ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.8, 5389–5400
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.8.10

COMPARISON OF VARIOUS METHODS FOR SOLVING REAL LIFE PROBLEMS RELATED TRANSPORTATION SCHEDULING

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ABSTRACT. Transportation problem is a special type of linear programming problem where the objective is to minimize the cost of distributing a product from a number of origins to a number of destinations. In this paper we find an optimal solution of transportation problem by using various methods and comparing these methods with each other.

1. INTRODUCTION

Transportation problem basically is a special case of linear programming in which we have some rows and some columns and a special kind of table formulation or we can say special kind of model. The main aim of transportation problem is to minimize the cost of distributing a product from number of sources to the number of destinations. The origin of transportation was first presented by F.L. Hitchcock [5] in 1941. Transportation network are large scale system and are in various forms such as rail, road, airways, ship, etc. In this paper we are comparing some new methods i.e. ASM Method, ATM Method and Stepping Stone Method with these methods in order to find the optimal solution of transportation problem. ASM Method gives an optimal solution directly, in few iteration it takes less time and is so easy to learn and apply.

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2010 *Mathematics Subject Classification.* 90B06.

Key words and phrases. Transportation Problem, Optimal solution, Linear Programming, Feasible Solution.

2. Mathematical Formulation

Let there are m sources and n destinations. Let a_i be the number of units at i sources where $(i = 1, 2, \dots m)$ and b_j be the number of units at destination $j(j = 1, 2, \dots n)$. Let c_{ij} be the unit transportation cost for transporting the units from source to destination j, so that the total transportation cost is minimum. If $x_{ij} \geq 0$ is the number of units shipped from source I to destination j, then equivalent linear transportation problem is to find $x_{ij}(i = 1, 2, \dots m, j = 1, 2, \dots n)$.

In order to minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} cijxij$
Subject to: $\sum_{j=1}^{n} xij = ai, i = 1, 2, \dots m$

$$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, \dots n$$

and $x_{ij} \geq 0$, for all i and j

The necessary and sufficient condition for transportation problem to have feasible solution is:

$$\sum_{i=1}^{m} ai = \sum_{j=1}^{n} bj.$$

3. Numerical Examples

Example 1. Let us consider the following transportation problem having three sources and four destinations.

Table 1

	D_1	D_2	D_3	D_4	Supply
S_1	16	14	10	11	11
S_2	13	9	15	14	8
S_3	10	17	12	18	10
Demand	12	7	4	6	29(Total)

Since the transportation problem is balanced, so we can solve it. Now we will use the following methods to solve this problem.

I. North West Corner Method (NWCM):

Table 2

	D_1	D_2	D_3	D_4	Supply
S_1	16(11)	14	10	11	11/0
S_2	13(1)	9(7)	15	14	8/7/0
S_3	10	17	12(4)	18(6)	10/6/0
Demand	12/1/0	7/0	4/0	6/0	29(Total)

$z = 16{*}11 + 13{*}1 + 9{*}7 + 12{*}4 + 18{*}6 = 408$

II. Least Cost Method (LCM):

Table 3

	D_1	D_2	D_3	D_4	Supply
S_1	16(1)	14	10(4)	11(6)	11/7/1/0
S_2	13(1)	9(7)	15	14	8/1/0
S_3	10(10)	17	12	18	10/0
Demand	12/2/1/0	7/0	4/0	6/0	29(Total)

Thus, $z = 16{*}1 + 10{*}4 + 11{*}6 + 13{*}1 + 9{*}7 + 10{*}10 = 298$

III. Vogel's Approximation Method (VAM):

Table 4

	D_1	D_2	D_3	D_4	Supply
S_1	16(1)	14	10(4)	11(6)	11/7/1/0
S_2	13(1)	9(7)	15	14	8/1/0
S_3	10(10)	17	12	18	10/0
Demand	12/2/1/0	7/0	4/0	6/0	29(Total)

$Z = 16{*}1 + 10{*}4 + 11{*}6 + 13{*}1 + 9{*}7 + 10{*}10 = 298$

IV. Stepping Stone Method:

Table 5

	D_1	D_2	D_3	D_4	Supply
S_1	16(1)	14	10(4)	11(6)	11/7/1/0
S_2	13(1)	9(7)	15	14	8/1/0
S_3	10(10)	17	12	18	10/0
Demand	12/2/1/0	7/0	4/0	6/0	29(Total)

Now, after applying VAM method we will find the value of the cells left without allocation, i.e. c(1,2), c(2,3), c(2,4), c(3,2), c(3,3), c(3,4).

Table 6

16(1)	14	10(4)	11(6)
13(1)	9(7)	15	14
10(10)	17	12	18

Cell Evaluation:

C(1,2)= 14-9+13-16 = 2
C(2,3) = 15-10+16-13 = 8
C(2,4) = 14-11+16-13 = 6
C(3,2) = 17-10+13-9 = 11
C(3,3) = 12-10+16-10 = 8
C(3,4) = 18-11+16-10 = 13

Since, all the cell evaluations are positive. Therefore, the initial basic feasible solution is optimal solution also.

Hence, $z = 16{*}1 + 10{*}4 + 11{*}6 + 13{*}1 + 9{*}7 + 10{*}10 = 298$.

V. ASM -Method:

Table 7

	D_1	D_2	D_3	D_4	Supply
S_1	16	14	10	11	11
S_2	13	9	15	14	8
S_3	10	17	12	18	10
Demand	12	7	4	6	29(Total)

Subtract each element of row from its row minima and after that subtract each element of column of the resulting transportation table from its column minima.

Table 8 (Reduced Row Matrix)

	D_1	D_2	D_3	D_4	Supply
S_1	6	4	0	1	11
S_2	4	0	6	5	8
S_3	0	7	2	8	10
Demand	12	7	4	6	29(Total)

Table 9 (Reduced Column Matrix)

	D_1	D_2	D_3	D_4	Supply
S_1	6	4	0	0	11
S_2	4	0	6	4	8
S_3	0	7	2	7	10
Demand	12	7	4	6	29(Total)

Since, the above table has at least one zero in each row and each column. So we allocate the values to all zeros.

Table 10

	D_1	D_2	D_3	D_4	Supply
S_1	16	14	10(4)	11(6)	11
S_2	13	9(7)	15	14	8
S_3	10(10)	17	12	18	10
Demand	12	7	4	6	29(Total)

Now, $z = 10^{*}4 + 11^{*}6 + 9^{*}7 + 10^{*}10 = 269$

VI. ATM - Method:

Table 11

	D_1	D_2	D_3	D_4	Supply
S_1	16	14	10	11	11
S_2	13	9	15	14	8
S_3	10	17	12	18	10
Demand	12	7	4	6	29(Total)

Select least odd cost from all cost cells of transportation table (TT), subtracts chosen least cost only from every odd cost esteemed cells of the transportation problem and then start allocating the values from least supply or demand to the resulting TT.

Table 12

	D_1	D_2	D_3	D_4	Supply
S_1	16(1)	14	10(4)	2(6)	11/5/1/0
S_2	4(1)	9(7)	6	14	8/1/0
S_3	10(10)	8	12	18	10/0
Demand	12/11/1/0	7/0	4/0	6/0	29

Since the number of allocations are 6 that is (= m+n-1), where m is number of rows and n is number of columns, hence initial basic feasible solution.

Table 13

	D_1	D_2	D_3	D_4	Supply
S_1	16(1)	14	10(4)	11(6)	11
S_2	13(1)	9(7)	15	14	8
S_3	10(10)	17	12	18	10
Demand	12	7	4	6	29

Thus, we have $z = 16{*}1 + 10{*}4 + 11{*}6 + 13{*}1 + 9{*}7 + 10{*}10 = 298$

Example 2. A company makes trucks and it has four factories F_1, F_2, F_3 and F_4 whose weekly manufacturing quantities are 8, 5, 12 and 6 numbers of trucks respectively. The company supplies trucks to its four showrooms positioned at D_1, D_2, D_3 and D_4 to whom weekly requirements are 5, 6, 11 and 9 no. of trucks respectively. The transportation cost for each truck is given in transportation table 14. Find the schedule of transferring trucks from factories to showrooms with least cost.

Table 14

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	15	6	8
F_2	12	9	30	13	5
F_3	10	22	17	21	12
F_4	25	16	8	11	6
Demand	5	6	11	9	31(Total)

I. North West Corner Method:

Table 15

	D_1	D_2	D_3	D_4	Supply
F_1	8(5)	14(3)	15	6	8/3/0
F_2	12	9(3)	30(2)	13	5/2/0
F_3	10	22	17(9)	21(3)	12/3/0
F_4	25	16	8	11(6)	6/0
Demand	5/0	6/3/0	11/9/0	9/6/0	31(Total)

Hence the total cost $= 8{*}5 + 14{*}3 + 9{*}3 + 30{*}2 + 17{*}9 + 21{*}3 + 11{*}6 = 451$

II. Least Cost Method (LCM):

Table 16

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	15	6(8)	8/0
F_2	12	9(5)	30	13	5/0
F_3	10(5)	22(1)	17(5)	21(1)	12/7/2/1/0
F_4	25	16	8(6)	11	6/0
Demand	5/0	6/1/0	11/5/0	9/1/0	31

$Z = 6{*}8 + 9{*}5 + 10{*}5 + 22{*}1 + 17{*}5 + 21{*}1 + 8{*}6 = 3194$

III. Vogel's Approximation Method (VAM):

Table 17

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	15	6(8)	8/0
F_2	12	9(5)	30	13	5/0
F_3	10(5)	22(1)	17(5)	21(1)	12/7/6/5/0
F_4	25	16	8(6)	11	6/0
Demand	5/0	6/1/0	11/5/0	9/1/0	31

Therefore, $z = 6{*}8 + 9{*}5 + 10{*}5 + 22{*}1 + 17{*}5 + 21{*}1 + 8{*}6 = 319$

IV. Stepping Stone Method:

Table 18

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	15	6(8)	8/0
F_2	12	9(5)	30	13	5/0
F_3	10(5)	22(1)	17(5)	21(1)	12/7/6/5/0
F_4	25	16	8(6)	11	6/0
Demand	5/0	6/1/0	11/5/0	9/1/0	31

Cell Evaluation: Now we will find the values of the cell without allocation.

C(1,1) = 8-10+21-6 = 13
C(1,2) = 14-22+21-6 = 7
C(1,3) = 15-17+21-6 = 13
C(2,1) = 12-10+22-9 = 15
C(2,3) = 30-17+22-9 = 26
C(2,4) = 13-21+22-9 = 5

C(4,1) = 25-10+17-8 = 24
C(4,2) = 16-8+17-22 = 3
C(4,4) = 11-21+17-8 = -1

Since there is a negative value in C(4,4) , therefore we can further improve the solution by adding and subtracting this negative value to all those cell allocations that gives the value of C(4,4).

Table 19

8	14	15	6(8)
12	9(5)	30	13
10(5)	22(1)	17(6)	21
25	16	8(5)	11(1)

Minimum cost, $z = 6{*}8 + 9{*}5 + 10{*}5 + 22{*}1 + 17{*}6 + 8{*}5 + 11{*}1 = 318$

Again cell evaluation is given as:

C(1,1)= 8-6+11-8+17-10=12
C(1,2)= 14-6+11-8+17-22=6
C(1,3)= 15-6+11-8=12
C(2,1)=12-9+22-10=15
C(2,3)=30-17+22-9=26
C(2,4)=13-9+22-17+8-11=6
C(3,4)=21-11+8-17=1
C(4,1)=25-8+17-10=24
C(4,2)=16-8+17-22=3

Since all the cell evaluations are positive, therefore optimal solution is obtained.

Hence minimum cost = 318

V. ASM - Method:

Table 20

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	15	6	8
F_2	12	9	30	13	5
F_3	10	22	17	21	12
F_4	25	16	8	11	6
Demand	5	6	11	9	31

Subtract all elements of row from its least row cost and then subtract all elements of the resulting TT from its column minima.

Table 21 Reduced Row Matrix

	D_1	D_2	D_3	D_4	Supply
F_1	2	8	9	0	8
F_2	3	0	21	4	5
F_3	0	12	7	11	12
F_4	17	8	0	3	6
Demand	5	7	11	9	31(Total)

Table 22 Reduced Column Matrix

	D_1	D_2	D_3	D_4	Supply
F_1	2	8	9	0	8
F_2	3	0	21	4	5
F_3	0	12	7	11	12
F_4	17	8	0	3	6
Demand	5	7	11	9	31(Total)

Since each row and column has at least one zero, therefore allocate the values to all zeros.

Table 23

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	15	6(8)	8
F_2	12	9(5)	30	13	5
F_3	10(5)	22	17	21	12
F_4	25	16	8(6)	11	6
Demand	5	7	11	9	31(Total)

Hence the total cost correlated with these allocations is

$$Z = 6^*8 + 9^*5 + 10^*5 + 8^*6 = 191.$$

VI. ATM - Method:

Table 24

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	15	6	8
F_2	12	9	30	13	5
F_3	10	22	17	21	12
F_4	25	16	8 11	6	
Demand	5	6	11	9	31

Table 25

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	6(5)	6(3)	8/5/0
F_2	12	9(5)	30	4 5/0	
F_3	10(5)	22(1)	8(6)	12	12/6/1/0
F_4	16	16	8	2(6)	6/0
Demand	5/0	6/1/0	11/6/0	9/3/0	

Allocation Table

Table 26

	D_1	D_2	D_3	D_4	Supply
F_1	8	14	15(5)	6(3)	8
F_2	12	9(5)	30	13	5
F_3	10(5)	22(1)	17(6)	21	12
F_4	25	16	8	11(6)	6
Demand	5	6	11	9	

Thus, $z = 15{*}5 + 6{*}3 + 9{*}5 + 10{*}5 + 22{*}1 + 17{*}6 + 11{*}6 = 378$

4. Result Analysis

After comparing all these methods, the solutions we obtained are given in table 27.

Table 27: Comparison of the results obtained by using various methods.

Methods	Example 1	Example 2
North West Corner Method	408	451
Least Cost Method	298	319
Vogel's Approximation Method	298	319
Stepping Stone Method	298	318
ASM Method	269	191
ATM Method	298	378
Optimal Solution	269	191

Since from table 27, we can say that ASM Method gives better result as compared to other methods. This method gives ideal arrangement directly, in fewer emphases. As this strategy devours less time and is straightforward and easy to apply, so it will be useful for those who are managing calculated and store network issues [2].

5. Conclusion

In this paper six methods are compared to each other to find an optimal solution of a transportation problem and it is observed that from table 27, ASM method provides comparatively better result than the results obtained by other methods, because ASM method provides an optimal solution directly, in fewer iterations. Also this method consumes less time and is very easy even for a layman to understand and apply.

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