

Example 3. *Consider W= {e, f},D= {g, h} and I=< w, (.2, .7), (.8, .2) >, J=< d, (.9, .3), (.1, .4) >. Then $\tau = \{0, I, 1\}, \sigma = \{0, J, 1\}$ are intuitionistic fuzzy topologies of W and D respectively. Define a transformation $h : (W, \tau) \to (D, \sigma)$ by h(e)=g and h(f)=h. Then IF-set V=< d, (.1, .4), (.9, .3) > is IF-closed in D. Hence $h^{-1}(V)$ is IFWg''CS but not an IFg''CS in W. Therefore h is IFWg''-continuous mapping but not IFg''-continuous mapping.*

Theorem 3.4. *Every IFWg''-continuity is IFWg-continuity, but the converse is not true.*

Proof. Consider $h : (W, \tau) \to (D, \sigma)$ is IFg''-continuous mapping and V is IF-closed in D. Hence $h^{-1}(V)$ is IFWg''-closed in W. We know that every IFWg''-closed set is IFWg-closed set, Therefore $h^{-1}(V)$ is IFWg-closed set in W. Hence h is IFWg-continuous mapping. □

Example 4. *Consider W= {e, f},D= {g, h} and I=< w, (.6, .5), (.4, .5) >, J=< d, (.7, .6), (.3, .4) >. Then $\tau = \{0, I, 1\}, \sigma = \{0, J, 1\}$ are intuitionistic fuzzy topologies of W and D respectively. Define a transformation $h : (W, \tau) \to (D, \sigma)$ by h(e)=g and h(f)=h. Then IF-set V=< d, (.3, .4), (.7, .6) > is IF-closed in D. Hence $h^{-1}(V)$ is IFWgCS but not an IFWg''CS in W. Therefore h is IFWg-continuous mapping but not IFWg''-continuity.*

Theorem 3.5. *If$h : (W, \tau) \to (D, \sigma)$ is IFWg''- continuity and $k : (D, \sigma) \to (E, \delta)$ is IF-continuous, then$koh : (W, \tau) \to (E, \delta)$ is IFWg'' -continuous.*

Proof. Consider$h : (W, \tau) \to (D, \sigma)$ is IFWg'' continuous and $k : (D, \sigma) \to (E, \delta)$ is IF-continuous. And V is IFCS in E.Thus $k^{-1}(V)$ is IFCS in D because D is IF-continuous and $h^{-1}(k^{-1}(V))$ is IFWg''-closed set in W because h is IFWg'' continuous. Hence $(koh)^{-1}(V) = h^{-1}$ $(k^{-1}(V))$ in W. Therefore koh is IFWg'' -continuous. □

4. INTUITIONISTIC FUZZY WEAKLY g''-IRRESOLUTE MAPPING

In this chapter intuitionistic fuzzy weakly mapping and its properties are introduced.

Definition 4.1. *A mapping $h : (W, \tau) \to (D, \sigma)$ is said to be intuitionistic fuzzy weaklyg''-irresolute mapping if $h^{-1}(V)$ is IFWg''CS in (W, τ) for everyIFWg''CS V of (D, σ).*

Theorem 4.1. *Let $h : (W, \tau) \to (D, \sigma)$ be an IFWg''-irresolute, then h isIFWg''-continuous mapping.*

Proof. Consider$h : (W, \tau) \to (D, \sigma)$ is IFWg''-irresolute mapping and V be an IFCS in D. We know that every IFCS is IFWg''CS. Hence V isIFWg''CS in D. By hypothesis $h^{-1}(V)$ is IFWg''CS in W. Therefore h is IFWg''-continuous mapping. □

Theorem 4.2. *If $h : (W, \tau) \to (D, \sigma)$ and $k : (D, \sigma) \to (E, \delta)$ IFWg''-irresolute mapping, then $koh : (W, \tau) \to (E, \delta)$ is IFWg''-irresolute mapping.*

Proof. Consider V beIFWg''CS in E. Then $k^{-1}(V)$ is IFWg''CS in D. We know that h is IFWg''-irresolute mapping,h^{-1} $(k^{-1}(V))$ is IFWg''CS in W. Hence koh is IFWg''-irresolute mapping. □

Theorem 4.3. *If $h : (W, \tau) \to (D, \sigma)$ be anIFWg''-irresolute mapping and $k : (D, \sigma) \to (E, \delta)$ be an IFWg''-continuous mapping, then $koh : (W, \tau) \to (E, \delta)$ is an IFWg''-continuous mapping.*

Proof. Consider V beIFCS in E. Then $k^{-1}(V)$ is IFWg''CS in D. We know that h is IFWg''-irresolute mapping, h^{-1} $(k^{-1}(V))$ is IFWg''CS in W. Hence koh is IFWg''-continuous mapping. □

5. Separation Axioms

In this chapter, I provide some separation Axioms of intuitionistic fuzzy weakly g''-closed sets.

Definition 5.1. *A space (W, τ) is called IFWg'' $T_{1/2}$ sapce if every IFWg''CS is IFCS in W.*

Definition 5.2. *A space (W, τ) is called IFWg'' $T^*_{1/2}$sapce if every IFWg''CS is IFRCS in W.*

Theorem 5.1. *EveryIFWg'' $T^*_{1/2}$ space is anIFWg'' $T_{1/2}$ space.*

Proof. Consider W be an IFWg'' $T^*_{1/2}$ space and let V be an IFWg''CS in W. By hypothesis V is IFRCS in W. We know that every IFRCS is IFCS, V is IFCS in W. Hence W is IFWg'' $T_{1/2}$. □

Theorem 5.2. *A space (W, τ) is IFWg'' $T_{1/2}$ space iff IFOS(W)=IFWg''OS(W).*

Proof. **Necessity:** Consider V is IFWg''OS in W. HenceV^c is IFWg''CS in W. By our hypothesis V^c is IFCS in W. Thus V is IFOS. Therefore IFOS(W)=IFWg''OS(W).
Sufficiency: Consider V is IFWg''CS in W. Hence$V^c$is IFWg''OS . By our hypothesis V^c is IFOS in W. Therefore V is IFCS in W. Hence W is IFWg'' $T_{1/2}$sapce. □

Theorem 5.3. *A space(W, τ) is IFWg'' $T^*_{1/2}$ space iff IFROS(W)=IFWg''OS(W).*

Proof. **Necessity:** Consider V is IFWg''OS in W. Hence V^c is IFWg''CS in W. By our hypothesis V^c is IFRCS in W. Hence V is an IFROS.
Therefore IFROS(W)=IFWg''OS(W).
Sufficiency: Consider V is IFWg''CS in W. ThusV^c is IFWg''OS . By our hypothesisV^c is IFROS in W. Therefore V is IFRCS in W. Hence W is IFWg'' $T_{1/2}$ space. □

6. CONCLUSION

The basic aim of this paper is to introduce Intuitionistic fuzzy weakly g''-continuity, Intuitionistic fuzzy weakly g''-irresolute and some of its properties.

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