ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal **9** (2020), no.8, 6397–6403
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.8.107 Special Issue on ICMA-2020

PRIESTLEY FUZZY SPACE

K. SUGAPRIYA[1] AND B. AMUDHAMBIGAI

ABSTRACT. The main purpose of understanding this study is to introduced the notions of fuzzy upset, fuzzy downset and Priestley fuzzy Spaces. Elaborating this study further, an equivalent condition of a fuzzy compact Hausdorff space with a fuzzy quasi order is given together with necessary and sufficient that is of prime important in the study of Priestly fuzzy space.

1. INTRODUCTION

Zadeh [7] is the founding father of fuzzy mathematics who introduced the fuzzy concepts like fuzzy set theory and fuzzy logic. In 1968, Chang [3] proposed the concept of fuzzy topological space with the aid of fuzzy concepts. In 1982, Priestly[6] introduced the concept of Priestly Space in topological space. The main purpose of understanding this study is to introduced the notions of fuzzy upset, fuzzy downset and Priestley fuzzy Spaces. Elaborating this study further, an equivalent condition of a fuzzy compact Hausdorff space with a fuzzy quasi order is given together with necessary and sufficient that is of prime important in the study of Priestly fuzzy space.

---

[1]*Corresponding author*

2010 *Mathematics Subject Classification.* 54A40, 03E72.

Key words and phrases. Fuzzy upset, Fuzzy downset, Priestley fuzzy Space, fuzzy one point compactification and Locally compact fuzzy order Hausdorff space.

2. PRELIMINARIES

Definition 2.1. *[3] A fuzzy topology on a set $\mathbb{X}$ is a collection $\mathbb{T}$ of fuzzy sets in $\mathbb{X}$ satisfying:*

(1) *$0 \in \mathbb{T}$ and $0 \in \mathbb{T}$*
(2) *If μ and γ belongs to $\mathbb{T}$, then does $\mu \cup \gamma$ and*
(3) *If μ_i belongs to $\mathbb{T}$ for each $i \in J$, then so does $\vee \mu_i$.*

If $\mathbb{T}$ is a fuzzy topology on $\mathbb{X}$, then the pair ($\mathbb{X}$,$\mathbb{T}$) is called a fuzzy topological space(in short., $\mathbb{FTS}$). The members of $\mathbb{T}$ are called fuzzy open sets. Fuzzy sets of the form (1-μ) where μ is a fuzzy open set, are called fuzzy closed sets.

Definition 2.2. *[1] A fuzzy ordered set on which there is given a $\mathbb{FTS}$ is called an ordered $\mathbb{FTS}$. In other words, ordered $\mathbb{FTS}$ is a triple $\mathbb{X}$ = ($\mathbb{X}$, $\mathbb{T}$, $\leq$), where ($\mathbb{X}$,$\mathbb{T}$) is a fuzzy topological space and $\leq$ is a fuzzy partial order on $\mathbb{X}$.*

Definition 2.3. *[5] A topological space ($\mathbb{X}$,$\mathbb{T}$) is a compact Hausdorff if it is compact and Hausdorff.*

Definition 2.4. *[6] Let $\mathbb{X}$ be a compact Hausdorff space and let $\mathbb{R}$ be a quasi-order on $\mathbb{X}$. Then ($\mathbb{X}$, $\mathbb{R}$) is a Priestley space if $\mathbb{R}$ satisfies the Priestley separation axiom: If a,b $\in \mathbb{X}$ with a $\not\mathbb{R}$ b then the clopen upset $\mathbb{U}$ with $a \in \mathbb{U}$ and $b \notin \mathbb{U}$.*

Definition 2.5. *[1] Let $\mathbb{X}$ and $\mathbb{Y}$ be nonempty sets. A fuzzy relation $\mathbb{R}$ is a fuzzy subset of $\mathbb{X} \times \mathbb{Y}$. In other words, $\mathbb{R} \in \mathbb{F}(\mathbb{X} \times \mathbb{Y})$. If $\mathbb{X} = \mathbb{Y}$ then we say that $\mathbb{R}$ is a binary fuzzy relation in $\mathbb{R}$. Let $\mathbb{R}$ be a binary fuzzy relation on $\mathbb{R}$. Then $\mathbb{R}$(u,v) is interpreted as the degree of membership of the ordered pair (u,v) in $\mathbb{R}$.*

Definition 2.6. *[2] Suppose $\mathbb{X}$ is a topological space, with topology $\mathbb{T}$. Let ∞ denote some abstract point that is not in $\mathbb{X}$ and let $\tilde{\mathbb{X}}$ be the set $\mathbb{X} \cup \{\infty\}$. Define a topology $\tilde{\mathbb{T}}$ on $\tilde{\mathbb{X}}$ as follows:*
(1) Each open set in $\mathbb{X}$ is included in $\tilde{\mathbb{T}}$, that is $\mathbb{T} \subseteq \tilde{\mathbb{T}}$.
(2) For each compact set $\mathbb{C} \subseteq \mathbb{C}$, define an element $\mathbb{U}_{\mathbb{C}} \in \tilde{\mathbb{T}}$ by $\mathbb{U}_{\mathbb{C}} = (\mathbb{X} - \mathbb{C}) \cup \{\infty\}$.

Definition 2.7. *[4] Let ($\mathbb{X}$, $\mathbb{T}$) be a topological space. Then ($\mathbb{X}$, $\mathbb{T}$) is a called locally compact, if every point of $\mathbb{X}$ has a compact neighbourhood.*

3. Priestley Fuzzy Space

The properties and the characterization of Priestly fuzzy space are discussed in this section. Throughout the section $\mathbb{X}$ denote the fuzzy relation $\leq$.

Definition 3.1. *Let ($\mathbb{X}$, $\mathbb{T}$) be a $\mathbb{FTS}$. Any fuzzy set $\lambda \in I^X$ is said to be a fuzzy upset with respect to the fuzzy relation $\mathbb{R}$ if for some μ, $\gamma \in I^X$ with $\mu \leq \lambda$ and μ $\mathbb{R}$ γ, then $\gamma \leq \lambda$. If λ is the fuzzy upset, then it is denoted by $\uparrow\lambda$. i.e., $\uparrow\lambda = \{ \gamma \in I^X$: if $\mu \; \mathbb{R} \; \gamma$ and $\mu \; \mathbb{R} \; \gamma$, then $\gamma \leq \lambda\}$. Any $\lambda \in I^X$ is said to be a fuzzy open upset with respect to the fuzzy relation $\mathbb{R}$ if λ is fuzzy open. The compliment of a fuzzy open upset is called a fuzzy closed upset.*

Definition 3.2. *Let ($\mathbb{X}$, $\mathbb{T}$) be a $\mathbb{FTS}$. Any $\lambda \in I^X$ is called a fuzzy downset with respect to $\mathbb{R}$ if for some μ, $\gamma \in I^X$ with $\mu \leq \lambda$ and $\gamma \; \mathbb{R} \; \mu$, then $\gamma \leq \lambda$. If λ is the fuzzy downset, then it is denoted by $\downarrow\lambda$. i.e., $\downarrow\lambda = \{ \gamma \in I^X$: if $\gamma \; \mathbb{R} \; \mu$ and $\mu \leq \lambda$, then $\gamma \leq \lambda\}$. Any $\lambda \in I^X$ is said to be a fuzzy open downset with respect to the fuzzy relation $\mathbb{R}$ if λ is fuzzy open. The compliment of a fuzzy open downet is called a fuzzy closed downset.*

Definition 3.3. *Let ($\mathbb{X}$, $\mathbb{T}$) be a $\mathbb{FTS}$. The fuzzy closure of a fuzzy upset $\lambda \in I^X$ defined by Fcl($\uparrow\lambda$) $= \cup\{\mu \in I^X : \mu \in \mathbb{T}'$ and $\lambda \leq \mu\}$. The fuzzy closure of a fuzzy upset $\lambda \in I^X$ is denoted by Fcl($\uparrow\lambda$) or $\uparrow^{-}\lambda$.*

Definition 3.4. *Let ($\mathbb{X}$, $\mathbb{T}$) be a $\mathbb{FTS}$. Any $\lambda \in I^X$, is said to be a fuzzy clopen upset with respect to the fuzzy relation $\mathbb{R}$ if it is both fuzzy open upset and fuzzy closed upset.*

Definition 3.5. *Let ($\mathbb{X}$, $\mathbb{T}$) be a fuzzy compact hausdorff topological space and let $\mathbb{R}$ be a fuzzy quasi-order over ($\mathbb{X}$, $\mathbb{T}$). Then ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) is said to be a Priestley fuzzy space if $\mathbb{R}$ satisfies the Priestley separation axiom: If λ, $\mu \in I^X$ with $\lambda \not\mathbb{R} \mu$, then the fuzzy clopen upset $\gamma \in I^X$ with $\lambda \leq \gamma$ and $\mu \; nleq \; \gamma$.*

Remark 3.1.

(1) *If ($\mathbb{X}$, $\mathbb{T}$) is a fuzzy compact hausdorff space. If $\lambda \in I^X$ is a fuzzy upset and $\mu \in I^X$ is any fuzzy set with $\lambda \not\mathbb{R} \mu$, then $\lambda \not\mathbb{R} \downarrow \mu$. Similarly, if $\lambda \in I^X$ is a fuzzy downset and $\mu \in I^X$ is any fuzzy set with $\lambda \not\mathbb{R} \mu$, then $\lambda \not\mathbb{R} \uparrow \mu$.*

(2) *The complement of a fuzzy upset is a fuzzy downset and vice-versa.*

Definition 3.6. *Any $\mathbb{FTS}$ ($\mathbb{X}$, $\mathbb{T}$) is said to be of fuzzy zero dimensional if for all λ, $\mu \in I^X$ with $\lambda \neq \mu$, then there is a fuzzy clopen set $\gamma \in I^X$ with $\lambda \leq \gamma$ and $\mu \nleq \gamma$.*

Proposition 3.1. *Let ($\mathbb{X}$, $\mathbb{T}$) be a fuzzy compact hausdorff space with a fuzzy quasi order $\mathbb{R}$ on ($\mathbb{X}$, $\mathbb{T}$). Then the following conditions are equivalent:*

(1) *$\lambda \in I^X$ is fuzzy closed in the fuzzy product space ($\mathbb{X}$, $\mathbb{T}$) $\times$ ($\mathbb{X}$, $\mathbb{T}$).*
(2) *For each fuzzy closed set $\mu \in I^X$, $\uparrow \mu$ and $\downarrow \mu$ are fuzzy closed.*
(3) *If $\mu \in I^X$, then $\overline{\uparrow \mu} \leq \uparrow \bar{\mu}$ and $\overline{\downarrow \mu} \leq \downarrow \bar{\mu}$.*
(4) *If λ, $\mu \in I^X$, with $\lambda \not{\mathbb{R}} \mu$, then there is a fuzzy open upset $\gamma \in I^X$ with $\lambda \leq \gamma$ and a fuzzy open downset $\eta \in I^X$ such that $\mu \leq \eta$ with $\gamma \not{q} \eta$.*

Proof.

(1)$\Rightarrow$(2) Suppose that $\lambda \in I^X$ is a fuzzy closed set in ($\mathbb{X}$, $\mathbb{T}$) $\times$ ($\mathbb{X}$, $\mathbb{T}$). Let $\mu \in I^X$be a fuzzy closed set. Consider two fuzzy projection mappings π_1, π_2 : ($\mathbb{X}$, $\mathbb{T}$) $\times$ ($\mathbb{X}$, $\mathbb{T}$) $\rightarrow$ ($\mathbb{X}$, $\mathbb{T}$). Since ($\mathbb{X}$, $\mathbb{T}$) is a fuzzy compact hausdorff, π_1, π_2 are fuzzy closed maps. So that $\uparrow \mu = \pi_2((\mu \times 1_X) \cup \lambda)$ and $\downarrow \mu = \pi_1((1_X \times \mu) \cup \lambda)$, both are fuzzy closed sets of ($\mathbb{X}$, $\mathbb{T}$).

(2)$\Rightarrow$(3) Suppose (2) holds. Let $\mu \in I^X$. Then $\bar{\mu} \in I^X$ is fuzzy closed. By (2), $\uparrow \bar{\mu}$ is also fuzzy closed in ($\mathbb{X}$, $\mathbb{T}$). Since $\uparrow \mu \leq \uparrow \bar{\mu}$, we have $\overline{\uparrow \mu} \leq \uparrow \bar{\mu}$. For fuzzy downsets the proof is similar.

(3)$\Rightarrow$(4) Let λ, $\mu \in I^X$, with $\lambda \not{\mathbb{R}} \mu$. By (3), $\overline{\uparrow \lambda} \leq \uparrow \lambda$, so $\uparrow \lambda$ is a fuzzy closed upset. Similarly, $\downarrow \mu$ is a fuzzy closed downset. Furthermore, $\uparrow \lambda \not{q} \downarrow \mu$. Since ($\mathbb{X}$, $\mathbb{T}$) is a fuzzy compact hausdorff space and hence ($\mathbb{X}$, $\mathbb{T}$) is a fuzzy normal space. Therefore, for any two fuzzy open sets γ, $\eta \in I^X$ with $\gamma \not{q} \eta$. We have $\uparrow \lambda \leq \gamma$ and $\downarrow \mu \leq \eta$. Since $\bar{\gamma}$ is fuzzy closed, condition(3) implies $\overline{\downarrow \bar{\gamma}} \leq \downarrow \bar{\gamma}$ and hence $\overline{\downarrow \bar{\gamma}} = \downarrow \bar{\gamma}$. Therefore, $\downarrow \bar{\gamma}$ is a fuzzy closed downset. So that $\overline{\downarrow \bar{\gamma}}$ is a fuzzy open upset. Since $\downarrow \bar{\gamma} \leq \bar{\gamma}$, we have $\overline{\downarrow \bar{\gamma}} \leq \gamma$. Since $\uparrow \lambda \not{\mathbb{R}} \bar{\gamma}$ and $\uparrow \lambda$ is a fuzzy upset, then $\uparrow \lambda \not{\mathbb{R}} \downarrow \bar{\gamma}$. Therefore, $\uparrow \lambda \leq \overline{\downarrow \bar{\gamma}}$, and so $\overline{\downarrow \bar{\gamma}}$ is a fuzzy open upset $\uparrow \lambda \leq \overline{\downarrow \bar{\gamma}} \leq \gamma$. By similar argument we can prove $\downarrow \mu \leq \eta$.

(4)$\Rightarrow$(1) Let λ, $\mu \in I^X$, with $\lambda \not{\mathbb{R}} \mu$, then there is a fuzzy open upset $\gamma \in I^X$ with $\lambda \leq \gamma$ and a fuzzy open downset $\eta \in I^X$ with $\mu \leq \eta$ such that $\gamma \not{q} \eta$. Then by the Definitions 3.1 and 3.2, (1) follows. □

Remark 3.2. *If ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) is a Priestley fuzzy space, then Proposition 3.1 holds.*

4. PRIESTLEY FUZZY ORDER COMPACTIFICATION

Let ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) be a $\mathbb{FTS}$. The collection of all fuzzy points in ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) is denoted by $\mathbb{FP}(\mathbb{X})$.

Definition 4.1. *Let ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) be a ordered $\mathbb{FTS}$ and let $\lambda \in I^X$ be fuzzy open of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$). Then for any $x_t \in \mathbb{FP}(\mathbb{X})$,*

(1) *λ is a fuzzy upset neighbourhood of x_t if for a fuzzy open upset $\mu \in I^X$, $x_t \leq \mu \leq \lambda$.*
(2) *λ is a fuzzy downset neighbourhood of x_t if for a fuzzy open downset upset $\gamma \in I^X$, $x_t \leq \gamma \leq \lambda$.*

Definition 4.2. *Let ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) be a ordered $\mathbb{FTS}$. Then ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) is said to be fuzzy order Hausdorff if for λ, $\mu \in I^X$ with $\lambda \nleq \mu$, a fuzzy upset neighbourhood γ of λ and a fuzzy downset neighbourhood η of μ such that $\gamma \not q \eta$.*

Definition 4.3. *Let ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) and ($\mathbb{Y}$, $\mathbb{S}$, $\mathbb{Q}$) be ordered $\mathbb{FTS}$. Then ($\mathbb{Y}$, $\mathbb{S}$, $\mathbb{Q}$) is a fuzzy order compactification of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) if ($\mathbb{Y}$, $\mathbb{S}$, $\mathbb{Q}$)is compact fuzzy order Hausdorff and ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) is fuzzy order homeomorphic to a dense fuzzy subspace of ($\mathbb{Y}$, $\mathbb{S}$, $\mathbb{Q}$).*

Definition 4.4. *Let ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) and ($\mathbb{Y}$, $\mathbb{S}$, $\mathbb{Q}$) be any two ordered $\mathbb{FTS}$s. Then ($\mathbb{Y}$, $\mathbb{S}$, $\mathbb{Q}$) is said to be a Priestley fuzzy order compactification of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) if ($\mathbb{Y}$, $\mathbb{S}$, $\mathbb{Q}$) is a Priestley fuzzy space which is a fuzzy order compactification of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$).*

Definition 4.5. *Let ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) be a $\mathbb{FTS}$. Let x_t be a fuzzy point such that $x_t\ notin$ $mathbbFP(X)$ and $\tilde{\mathbb{X}} = \mathbb{X} \cup \{x_t\}$. Define a fuzzy topology $\tilde{\mathbb{T}}$ on $\tilde{\mathbb{X}}$ is as follows: (1) Each fuzzy set in $\mathbb{T}$ is a fuzzy set in $\tilde{\mathbb{T}}$. (2) For each fuzzy compact set $\lambda \in I^X$ define $\mu \in I^{\tilde{X}}$ such that μ = (1_X-λ) $\cup \{x_t\}$. Then ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) is the fuzzy one-point compactification of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$).*

Remark 4.1. *A ordered $\mathbb{FTS}$ has a Priestley fuzzy order compactification iff it is of fuzzy order zero dimensional.*

Proposition 4.1. *Let ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) be a locally compact fuzzy order Hausdorff space, and let ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) be a fuzzy one point compactification of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$). Then ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) is a Priestley fuzzy space iff ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) is a fuzzy order zero dimensional.*

Proof. If ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) is a Priestley fuzzy space, then ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) has a Priestley fuzzy order-compactification. Thus, by Remark 4.1, ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) is of fuzzy order zero dimensional. Conversely, suppose that ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) is a fuzzy order zero dimensional. We need to show that ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) satisfies the Priestley fuzzy separation axiom. Let λ, $\mu \in I^X$ with $\lambda\ n\tilde{l}eq\ \mu$. Since ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) is fuzzy compact and fuzzy order Hausdorff, there is a fuzzy open upset $\delta^{'} \in I^{\tilde{X}}$ and a fuzzy open downset $\gamma^{'} \in I^{\tilde{X}}$ so that $\lambda \leq \delta^{'}$, $\mu \leq \gamma^{'}$ and $\delta^{'} \not\leq \gamma^{'}$. First suppose that $\lambda = x_t$. Then $1_{\tilde{X}}$ - $\delta^{'}$ is a fuzzy compact downset of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) such that $1_{\tilde{X}}$ - $\delta^{'} \geq \mu^{'}$. Therefore, γ = $\bar{\gamma}^{'} \leq 1_{\tilde{X}}$ - $\delta^{'}$ is a fuzzy compact subset of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$). Moreover, since ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) satisfies the Priestley fuzzy separation axiom, so does γ. Thus, γ is a Priestley fuzzy space and $\gamma^{'}$ is a fuzzy open downset of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$). Therefore, $\gamma^{'} = \vee_{i=1}^{n}$ γ_i, where γ_i denotes the fuzzy clopen downsets of γ. Let ϑ be a fuzzy clopen downset of γ such that $\mu \leq \vartheta \leq \gamma^{'}$. Since $\gamma^{'}$ is a fuzzy open downset of $\mathbb{X}$ and ϑ $\leq \gamma^{'}$, then ϑ is a fuzzy compact open downset of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$). Let $\delta = 1_{\tilde{X}}$ - ϑ. Then δ is a fuzzy clopen upset of ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) for $x_t \leq \delta$ and $\mu \not\leq \delta$. A similar argument shows that if $\mu = x_t$, then there is a fuzzy clopen upset δ of ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) such that $\lambda \leq \delta$ and $x_t \not\leq \delta$. Finally, suppose λ, $\mu \in$ ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$). Because $\lambda \not\leq \mu$, we have $\lambda \not\leq \mu$, and since ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) satisfies the Priestley fuzzy separation axiom, there is a fuzzy clopen upset $\lambda^{'}$ of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$) such that $\lambda \leq \lambda^{'}$ and $\mu \not\leq \lambda^{'}$. From λ $\tilde{\not\leq}$ μ it follows that either $\lambda \tilde{\not\leq} x_t$ or $x_t \tilde{\not\leq} \mu$. Suppose$\lambda \tilde{\not\leq} x_t$. Since ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) is fuzzy compact and fuzzy order Hausdorff, then there is a fuzzy open upset δ_1 and a fuzzy open downset γ_1 of ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) such that $\lambda \leq \delta_1$, $x_t \leq \gamma_1$, and $\delta_1 \not\leq$ γ_1. Therefore, $1_{\tilde{\mathbb{X}}}$- γ_1 is a fuzzy compact subset of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$). Thus, $\delta' \bar{\cup} \delta_1 \leq 1_{\tilde{\mathbb{X}}}$- γ_1 is a fuzzy compact subset of ($\mathbb{X}$, $\mathbb{T}$, $\mathbb{R}$), and since $\delta' \bar{\cup} \delta_1$ satisfies the Priestley fuzzy separation axiom, $\delta' \bar{\cup} \delta_1$ is a Priestley fuzzy space. Moreover, $\delta^{'} \cup \delta_1$ is a fuzzy open upset of $\delta' \bar{\cup} \delta_1$. Therefore, there exists a fuzzy clopen upset δ of $\delta' \bar{\cup} \delta_1$ such that $\lambda \leq \delta \leq \delta^{'} \cup \delta_1$. But then δ is a fuzzy clopen upset of ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) with $\lambda \leq \delta$ and $\mu \not\leq \delta$. The case when $x_t \tilde{\not\leq} \mu$ is similar. Thus, ($\tilde{\mathbb{X}}$, $\tilde{\mathbb{T}}$, $\tilde{\mathbb{R}}$) is a Priestley fuzzy space. □

ACKNOWLEDGMENT

The authors would like to extend their gratitude to the anonymous referees for their valuable and constructive suggestions which have improved this paper.

REFERENCES

[1] A. AMROUNE, B. DAVVAZ: *Fuzzy ordered sets and dually for finite fuzzy distributive lattices*, Iranian journal of fuzzy systems, **8**(5) (2011), 1-12.

[2] G. BEZHANISHVILI, P.J. MORANDI: *Priestley Rings and Priestley Order-Compactifications*, Order, **28** (2011), 399-413. https://doi.org/10.1007/s11083-010-9180-2

[3] C.L. CHANG: *Fuzzy topological spaces*, J. Math. Anal. Appl., **24**(1) (1968), 182–190.

[4] P.T. JOHNSTONE: *Stone Spaces*, Cambridge University Press, Cambridge, (1982).

[5] L. NACHBIN: *Topology and order*, Van Nostrand Inc., Princeton, NJ, (1965).

[6] H.A. PRIESTLY: *Ordered topological spaces and the representation of distributive lattices*, Proc. London Math. Soc., **s3-24** (1972), 507-530.

[7] L.A. ZADEH: *Fuzzy Sets*, Information and Control, **8** (1965), 338-353.

DEPARTMENT OF MATHEMATICS
SRI SARADA COLLEGE FOR WOMEN(AUTONOMOUS)
SALEM, TAMILNADU, INDIA
Email address: nirmalapriya97@gmail.com

DEPARTMENT OF MATHEMATICS
SRI SARADA COLLEGE FOR WOMEN(AUTONOMOUS)
SALEM, TAMILNADU, INDIA
Email address: rbamudha@yahoo.co.in