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K-ODD SEQUENTIAL HARMONIOUS LABELING OF DOUBLE M-TRIANGULAR SNAKES

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ABSTRACT. Harmonious graphs introduced by Graham and sloane [1] and singh and Varkey [3] presented the odd sequential graphs. The main objective of this paper we have shown that the double m triangular snakes, and alternative double m triangular snakes are odd sequential harmonious graph for every m.

1. INTRODUCTION

All graphs considered as finite, simple and undirected during this paper. A graph with $p-$ vertices and $q-$ edges are referred to as a (p, q) graph. Indicate the set of vertex and edge symbols $V(G)$ and $E(G)$. An arrangement of whole numbers at the vertices or edges or both according to unique conditions is understood as a graph labeling. Graph labeling presented in late $1960's$. Graph labeling is in many applications like coding theory. $X-$ beam Crystallography, radar, space science, circuit structure, correspondence organize tending to, information base administration.

Definition 1.1 ($k-$ Odd sequential harmonious labeling $(k - OSHL)$:). *A graph G is supposed to be $k-$ odd sequential harmonious labeling if there exist a*

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one - one function $h : V(G) \to \{k-1, k, k+1, ..., k+2q-1\}$ *specified the actuated mapping* $h^* : E(G) \to \{2k-1, 2k+1, 2k+3, ..., 2k+2q-3\}$ *defined by*

$$h^*(uv) = \begin{cases} h(u) + h(v) + 1, & if \;\; h(u) + h(v) \;\; iseven \\ h(u) + h(v), & if \;\; h(u) + h(v) \;\; isodd \end{cases}$$

are distinct.

Definition 1.2 ($k-$ Odd sequential harmonious graph ($k - OSHG$))**.** *A graph is called* $k-$ *odd sequential harmonious graph if it has* $k-$ *odd sequential harmonious labeling.*

2. Main Results

Definition 2.1 (Double m triangular snake ($2mTS_n$))**.** *A double* m *triangular snake comprises of* m *triangular snakes that have path, in like manner, i.e., a* ($2mTS_n$) *is gotten from a path* $v_1, v_2, \ldots, v_n$ *by joining* v_i *and* v_{i+1} *to a different vertex* u_i^j *for* $i = 1, 2, \ldots, n-1, j = 1, 2, \ldots, m$ *to a different vertex* w_i^j *for* $i = 1, 2, \ldots, n-1, j = 1, 2, \ldots, m$. *Specifically* $m = 1$ *is named double triangular snake.*

Theorem 2.1. *Double* m *- triangular snake is an* $OSHG$ *for every* m.

Proof. Let the vertices of $2m(TS_n)$ is:

$$\{v_i : 1 \leq i \leq n\} \cup \{u_i^j, w_i^j; 1 \leq i \leq n-1, \leq j \leq m\}.$$

Then the edges labels of $2m(TS_n)$ are:

$$\begin{aligned} &\{v_i v_{i+1}; 1 \leq i \leq n-1\} \cup \{v_i u_i^j; 1 \leq i \leq n-1, \leq j \leq m\} \\ &\cup \{v_{i+1} w_i^j; 1 \leq i \leq n-1, \leq j \leq m\} \cup \{v_{i+1} u_i^j; 1 \leq i \leq n-1, \leq j \leq m\} \\ &\cup \{v_i w_i^j; 1 \leq i \leq n-1, 1 \leq j \leq m\} \end{aligned}$$

and are denoted as the following figure:

The vertices are first labelled as follows:

Let $h : V \to \{k-1, k, k+1, k+2, \ldots, k+2q-1\}$ be defined by

$$\begin{cases} h(v_i) & = i + k - 2, 1 \leq i \leq n \\ h(u_i^j) & = (8j-2)n + 3i + k - (8j+1) \;\; i \leq j \leq m, 1 \leq i \leq n-1 \\ h(w_i^j) & = (8j-6)n + 3i + k - (8j-3) \end{cases}$$

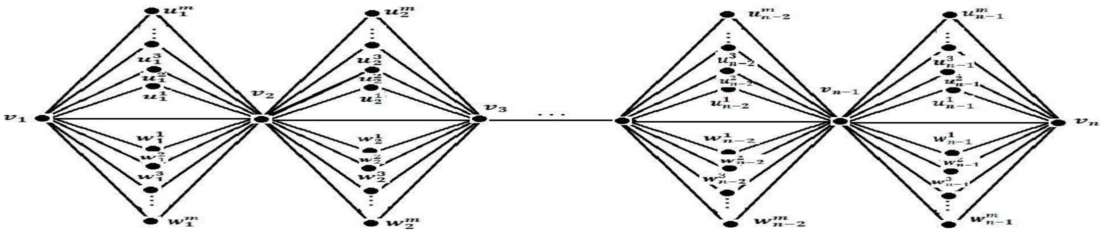


FIGURE 1. $2m(TS_n)$

At that point the incited edge labels are:

$h^*(v_iv_{i+1}) = 2i + 2k - 3, 1 \leq i \leq n-1$

$$\begin{cases} h^*(v_iu_i^j) & = (8j-2)n + 4i + 2k - (8j+3) \\ h^*(v_{i+1}u_i^j) & = (8j-2)n + 4i + 2k - (8j+1) \\ h^*(v_iw_i^j) & = (8j-6)n + 4i + 2k - (8j-1) \\ h^*(v_{i+1}w_i^j) & = (8j-6)n + 4i + 2k - (8j-3), \end{cases} \quad 1 \leq j \leq m, 1 \leq i \leq n-1$$

Obviously, we see that the edge labels are distinct. Along these lines, the graph $(2mTS_n)$ is a $(k-OSHL)$, for every m. Thus the graph $2m(TS_n)$ is an $k-OSHG$, for every m. □

Example 1.

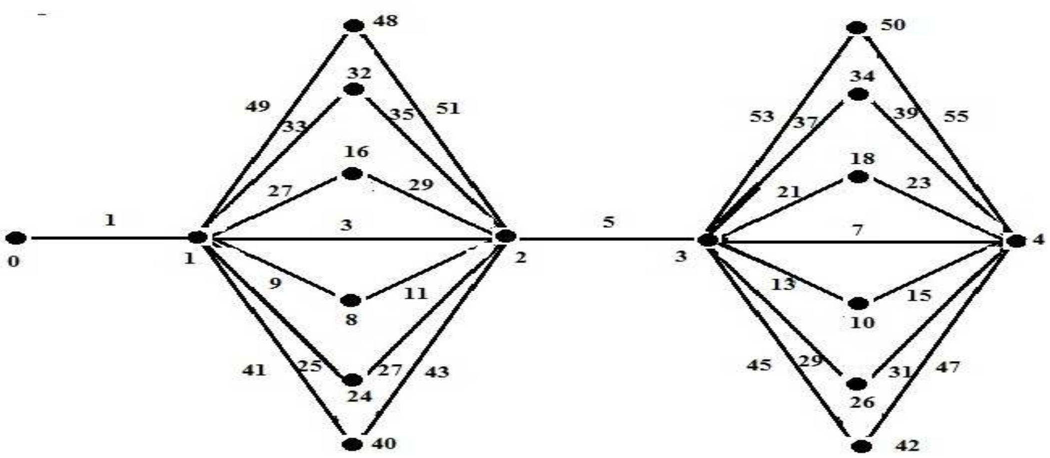
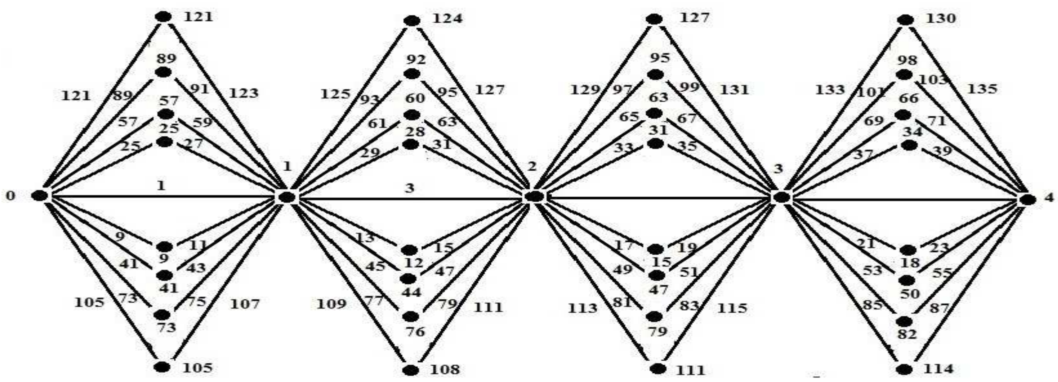


FIGURE 2. $1 - OSHL$

Example 2.



$1-OSHL$

Theorem 2.2. *Alternate double triangular snake $A(2mTS_n)$ starting with an edge is an $k-$ odd sequential harmonious graph for every m.*

Proof. There are two other cases.

Case(i): $n-$ is even.

Let the vertices of $A(2mTS_n)$ is

$$\{v_i; 1 \leq i \leq n\} \cup \{u_i^j, w_i^j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\},$$

and the edges of $A(2mTS_n)$ is

$$\begin{aligned}
&\{v_i v_{i+1}; 1 \leq i \leq n-1\} \\
&\cup \{v_{2i} u_i^j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\} \\
&\cup \{v_{2i+1} u_i^j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\} \\
&\cup \{v_{2i} w_i^j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\} \\
&\cup \{v_{2i-1} w_i^j; 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m\},
\end{aligned}$$

which are denoted as the following figure.

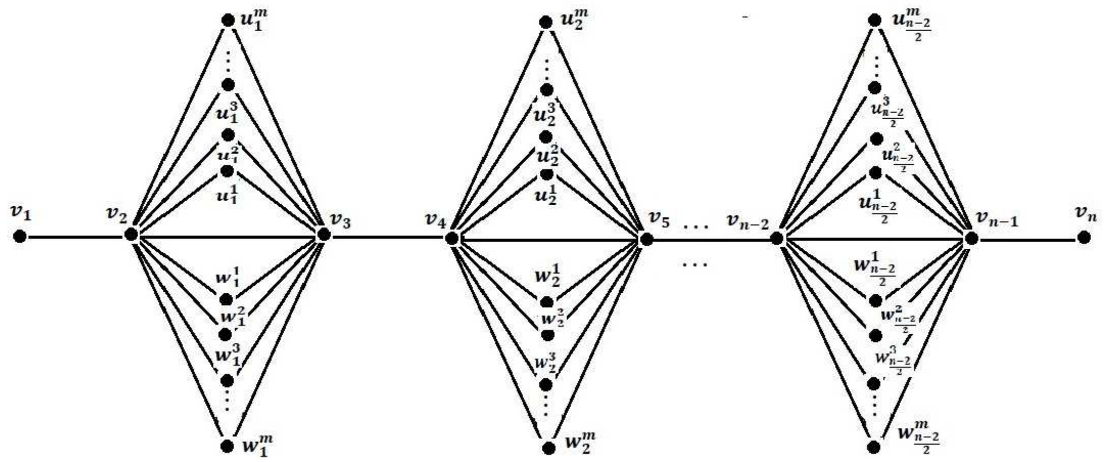


FIGURE 3. $A(2m(TS_n))$

The vertices are first labelled as follows. Let $h : V \to \{k-1, k, k+1, \dots, k+2q-1\}$ be defined by

$$h(v_i) = i + k - 2, 1 \le i \le n,$$

$$\begin{cases} h(u_i^j) & = 4jn + 2i + k - (8j+1), \\ h(w_i^j) & = (4j-2)n + 2i + k - (8j-3), \end{cases} \quad 1 \le i \le \frac{n-2}{2}, 1 \le j \le m.$$

At that point the prompted edge labels are

$$h^*(v_i v_{i+1}) = 2i + 2k - 3, \quad 1 \le i \le n-1,$$

$$\begin{cases} h^*(v_{2i} w_i) & = (4j-2)n + 4i + 2k - (8j-1), \\ h^*(v_{2i+1} w_i^j) & = (4j-2)n + 4i + 2k - (8j-3) \\ h^*(v_{2i} u_i^j) & = 4jn + 4i + 2k - (8j+3) \\ h^*(v_{2i+1} u_i^j) & = 4jn + 4i + 2k - (8j+1) \end{cases} \quad 1 \le i \le \frac{n-2}{2}, 1 \le j \le m$$

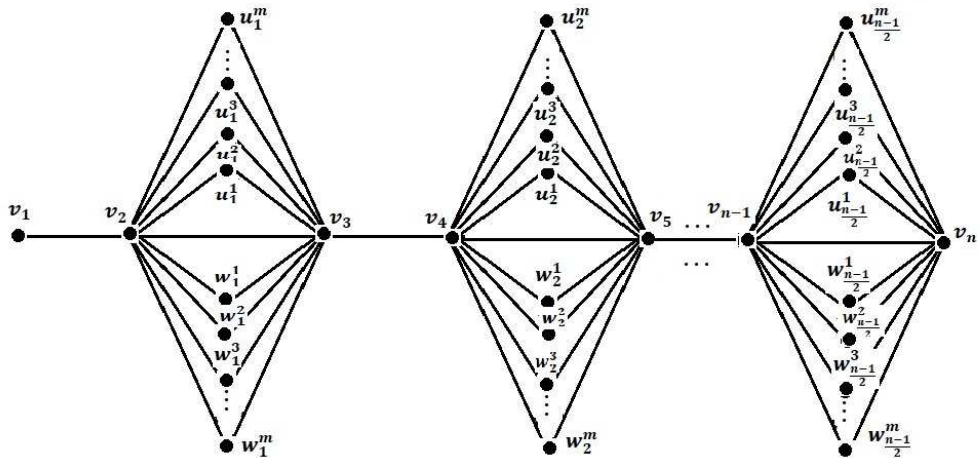
Case (ii): $n-$ is odd.
Let the vertices of $A(2mTS_n)$ are

$$\{v_i; 1 \le i \le n\} \cup \{u_i^j, w_i^j; 1 \le i \le \frac{n-1}{2}, 1 \le j \le m\}$$

and the edges of $A(2mTS_n)$ are

$$\begin{aligned}\{v_iv_{i+1}; 1 \leq i \leq n\} &\cup \{v_{2i}u_i^j; 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m\}\\ &\cup \{v_{2i+1}u_i^j; 1 \leq j \leq m\}\\ &\cup \{v_{2i}w_i^j; 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m\}\\ &\cup \{v_{2i+1}w_i^j; 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m\}.\end{aligned}$$

which are denoted in figure:



The vertices are first labeled as follows:

$$h(v_i) = i + k - 2, \quad 1 \leq i \leq n,$$

$$\begin{cases} h(u_i^j) & = 4jn + 2i + k - (4j+3), \\ h(w_i^j) & = (4j-2)n + 2i + k - (4j+1), \end{cases} \quad 1 \leq i \leq \frac{n-1}{2}, 1 \leq j \leq m.$$

The edge labels then are,

$$h^*(v_iv_{i+1}) = 2i - 1, \quad 1 \leq i \leq n-1,$$

$$\begin{cases} h^*(v_{2i}u_i) & = 4jn + 4i + 2k - (4j+5), \\ h^*(v_{2i+1}u_i^j) & = 4jn + 4i + 2k - (4j+3), \\ h^*(v_{2i}w_i^j) & = (4j-2)n + 4i + 2k - (4j+3), \\ h^*(v_{2i+1}w_i^j) & = (4j-2)n + 4i + 2k - (4j+1), \end{cases} \quad 1 \leq i \leq \frac{n-2}{2}, 1 \leq j \leq m.$$

In both case, we see that the edge labels are distinct. So, $h-$ admits $k-$ odd sequential harmonious labeling. Hence the alternating double triangular graph $A(2mTS_n)$ is $k - OSHG$ for every m. □

Example 3. *$n-$ is even.*

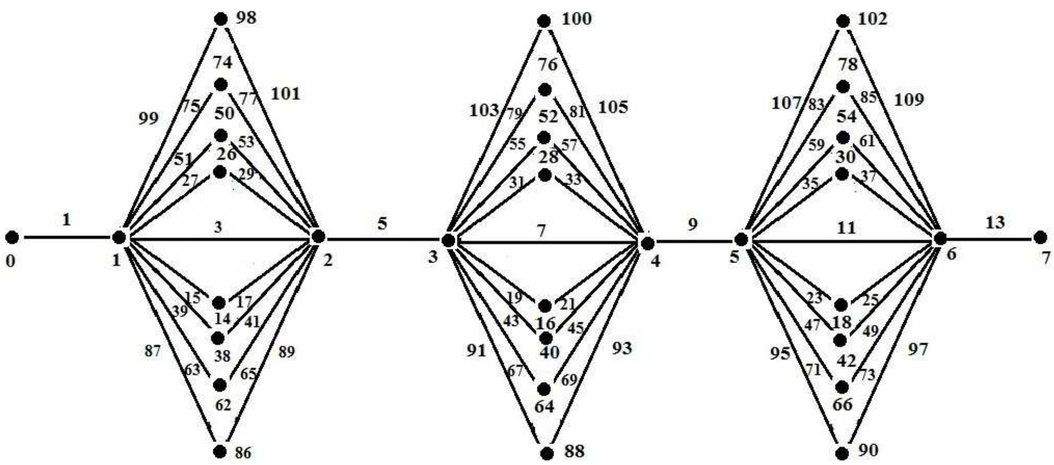


FIGURE 4. $1 - OSHL$

Example 4.

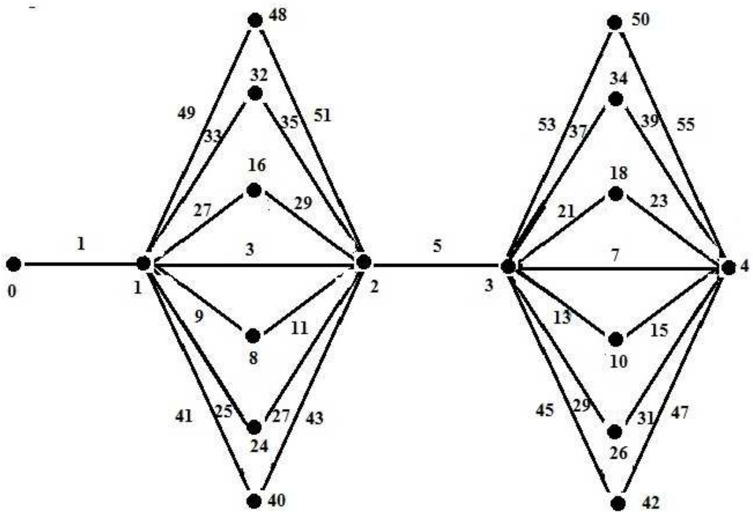


FIGURE 5. $1 - OSHL$

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