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ALGEBRAIC OPERATIONS ON PICTURE FUZZY SOFT MATRICES

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ABSTRACT. In this paper, we first define the Picture fuzzy Soft matrices and study some of their relevant properties. We construct nP and P^n of a Picture fuzzy soft matrices P and studied the algebraic properties of these operations.

1. INTRODUCTION

Fuzzy set theory was introduced by Zadeh, [12] to handle uncertainty in real-life situation. Atanassov in [4] proposed the intuitionistic fuzzy set (IFS). IFS is an extension of fuzzy set (FS), [5], which is characterized by a positive membership degree and a negative membership degree, satisfying the condition that the sum of these two degrees is equal to or less than one; therefore, IFS is a useful tool in processing fuzzy and uncertainty information. Chetia and Das in [6] defined intuitionistic fuzzy soft matrices and their operations which are more functional to make theoretical studies in the intuitionistic fuzzy soft set theory. Yager at al. in [11] proposed the concept of the Pythagorean fuzzy set (PyFS), which is also characterized by the positive membership degree and the non-membership degree, but the square sum of these two degrees is equal to or less than one. Arikrishnan and Sriram in [3] defined necessity and possibility operators on Pythagorean fuzzy soft matrices and discussed there properties. Another extension of fuzzy sets was the introduction of neutrosophic

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sets, see [10], by Smarandache, who postulates that a neutrosophic set is characterized by positive, indeterminate and negative membership functions each assuming membership values in the interval $[0,1]$ such that the sum of these membership values lies between 0 and 3. The introduction of neutrosophic sets opened up the possibility of including more cases of uncertainty and has found place in many applications, see [1,2].

Cuong in [7] introduced the core idea of the Picture fuzzy set (PFS). PFS is an extension of IFS and PyFS. In the concept of the PFS, Cuong added the neutral term along with the positive membership and negative membership degrees and the sum of their membership degrees is equal to or less than one. Many researchers have attempted to contribute to the application of PFS. Cuong in [8] introduced the concept of picture fuzzy soft sets (PFSSs) and some operations on PFSS with some properties are considered. Dogra and Pal in [9] introduced picture fuzzy matrix and some properties are investigated. An application of picture fuzzy matrix in decision-making problem was discussed. In this paper, we introduced Picture fuzzy soft matrices and its various types operations are analogously proposed for the picture fuzzy soft matrices.

2. Main Results For Picture Fuzzy Soft Matrices

Similar to the definition of PFSSs, in the following we introduce the definition of Picture fuzzy soft matrices (PFMSs).

Definition 2.1. *If (F_A, E) be a Picture fuzzy soft set over X, then the subset, $X \times E$ is uniquely defined by $R_A = \{(x,e), e \in A, x \in F_A(e)\}$. The R_A can be characterized by its positive membership, neutral and negative membership given by $\mu_R : X \times E \to [0,1]$, $\eta_R : X \times E \to [0,1]$ and $\nu_R : X \times E \to [0,1]$, respectively. If $(\mu_{ij}, \eta_{ij}, \nu_{ij}) = (\mu_{R_A}(x_i, e_j), \eta_{R_A}(x_i, e_j, \nu_{R_A}(x_i, ej))$, where $\mu_{R_A}(x_i, e_j)$ is the positive membership of x_i in the Picture fuzzy set $F(e_j)$, $\eta_{R_A}(x_i, e_j)$ is the neutral membership of x_i in the Picture fuzzy set $F(e_j)$ and $\nu_{R_A}(x_i, e_j)$ is the negative membership of x_i in the Picture fuzzy set $F(e_j)$, respectively, then we define a matrix given by*

$$[M]_{m\times n} = \begin{bmatrix} (\mu_{11}, \eta_{11}, \nu_{11}) & (\mu_{12}, \eta_{12}, \nu_{12}) & \cdots & (\mu_{1n}, \eta_{1n}, \nu_{1n}) \\ (\mu_{21}, \eta_{21}, \nu_{21}) & (\mu_{21}, \eta_{21}, \nu_{21}) & \cdots & (\mu_{2n}, \eta_{2n}, \nu_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\mu_{m1}, \eta_{m1}, \nu_{m1}) & (\mu_{m2}, \eta_{m2}, \nu_{m2}) & \cdots & (\mu_{mn}, \eta_{mn}, \nu_{mn}) \end{bmatrix},$$

*which is called **Picture fuzzy soft matrix** of order $m \times n$ over X.*

Example 1. *Let us $U = \{u_1, u_2, u_3, u_4\}$ as a universal set and $E = \{e_1, e_2, e_3, e_4\}$ be the set of parameters. If $P = \{e_1, e_2, e_3\} \subseteq E$ and*

$$F_P(e_1) = \{(u_1, 0.5, 0.2, 0.1), (u_2, 0.1, 0.5, 0.3), (u_3, 0.2, 0.6, 0.1), (u_4, 0.5, 0.1, 0.3)\},$$

$$F_P(e_2) = \{(u_1, 0.7, 0.1, 0.2), (u_2, 0.4, 0.3, 0.2), (u_3, 0.2, 0.1.0.6), (u_4, 0.1, 0.1, 0.7)\},$$

$$F_P(e_3) = \{(u_1, 0.1, 0.4, 0.3), (u_2, 0.6, 0.1, 0.2), (u_3, 0.1, 0.7, 0.1), (u_4, 0.4, 0.3, 0.1)\}$$

then (F_P, E) is the parameterized family of $F_P(e_1), F_P(e_2), F_P(e_3)$ over X. Hence, the Picture fuzzy soft matrix $[M(F_P, E)]$ can be written as

$$[M] = [(\mu_{ij}^M, \eta_{ij}^M, \nu_{ij}^M)]_{m\times n} = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \end{array}\begin{array}{c} \begin{array}{ccc} e_1 & e_2 & e_3 \end{array} \\ \begin{pmatrix} (0.5, 0.2, 0.1) & (0.7, 0.1, 0.2) & (0.2, 0.4, 0.3) \\ (0.1, 0.5.0.3) & (0.4, 0.3, 0.2) & (0.6, 0.1, 0.2) \\ (0.2, 0.6, 0.1) & (0.2, 0.1, 0.6) & (0.1, 0.7, 0.1) \\ (0.5, 0.1, 0.3) & (0.1, 0.1, 0.7) & (0.4, 0.3, 0.1) \end{pmatrix} \end{array}.$$

Suppose $PFSM_{m\times n}$ is a collection of all Picture fuzzy soft matrices over X. Subsequently, various kinds of Picture fuzzy soft matrices have been analogously proposed.

Definition 2.2. *For any two Picture fuzzy soft matrices $P = \left[(\mu_{ij}^P, \eta_{ij}^P, \mu_{ij}^P)\right]$ and $Q = \left[(\mu_{ij}^Q, \eta_{ij}^Q, \mu_{ij}^Q)\right] \in PFSMs_{m\times n}$ can be defined as follows:*

(i) $P^C = \left[\left(\nu_{ij}^P, \eta_{ij}^P, \mu_{ij}^P\right)\right]$;

(ii) $P \cup Q = \left[\left(\max(\mu_{ij}^P, \mu_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^Q), \min(\nu_{ij}^P, \nu_{ij}^Q)\right)\right]$;

(iii) $P \cap Q = \left[\left(\min(\mu_{ij}^P, \mu_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^Q), \max(\nu_{ij}^P, \nu_{ij}^Q)\right)\right]$;

(iv) $P \oplus Q = \left[\left(\mu_{ij}^P + \mu_{ij}^Q - \mu_{ij}^P \cdot \mu_{ij}^Q, \eta_{ij}^P \cdot \mu_{ij}^Q, \nu_{ij}^P \cdot \nu_{ij}^Q\right)\right]$;

(v) $P \odot Q = \left[\left(\mu_{ij}^P \cdot \mu_{ij}^Q, \eta_{ij}^P + \eta_{ij}^Q - \eta_{ij}^P \cdot \eta_{ij}^Q, \nu_{ij}^P + \nu_{ij}^Q - \nu_{ij}^P \cdot \nu_{ij}^Q\right)\right]$;

(vi) $P@Q = \left[\left(\frac{\mu_{ij}^P + \mu_{ij}^Q}{2}, \frac{\eta_{ij}^P + \eta_{ij}^Q}{2}, \frac{\nu_{ij}^P + \nu_{ij}^Q}{2}\right)\right], \forall\, i$ *and* j;

(vii) $P\$Q = \left[\left(\sqrt{\mu_{ij}^P \cdot \mu_{ij}^Q}, \sqrt{\eta_{ij}^P \cdot \eta_{ij}^Q}, \sqrt{\nu_{ij}^P \cdot \nu_{ij}^Q}\right)\right], \forall\, i$ *and* j;

(viii) $P \bowtie Q = \left[\left(2 \cdot \frac{\mu_{ij}^{P} \cdot \mu_{ij}^{Q}}{\mu_{ij}^{P} + \mu_{ij}^{Q}}, 2 \cdot \frac{\eta_{ij}^{P} \cdot \eta_{ij}^{Q}}{\eta_{ij}^{P} + \eta_{ij}^{Q}}, 2 \cdot \frac{\nu_{ij}^{P} \cdot \nu_{ij}^{Q}}{\nu_{ij}^{P} + \nu_{ij}^{Q}} \right) \right]$, $\forall$ i *and* j.

Using these operations, (iv),(v) the following equations are obtained for any integer $n > 0$.

$P \oplus P = \left[\left(2\mu_{ij}^{P} - (\mu_{ij}^{P})^2, (\eta_{ij}^{P})^2, (\nu_{ij}^{P})^2 \right) \right]$

$2P = \left[\left((1 - (1 - \mu_{ij}^{P})^2, (\eta_{ij}^{P})^2, (\nu_{ij}^{P})^2 \right) \right]$

Similarly,

$3P = \left[\left((1 - (1 - \mu_{ij}^{P})^3, (\eta_{ij}^{P})^3, (\nu_{ij}^{P})^3 \right) \right]$

In general,

$nP = P \oplus P \oplus .... \oplus P = \left[\left(1 - \left(1 - \mu_{ij}^{P}\right)^n, \left(\eta_{ij}^{P}\right)^n, \left(\nu_{ij}^{P}\right)^n \right) \right]$,

$......P \odot P = \left[\left((\mu_{ij}^{P})^2, 2\eta_{ij}^{P} - (\eta_{ij}^{P})^2, 2\nu_{ij}^{P} - (\nu_{ij}^{P})^2 \right) \right]$

$P^2 = \left[\left((\mu_{ij}^{P})^2, 1 - (1 - \eta_{ij}^{P})^2, 1 - (1 - \nu_{ij}^{P})^2 \right) \right]$

Similarly,

$P^3 = \left[\left((\mu_{ij}^{P})^3, 1 - (1 - \eta_{ij}^{P})^3, 1 - (1 - \nu_{ij}^{P})^3 \right) \right]$

In general,

$P^n = P \odot P \odot .... \odot P = \left[\left((\mu_{ij}^{P})^n, 1 - (1 - \eta_{ij}^{P})^n, 1 - (1 - \nu_{ij}^{P})^n \right) \right]$

It can be easily verified that,

$0 \leq 1 - \left(1 - \mu_{ij}^{P}\right)^n + \left(\eta_{ij}^{P}\right)^n + \left(\nu_{ij}^{P}\right)^n \leq 1$

and $0 \leq (\mu_{ij}^{P})^n + 1 - (1 - \eta_{ij}^{P})^n + 1 - (1 - \nu_{ij}^{P})^n \leq 1$.

Theorem 2.1. *$P = [(\mu_{ij}^{P}, \eta_{ij}^{P}, \nu_{ij}^{P}]$ and $Q = [(\mu_{ij}^{Q}, \eta_{ij}^{Q}, \nu_{ij}^{Q})] \in PFSM_{m \times n}$. If $n > 0$ are integer, then*

(i) $n(P \oplus Q) = nP \oplus nQ$

(ii) $n(P \odot Q) = nP \odot nQ$

Proof. Let $P = [(\mu_{ij}^{P}, \eta_{ij}^{P}, \nu_{ij}^{P}]$ and $Q = [(\mu_{ij}^{Q}, \eta_{ij}^{Q}, \nu_{ij}^{Q})] \in PFSM_{m \times n}$. for all i and j,

(i) $n(P \oplus Q)$

$$= n\left(\left[\left(\mu_{ij}^{P} + \mu_{ij}^{Q} - \mu_{ij}^{P} \cdot \mu_{ij}^{Q}, \eta_{ij}^{P} \cdot \mu_{ij}^{Q}, \nu_{ij}^{P} \cdot \nu_{ij}^{Q} \right) \right] \right)$$

$$= \left[\left(1 - \left(\left(1 - \mu_{ij}^{P}\right) + \left(1 - \mu_{ij}^{Q}\right) - \left(1 - \mu_{ij}^{P}\right) \cdot \left(1 - \mu_{ij}^{Q}\right) \right)^n, \left(\left(\eta_{ij}^{P}\right) \cdot \left(\eta_{ij}^{Q}\right) \right)^n, \left(\left(\nu_{ij}^{P}\right) \cdot \left(\nu_{ij}^{Q}\right) \right)^n \right) \right]$$

$$= \left[\left(1 - \left(1 - \mu_{ij}^{P}\right)^n \cdot \left(1 - \mu_{ij}^{Q}\right)^n, \left(\left(\eta_{ij}^{P}\right) \cdot \left(\eta_{ij}^{Q}\right) \right)^n, \left(\left(\nu_{ij}^{P}\right) \cdot \left(\nu_{ij}^{Q}\right) \right)^n \right) \right] \quad (2.1)$$

$nP \oplus nB$

$$= \left[\left(1 - \left(1 - \mu_{ij}^{P}\right)^n + 1 - \left(1 - \mu_{ij}^{Q}\right)^n - \left(1 - \left(1 - \mu_{ij}^{P}\right)^n\right) + \left(1 - \left(1 - \mu_{ij}^{Q}\right)^n\right), \left(\eta_{ij}^{P}\right)^n \cdot \left(\eta_{ij}^{Q}\right)^n, \left(\nu_{ij}^{P}\right)^n \cdot \left(\nu_{ij}^{Q}\right)^n \right) \right]$$

$$= \left[\left(1 - \left(1 - \mu_{ij}^P\right)^n \cdot \left(1 - \mu_{ij}^Q\right)^n, \left(\left(\eta_{ij}^P\right) \cdot \left(\eta_{ij}^Q\right) \right)^n, \left(\left(\nu_{ij}^P\right) \cdot \left(\nu_{ij}^Q\right) \right)^n \right) \right] \quad (2.2)$$

Hence, from (2.1) and (2.2), we get the result (i)

(ii) It can be proved analogously. □

Theorem 2.2. *For any PFSM P. If $n_1, n_2 > 0$ are integer, then*

(i) $n_1 P \oplus n_2 P = (n_1 + n_2)P$

(ii) $P^{n_1} \odot P^{n_2} = P^{(n_1+n_2)}$

Proof. (i) $n_1 P \oplus n_2 P$

$$= \left[\left(1 - \left(1 - \mu_{ij}^P\right)^{n_1} + 1 - \left(1 - \mu_{ij}^P\right)^{n_2} - \left(1 - \left(1 - \mu_{ij}^P\right)^{n_1}\right) \cdot \left(1 - \left(1 - \mu_{ij}^P\right)^{n_2}\right), \left(\eta_{ij}^P\right)^{n_1} \cdot \left(\eta_{ij}^P\right)^{n_2}, \left(\nu_{ij}^P\right)^{n_1} \cdot \left(\nu_{ij}^P\right)^{n_2} \right) \right]$$

$$= \left[\left(1 - \left(1 - \mu_{ij}^P\right)^{n_1+n_2}, \left(\eta_{ij}^P\right)^{n_1+n_2}, \left(\nu_{ij}^P\right)^{n_1+n_2} \right) \right]$$

$$= (n_1 + n_2)P$$

(ii) It can be proved analogously. □

Theorem 2.3. *For any PFSM P. If $n > 0$ are integer, then*

(i) $(P^C)^n = (nP)^C$

(ii) $n(P^C) = (P^n)^C$

Proof. (i) $(P^C)^n = \left[\left(\left(\nu_{ij}^P\right)^n, \left(\eta_{ij}^P\right)^n, 1 - \left(1 - \mu_{ij}^P\right)^n \right) \right]$

$$(nP)^C = \left[\left(\left(\nu_{ij}^P\right)^n, \left(\eta_{ij}^P\right)^n, 1 - \left(1 - \mu_{ij}^P\right)^n \right) \right]$$

(ii) It can be proved analogously. □

Theorem 2.4. *Let P and Q are two PFSMs. If $n > 0$ are integer, then*

(i) $n(P \cup Q) = nP \cup nQ$

(ii) $(P \cup Q)^n = P^n \cup Q^n$

Proof. (i) $n(P \cup Q) = n \left(\left[\max(\mu_{ij}^P, \mu_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^Q), \min(\nu_{ij}^P, \nu_{ij}^Q) \right] \right)$

$$= \left[\left(1 - \left(1 - \max(\mu_{ij}^P, \mu_{ij}^Q)\right)^n, min\left(\left(\eta_{ij}^P\right)^n, \left(\eta_{ij}^Q\right)^n \right), min\left(\left(\nu_{ij}^P\right)^n, \left(\nu_{ij}^Q\right)^n \right) \right) \right] (2.3)$$

$$nP \cup nQ = \left[\left(1 - \left(1 - \mu_{ij}^P\right)^n, \left(\eta_{ij}^P\right)^n, \left(\nu_{ij}^P\right)^n \cup 1 - \left(1 - \mu_{ij}^Q\right)^n, \left(\eta_{ij}^Q\right)^n, \left(\nu_{ij}^Q\right)^n \right) \right]$$

$$= \left[\left(max\left(1 - \left(1 - \mu_{ij}^P\right)^n, 1 - \left(1 - \mu_{ij}^Q\right)^n \right), min\left(\left(\eta_{ij}^P\right)^n, \left(\eta_{ij}^Q\right)^n \right), \min\left(\left(\nu_{ij}^P\right)^n, \left(\nu_{ij}^Q\right)^n \right) \right) \right]$$

$$= \left[\left(1 - \left(1 - \max(\mu_{ij}^P, \mu_{ij}^Q)\right)^n, min\left(\left(\eta_{ij}^P\right)^n, \left(\eta_{ij}^Q\right)^n \right), min\left(\left(\nu_{ij}^P\right)^n, \left(\nu_{ij}^Q\right)^n \right) \right) \right] (2.4)$$

From (2.3) and (2.4), we get the result (i)

(ii) It can be proved analogously. □

The following theorem is obvious. The operations $\cup$ and $\cap$ obey the De Morgan's laws.

Theorem 2.5. *If P and Q are two PFSMs, then*
(i) $P^C \cup Q^C = (P \cup Q)^C$ $\quad (ii)$ $P^C \cap Q^C = (P \cap Q)^C$
The operations $\oplus$ and $\odot$ obey the De Morgan's laws.

Theorem 2.6. *If P and Q are two PFSMs, then*
(i) $P^C \oplus Q^C = (P \odot Q)^C$ $\quad (ii)$ $P^C \odot Q^C = (P \oplus Q)^C$

Proof. (i) $P^C \oplus B^C$

$$= \left[\left(\nu_{ij}^P, \eta_{ij}^P, \mu_{ij}^P\right)\right] \oplus \left[\left(\nu_{ij}^Q, \eta_{ij}^Q, \mu_{ij}^Q\right)\right]$$
$$= \left[\left(\nu_{ij}^P + \nu_{ij}^Q - (\nu_{ij}^P) \cdot \left(\nu_{ij}^Q\right), (\eta_{ij}^P) \cdot \left(\eta_{ij}^Q\right), (\mu_{ij}^P) \cdot \left(\mu_{ij}^Q\right)\right)\right]$$

$(P \odot Q)$

$$= \left[\left((\mu_{ij}^P) \cdot \left(\mu_{ij}^Q\right), \eta_{ij}^P + \eta_{ij}^Q - \eta_{ij}^P \cdot \eta_{ij}^Q, \nu_{ij}^P + \nu_{ij}^Q - \nu_{ij}^P \cdot \nu_{ij}^Q\right)\right]$$

$(P \odot B)^C$

$$= \left[\left(\nu_{ij}^P + \nu_{ij}^Q - (\nu_{ij}^P) \cdot \left(\nu_{ij}^Q\right), \eta_{ij}^P + \eta_{ij}^Q - (\eta_{ij}^P) \cdot (\eta ij^Q), (\mu_{ij}^P) \cdot \left(\mu_{ij}^Q\right)\right)\right]$$
$$= P^C \oplus Q^C$$

(ii) It can be proved analogously. □

Theorem 2.7. *Let P and Q are two PFSMs, then*
(i) $(P \cup Q) \oplus (P \cap Q) = P \oplus Q$ $\quad (ii)$ $(P \cup Q) \odot (P \cap Q) = P \odot Q$

Proof. (i) $(P \cup Q) \oplus (P \cap Q)$

$$= \left[\left(\max(\mu_{ij}^P, \mu_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^Q), \min(\nu_{ij}^P, \nu_{ij}^B)\right)\right] \oplus \left[\left(\min(\mu_{ij}^P, \mu_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^Q), \max(\nu_{ij}^P, \nu_{ij}^Q)\right)\right]$$
$$= \left[\left(\max(\mu_{ij}^P, \mu_{ij}^Q) + \min(\mu_{ij}^P, \mu_{ij}^Q) - \max(\mu_{ij}^P, \mu_{ij}^Q) \cdot \min(\mu_{ij}^P, \mu_{ij}^Q)\right), \left(\min(\eta_{ij}^P, \eta_{ij}^Q) \cdot \min(\eta_{ij}^P, \eta_{ij}^Q)\right), \left(\min(\nu_{ij}^P, \nu_{ij}^Q) \cdot \max(\eta_{ij}^P, \eta_{ij}^Q)\right)\right]$$
$$= \left[\left(\mu_{ij}^P + \mu_{ij}^Q - \mu_{ij}^P \cdot \mu_{ij}^Q, \eta_{ij}^P \cdot \mu_{ij}^Q, \nu_{ij}^P \cdot \nu_{ij}^Q\right)\right]$$
$$= P \oplus Q$$

(ii) It can be proved analogously. □

Theorem 2.8. *Let P, Q and R are three PFSMs, then*
(i) $(P \cup Q) \cap R = (P \cap R) \cup (Q \cap R)$
(ii) $(P \cap Q) \cup R = (P \cup R) \cap (Q \cup R)$
(iii) $(P \cup Q) \oplus R = (P \oplus R) \cup (Q \oplus R)$

(iv) $(P \cap Q) \oplus R = (P \oplus R) \cap (Q \oplus R)$
(v) $(P \cup Q) \odot R = (P \odot R) \cup (Q \odot R)$
(vi) $(P \cap Q) \odot R = (P \odot R) \cap (Q \odot R)$

Proof. (i) $(P \cup Q) \cap R$

$$= \Big[\Big(\max(\mu_{ij}^P, \mu_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^Q), \min(\nu_{ij}^P, \nu_{ij}^Q)\Big)\Big] \cap \Big[\Big(\mu_{ij}^R, \eta_{ij}^R, \nu_{ij}^R\Big)\Big]$$
$$= \Big[\Big(\min\Big(\max(\mu_{ij}^P, \mu_{ij}^Q), \mu_{ij}^R\Big), \min\Big(\min(\eta_{ij}^P, \eta_{ij}^Q), \eta_{ij}^R\Big), \max\Big(\min(\nu_{ij}^P, \nu_{ij}^Q), \nu_{ij}^R\Big)\Big)\Big]$$
$$= \Big[\Big(\max\Big(\min(\mu_{ij}^P, \mu_{ij}^Q), \min(\mu_{ij}^P, \mu_{ij}^R)\Big), \min\Big(\min(\eta_{ij}^P, \eta_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^R)\Big), \min\Big(\max(\nu_{ij}^P, \nu_{ij}^Q), \max(\nu_{ij}^P, \nu_{ij}^R)\Big)\Big)\Big]$$
$$= \Big[\Big(\big(\min(\mu_{ij}^P, \mu_{ij}^R), \min(\eta_{ij}^P, \eta_{ij}^R), \max(\nu_{ij}^P, \nu_{ij}^R)\big) \cup \big(\min(\mu_{ij}^Q, \mu_{ij}^R), \min(\eta_{ij}^Q, \eta_{ij}^R), \max(\nu_{ij}^Q, \nu_{ij}^R)\big)\Big)\Big]$$
$$= (P \cup R) \cap (Q \cup R)$$

(ii) It can be proved analogously.

(iii) $(P \cup Q) \oplus R$

$$= \Big[\Big(\max(\mu_{ij}^P, \mu_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^Q), \min(\nu_{ij}^P, \nu_{ij}^Q)\Big)\Big] \oplus \Big[\Big(\mu_{ij}^R, \eta_{ij}^R, \nu_{ij}^R\Big)\Big]$$
$$= \Big[\Big(\max(\mu_{ij}^P, \mu_{ij}^Q) + \mu_{ij}^R - \max(\mu_{ij}^P, \mu_{ij}^Q) \cdot \mu_{ij}^R, \min(\eta_{ij}^P, \eta_{ij}^Q) \cdot \eta_{ij}^R, \min(\nu_{ij}^P, \nu_{ij}^Q) \cdot \nu_{ij}^R\Big)\Big] \tag{2.5}$$

$(P \oplus R) \cup (Q \oplus R)$

$$= \Big[\Big(\mu_{ij}^P + \mu_{ij}^R - \mu_{ij}^P \cdot \mu_{ij}^R, \eta_{ij}^P \cdot \mu_{ij}^R, \nu_{ij}^P \cdot \nu_{ij}^R\Big)\Big] \cup \Big[\Big(\mu_{ij}^P + \mu_{ij}^R - \mu_{ij}^P \cdot \mu_{ij}^R, \eta_{ij}^P \cdot \mu_{ij}^R, \nu_{ij}^P \cdot \nu_{ij}^R\Big)\Big]$$
$$= \Big[\Big(\max(\mu_{ij}^P + \mu_{ij}^R - \mu_{ij}^P \cdot \mu_{ij}^R, \Big(\mu_{ij}^Q + \mu_{ij}^R - \mu_{ij}^Q \cdot \mu_{ij}^R\Big), \min\Big(\eta_{ij}^P \cdot \eta_{ij}^R, \eta_{ij}^Q \cdot \eta_{ij}^R\Big), \min\Big(\nu_{ij}^P \cdot \nu_{ij}^R, \nu_{ij}^Q \cdot \nu_{ij}^R\Big)\Big)\Big]$$
$$= \Big[\Big(\max\Big((1 - \mu_{ij}^R)\mu_{ij}^P + \mu_{ij}^R, (1 - \mu_{ij}^R)\mu_{ij}^Q + \mu_{ij}^R\Big), \min\Big(\eta_{ij}^P \cdot \eta_{ij}^R, \eta_{ij}^Q \cdot \eta_{ij}^R\Big), \min\Big(\nu_{ij}^P \cdot \nu_{ij}^R, \nu_{ij}^Q \cdot \nu_{ij}^R\Big)\Big)\Big]$$
$$= \Big[\Big((1-\mu_{ij}^R) \max\Big(\mu_{ij}^P, \mu_{ij}^Q\Big) + \mu_{ij}^R, \min\Big(\eta_{ij}^P \cdot \eta_{ij}^R, \eta_{ij}^Q \cdot \eta_{ij}^R\Big), \min\Big(\nu_{ij}^P \cdot \nu_{ij}^R, \nu_{ij}^Q \cdot \nu_{ij}^R\Big)\Big)\Big] \tag{2.6}$$
$$= (P \oplus R) \cup (Q \oplus R)$$

From (2.5) and (2.6), we get the result (iii)

(iv) It can be proved analogously.

(v) $(P \cup B) \odot R$

$$= \Big[\Big(\max(\mu_{ij}^P, \mu_{ij}^Q), \min(\eta_{ij}^P, \eta_{ij}^Q), \min(\nu_{ij}^P, \nu_{ij}^Q)\Big)\Big] \odot \Big[\Big(\mu_{ij}^R, \eta_{ij}^R, \nu_{ij}^R\Big)\Big]$$
$$= \Big[\Big(\max(\mu_{ij}^P, \mu_{ij}^Q) \cdot \mu_{ij}^R, \min(\eta_{ij}^P, \eta_{ij}^Q) + \eta_{ij}^R - \min(\eta_{ij}^P, \eta_{ij}^Q) \cdot \eta_{ij}^R,$$

$$\min(\nu_{ij}^P,\nu_{ij}^Q)+\nu_{ij}^R-\min(\nu_{ij}^P,\nu_{ij}^Q)\cdot\nu_{ij}^R\Big)\Big]$$

$$=\Big[\Big(\max(\mu_{ij}^P,\mu_{ij}^Q)\cdot\mu_{ij}^R,(1-\eta_{ij}^R)\min(\eta_{ij}^P,\mu_{ij}^Q)+\eta_{ij}^R\Big)\Big] \tag{2.7}$$

$(P\odot R)\cup(Q\odot R)$

$$=\Big[\Big(\mu_{ij}^P\cdot\mu_{ij}^R,\eta_{ij}^P+\eta_{ij}^R-\eta_{ij}^P\cdot\eta_{ij}^R,\nu_{ij}^P+\nu_{ij}^R-\nu_{ij}^P\cdot\nu_{ij}^R\Big)\Big]\cup\Big[\Big(\mu_{ij}^Q\cdot\mu_{ij}^R,\eta_{ij}^Q+\eta_{ij}^R-\eta_{ij}^Q\cdot\eta_{ij}^R,$$
$$\nu_{ij}^Q+\nu_{ij}^R-\nu_{ij}^Q\cdot\nu_{ij}^R\Big)\Big]$$

$$=\Big[\Big(\max\left(\mu_{ij}^P\cdot\mu_{ij}^R,\mu_{ij}^Q\cdot\mu_{ij}^R\right),\min\left(\eta_{ij}^P+\eta_{ij}^R-\eta_{ij}^P\cdot\eta_{ij}^R,\eta_{ij}^Q+\eta_{ij}^R-\eta_{ij}^Q\cdot\eta_{ij}^R\right),$$
$$\min\left(\nu_{ij}^P+\nu_{ij}^R-\nu_{ij}^P\cdot\nu_{ij}^R,\nu_{ij}^Q+\nu_{ij}^R-\nu_{ij}^Q\cdot\nu_{ij}^R\right)\Big)\Big]$$

$$=\Big[\Big(\max(\mu_{ij}^P,\mu_{ij}^Q)\cdot\mu_{ij}^R,(1-\eta_{ij}^R)\min(\eta_{ij}^P,\mu_{ij}^Q)+\eta_{ij}^R\Big)\Big] \tag{2.8}$$

$=(P\cup Q)\odot R$

From (2.7) and (2.8), we get the result (v)

(vi) It can be proved analogously. □

Theorem 2.9. *Let* $P=[(\mu_{ij}^P,\eta_{ij}^P,\nu_{ij}^P]$ *and* $Q=[(\mu_{ij}^Q,\eta_{ij}^Q,\nu_{ij}^Q)]\in PFSM_{m\times n}$. *Then*
$(i)(P\cap Q)@(P\cup Q)=P@Q$ $\quad(ii)(P\cap Q)\$(P\cup Q)=P\$Q$
$(iii)(P\cap Q)\bowtie(P\cup Q)=P\bowtie Q$ $\quad(iv)(P\oplus Q)@(P\odot Q)=P@Q$

Proof. (i) $(P\cap Q)@(P\cup Q)$

$$=\Big[\Big(\max(\mu_{ij}^P,\mu_{ij}^Q),\min(\eta_{ij}^P,\eta_{ij}^Q),\min(\nu_{ij}^P,\nu_{ij}^Q)\Big)\Big]\ @\ \Big[\Big(\min(\mu_{ij}^P,\mu_{ij}^Q),\min(\eta_{ij}^P,\eta_{ij}^Q),$$
$$\max(\nu_{ij}^P,\nu_{ij}^Q)\Big)\Big]$$

$$=\Big[\Big(\frac{min\{\mu_{ij}^P,\mu_{ij}^Q\}+max\{\mu_{ij}^P,\mu_{ij}^Q\}}{2},\frac{min\{\eta_{ij}^P+\eta_{ij}^Q\}+min\{\eta_{ij}^P+\eta_{ij}^Q\}}{2},$$
$$\frac{max\{\nu_{ij}^P+\nu_{ij}^Q\}+min\{\nu_{ij}^P+\nu_{ij}^Q\}}{2}\Big)\Big]$$

$$=\Big[\Big(\frac{\mu_{ij}^P+\mu_{ij}^Q}{2},\frac{\eta_{ij}^P+\eta_{ij}^Q}{2},\frac{\nu_{ij}^P+\nu_{ij}^Q}{2}\Big)\Big]=P@Q.$$

(ii),(iii),(iv) can be proved similarly. □

Theorem 2.10. *Let* $P=[(\mu_{ij}^P,\eta_{ij}^P,\nu_{ij}^P)]$, $Q=[(\mu_{ij}^Q,\eta_{ij}^Q,\nu_{ij}^Q)]$ *and*
$R=[(\mu_{ij}^R,\eta_{ij}^P,\nu_{ij}^R)]\in PFSM_{m\times n}$ *be a Picuture fuzzy soft matrix. Then, in view of the defination, the following resultsmay easily be verfied:*
(i) $(P\cup Q)@R=(P@R)\cup(Q@R)$
(ii) $(P\cap Q)@R=(P@R)\cap(Q@R)$
(iii) $(P\cup Q)\bowtie R=(P\bowtie R)\cup(Q\bowtie R)$

(iv) $(P \cap Q) \bowtie R = (P \bowtie R) \cap (Q \bowtie R)$
(v) $(P@Q) \odot R = (P \odot R)@(Q \odot R)$
(vi) $(P@Q) \oplus R = (P \oplus R)@(Q \oplus R)$

3. CONCLUSIONS

In this work, we defined of the Pictute fuzzy Soft matrix and studied basic operations. We constructed nP and P^n of an Picture fuzzy soft matrix P and investigated their algebraic properties. Some results on nP and P^n combined with max-min, min-max, algebraic sum, algebraic product and complement of Picture fuzzy soft matrices are presented.

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