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COMPLETE COTOTAL DOMINATION NUMBER OF CENTRAL GRAPHS

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ABSTRACT. A total dominating set D is said to be a complete cototal dominating set if $\langle V-D\rangle$ has no isolated nodes and it is represented by $\gamma_{cctd}(G)$. The complete cototal domination number, represented by $\gamma_{cc}(G)$, is the minimum cardinality of a $\gamma_{cctd}-$ set of G . Our aim is to determine the complete cototal domination number of central graphs and its bounds.

1. INTRODUCTION AND PRELIMINARIES

Domination theory in graph was developed by Claude Berge around 1960's with the problem of placing minimum number of queens on a $n \times n$ chess board to dominate every square by atleast one queen. After that Oystein Ore developed the concept dominating set and domination number [6]. A set S of nodes of G is a dominating set of G if every node of G is dominated by some node in S. Cockayne and Hedetniemi issued several research papers in this topic [2]. Cockayne, Dawes and Hedetniemi presented total domination in [3]. A subset D of V is called an dominating set of G if every node not in D is adjacent to some node in D [2]. A total dominating set for a graph G is a dominating set M for G with the property that every node in M has a neighbor in M. Note that total dominating sets are not defined for graphs with isolated nodes. Kulli, Janakiram and Iyer presented cototal dominating set in [4]. A dominating

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set D is said to be a cototal dominating set if $\langle V - D\rangle$ has no isolated nodes. The cototal domination number of G is the minimum cardinality of a cototal dominating set of G and it is represented by $\gamma_{ctd}(G)$ [1]. This concept motivate us to do research under this topic.

Throughout this paper we considered a simple connected graph. Let $G = (V(G), E(G))$ where $V(G)$ denotes the node set and $E(G)$ denotes the edge set. The total number of nodes and edges are denoted by p and q respectively.

Definition 1.1. *[1] A total dominating set D is said to be a $\gamma_{cctd}-$ set if the $\langle V - D\rangle$ has no isolated nodes. The complete cototal domination number $\gamma_{cc}(G)$ is the minimum cardinality of a $\gamma_{cctd}-$ set of G.*

Definition 1.2. *[5] Let G be a simple and undirected graph. The Central graph of G, denoted by $C(G)$ is obtained by subdividing every edge of G exactly once and joining all the non-adjacent nodes of G in $C(G)$.*

2. Main Results

Theorem 2.1. *For a Path graph P_n, $\gamma_{cc}(C(P_n)) = \lceil \frac{n+3}{2} \rceil$.*

Theorem 2.2. *For a Star graph $K_{1,n}$, $\gamma_{cc}(C(K_{1,n})) = 3$.*

Theorem 2.3. *For a Complete graph K_n, $\gamma_{cc}(C(K_n)) = p - n + 1$ where p is the total number of nodes on $C(K_n)$.*

Theorem 2.4. *For a Ladder graph L_n,* $\gamma_{cc}(C(L_n)) = \begin{cases} 5 \text{ if } n = 2 \\ 4 \text{ if } n = 3 \\ n \text{ if } n \geq 4 \end{cases}$.

Theorem 2.5. *For a Fan graph f_n,* $\gamma_{cc}(C(f_n)) = \begin{cases} 2n \text{ if } n = 2,3 \\ 6 \text{ if } n = 4 \\ n+2 \text{ if } n \text{ is odd}(n \geq 5) \\ n+1 \text{ if } n \text{ is even}(n \geq 6) \end{cases}$.

Theorem 2.6. *For any connected graph $C(G)$, $\left\lceil \frac{diam(C(G))+1}{3} \right\rceil < \gamma_{cc}(C(G))$.*

Proof. Let S be a γ_{cctd}- set of a connected graph $C(G)$. Consider an arbitrary path of length $diam(C(G))$. This diametral path includes at most two edges from the induced subgraph $\langle N[u]\rangle$ for each $v \in S$. Furthermore, since S is a γ_{cctd}- set, the diametral path includes at most $\gamma_{cctd}(C(G))-1$ edges joining the neighborhoods of the nodes of S. Hence,

$$diam(C(G)) < 2\gamma_{cc}(C(G)) + \gamma_{cc}(C(G)) - 1 = 3\gamma_{cc}(C(G)) - 1\,.$$

Therefore $\left\lceil \frac{diam(C(G))+1}{3} \right\rceil < \gamma_{cc}(C(G))$. □

Theorem 2.7. *Let $C(G)$ be a graph with no isolated nodes. Then $\gamma_{cc}(C(G)) \geq \left\lceil \frac{n}{2} \right\rceil$ iff $C(G)$ is not a Star graph $(n \geq 6)$.*

Proof. Let $D \subseteq V(C(G))$ be a γ_{cctd}- set. Since $C(G)$ has no isolated nodes, every $v \in D$ has at least one neighbor in $V - D$. This means that $V - D$ is also a complete cototal dominating set. If $|D| < \left\lceil \frac{n}{2} \right\rceil$, then $V - D$ is the smallest complete cototal dominating set, contradicting the choice of D as a complete cototal dominating set. Thus $\gamma_{cc}(C(G)) = |D| \geq \left\lceil \frac{n}{2} \right\rceil$.

Conversely, Suppose $C(G) = K_{1,n}(n \geq 6)$, be a Star graph with no isolated nodes, by using theorem 2.2, $\gamma_{cc}(C(G)) = 3$. Therefore $\gamma_{cc}(G) \leq \left\lceil \frac{n}{2} \right\rceil$.

□

Result 1. The above bound is sharp for $C(K_{1,5})$ and $C(K_{1,6})$ since $\gamma_{cc}(C(K_{1,5})) = 3$ and $\gamma_{cc}(C(K_{1,6})) = 3$.

Theorem 2.8. *For a connected graph $C(G)$, $\gamma_{cc}(C(G)) > \left\lceil \frac{n}{\Delta(C(G))} \right\rceil$.*

Proof. Let the complete cototal dominating set $S \subseteq V(C(G))$ in $C(G)$. Every node in S dominate at least one of the nodes in S and dominates at most $\Delta(C(G)) - 1$ nodes of $V(C(G)) - S$. Therefore, $|S|\,(\Delta(C(G)) - 1) + |S| > n$. Hence $\gamma_{cc}(C(G)) > \left\lceil \frac{n}{\Delta(C(G))} \right\rceil$, as S is an arbitrary complete cototal dominating set. □

Theorem 2.9. *For a graph $C(G)$ with $diam(C(G)) \geq 2$, $\gamma_{cc}(C(G)) \geq \delta(C(G))+1$.*

Proof. Let $u \in V(C(G))$ and $deg(u) = \delta(C(G))$. Since $diam(C(G)) \geq 2$, then $N(u)$ is a cototal dominating set for $C(G)$. Now $S = N(u) \bigcup \{u\}$ is a complete cototal dominating set for $C(G)$ and $|S| = \delta(C(G)) + 1$. Hence, $\gamma_{cc}(C(G)) \geq \delta(C(G)) + 1$. □

Result 2. *The above bound is sharp for $C(P_3)$ and $C(K_{1,n})$ since $\gamma_{cc}(C(P_3)) = 3$ and $\gamma_{cc}(C(K_{1,n})) = 3$ and also $\delta(C(P_3)) = 2$ and $\delta(C(K_{1,n}))(n \geq 2) = 2$.*

Theorem 2.10. *For a connected graph $C(G)$ with the girth of length $g(C(G)) \geq 3$ and $\delta(C(G)) \geq 2$, $\gamma_{cc}(C(G)) \geq n - \left\lceil \frac{g(C(G))}{2} \right\rceil + 1$ iff $C(G)$ is not a Star graph $(n \geq 5)$.*

Proof. Consider a connected graph $C(G)$ with girth of length ≥ 3 and the cycle of length $g(C(G))$ is denoted as C. Construct a graph $C(G')$ by removing C from $C(G)$. Let us choose a node $v \in V(C(G'))$. Here the node v has at least two neighbors say x and y because the minimum degree of $C(G)$ is at least two. Let $x, y \in C$. Suppose $d(x, y) \geq 3$. Then the path from x to y on C is replaced by the path x, v, y which reduces the girth of $C(G)$, which is a contradiction. Suppose $d(x, y) \leq 2$. Then the nodes x, y, v lie on either C_3 or C_4 in $C(G)$, which is a contradiction to $g(C(G)) \geq 3$. Therefore, every node in $C(G')$ has at most one neighbour on C. Hence the graph $C(G')$ has minimum degree at least $\delta(C(G)) - 1 \geq 1$ because $\delta(C(G)) \geq 2$. Therefore $C(G)$ contains no isolated node. Let S' be the complete cototal dominating set for C. Then $S = S' \bigcup V(C(G'))$ is a complete cototal dominating set for $C(G)$. Hence, $\gamma_{cc}(C(G)) \geq n - \left\lceil \frac{g(C(G))}{2} \right\rceil + 1$.

Conversely, Suppose $C(G) = C(K_{1,n})(n \geq 5)$, be a Star graph with the girth of length $g(C(G)) \geq 3$ and $\delta(C(G)) \geq 1$, by using theorem 2.2, $\gamma_{cc}(C(G)) = 3$. Therefore $\gamma_{cc}(C(G)) < n - \left\lceil \frac{g(C(G))}{2} \right\rceil + 1$. □

Result 3. The above bound is sharp for $C(K_{1,4})$ since $\gamma_{cc}(C(K_{1,4})) = 3$.

Theorem 2.11. *For any graph $C(G)$, $\gamma_{cc}(C(G)) > n - \Delta(C(G))$.*

Proof. Let $C(G)$ be a γ_{cc}-set of $C(G)$. Let v be a node of maximum degree $\Delta(C(G))$. Then v dominates $N[u]$ and the nodes in $V - N[u]$ dominate themselves. Hence $V - N[u]$ is a γ_{cc} dominating set of cardinality $n - \Delta(C(G))$, so $\gamma_{cc}(C(G)) > n - \Delta(C(G))$. □

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