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FELICITOUS FUZZY GRAPH

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ABSTRACT. The conception of felicitous fuzzy graph is introduced. Felicitous fuzzy graph for cycle related graphs and others graphs are defined. Every felicitous fuzzy graph is fuzzy labeling graph but converse is not true.

1. INTRODUCTION

The history of graph theory may be especially traced in 1735 [2], when the Switzerland mathematician Leonhard Euler clarified the Königsberg bridge problem. The Königsberg bridge problem was a puzzle regarding the probability of finding a route over every one of 7 bridges that reach a branched river falling past an island but without crossing any bridge two times. Euler disputed that no such path occurs. His proof involved only hits to the substantial plan to the bridges, but approximately he proved the initial theorem in graph theory. As handled in graph theory, the word graph does not refer to data charts, such as bar graphs. Rather, it refers to a set of nodes (that is, points or nodes) and of lines (or lines) that connect the nodes. In 1973, Initially Kausfmann introduced fuzzy graph. In 1975 [5] the theory of fuzzy graph developed by Azrief Rosenfeld. Fuzzy graph theory, a merge of fuzzy set theory and graph theory have been enforced in various fields of engineering and science. In 1975, Rosenfeld considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs .

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Sloane and Graham [1] introduced the perception of a harmonious graph. A connected graph with nodes and lines is said to be harmoniuos if it feasible to label the nodes with distinct elements. Hsu and rogers call a graph strongly $c-$elegent if the both node labels and line labels are distinct. To generalized both the strongly $c-$ elegent labeling and harmonious labeling at the same time, and keep the number of node labels that can be repeated to a minimum, E. Schmeichel and Sin-Min Lee introduce the felicitous labeling. In this paper section 2 involves basic definitions and in section 3 involves a new conception of felicitous fuzzy graph, and some results are discussed and in section 4 involves conclusion.

2. Preliminaries

Definition 2.1. *Suppose that $(s,t)-$graph G has a proper labeling $f : v(G) \rightarrow [0,t]$, where s and t represents the number of node and lines respectively. The line label $\beta(uv)$ of each line $uv \in E(G)$ is defined as $\beta(uv) = (\eta(u) + \eta(v))(mod\ t)$. If the line label set $\beta(uv) : uv \in E(G) = [0, t-1]$ then we say both G and f to be felicitous.*

Definition 2.2. *Suppose that $(s,t)-$ graph G has a proper labeling $f : V(G) \rightarrow \{t+1, t+2, ..., s+t\}$, where s and t represents the number of node and number of lines respectively. The line label $\beta(uv)$ of each line $uv \in E(G)$ is defined as $\beta(uv) = (\eta(u)+\eta(v))(mod(s.h)$. Where $h = 0.01$ if $n \leq 3$ and $h = 0.001$ if $n \geq 50$. If the line label set $\{\beta(uv) : uv \in E(G)\} = [1,t]$, then we can say G is said to be felicitous fuzzy graph.*

3. Main Result

Theorem 3.1. *For n is even, path P_n admits a felicitous fuzzy graph.*

Proof. Let P_n be the path with even number of nodes and lines $v_1, v_2, \ldots, v_n$ and $v_1v_2, v_2v_3, v_3v_4, \ldots, v_nv_{n-1}$ be the nodes and lines of P_n respectively. Let $h = 0.01$, if $n \leq 3$ and $h = 0.001$ if $n \geq 51$. If the length of the path is even, the felicitous fuzzy graph is defined as follows:

$$\begin{aligned} \eta(v_{2(i+1)}) &= ((n+1)+2i)h, \left(0 \leq i \leq \frac{n-1}{2}\right) \\ \eta(v_{2i+1}) &= \eta(v_2) + (2i+1)h, \left(0 \leq i \leq \frac{n-1}{2}\right) \end{aligned}$$

$$\beta(u_{i+1}, v_{i+2}) = (2i+1)h, \qquad \left(0 \leq i \leq \frac{n-1}{2}\right)$$
$$\beta(u_{n-i}, v_{(n-1)-i}) = (n-(2i+1))h, \quad \left(0 \leq i \leq \frac{n-3}{2}\right)$$

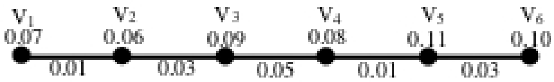

FIGURE 1. Felicitous fuzzy graph for path graph

Therefore, from the above we verified that G is a felicitous fuzzy graph if G has an even number of nodes. □

Corollary 3.1. *P_{2k+1} is not a felicitous fuzzy graph.*

Proof. Let us assume that P_{2k+1} is felicitous fuzzy graph. The number of nodes of P_{2k+1} is odd. From the definition of fuzzy graph, we will get the line values either 0 or even. Therefore P_{2k+1} is not a felicitous fuzzy graph. □

Theorem 3.2. *For n is even, cycle C_n admits a felicitous fuzzy graph.*

Proof. Let C_n be the cycle with even number of nodes and lines $v_1, v_2, \ldots, v_n$ and $v_1, v_2, \ldots, v_n v_{n-1}$ be the nodes and lines of C_n respectively. Let $h = 0.01$, if $n \leq 3$ and $h = 0.001$ if $n \geq 50$. If the length of the cycle is even, the Felicitous Fuzzy Graph is defined as follows:

$$\eta(v_{2(i+1)}) = (n+2(i+1))h, \qquad \left(0 \leq i \leq \frac{n-2}{2}\right)$$
$$\eta(v_{2i+1}) = \eta(v_2) + (2i+1)h, \qquad \left(0 \leq i \leq \frac{n-2}{2}\right)$$
$$\beta(v_{i+1}, v_{i+2}) = (\eta(v_i) + \eta(v_{i+1}))mod(s.h), \left(0 \leq i \leq s\right)$$

Therefore, it is verified that G is a felicitous fuzzy graph, if G has a even number of nodes. □

Corollary 3.2. *$C_{(2k+1)}$ is not a felicitous fuzzy graph.*

Proof. Let us assume that $C_{(2k+1)}$ is felicitous fuzzy graph. The number of nodes of C_{2k+1} is odd. From the definition fuzzy graph, we will get the line values either 0 or even. Therefore C_{2k+1} is not a felicitous fuzzy graph. □

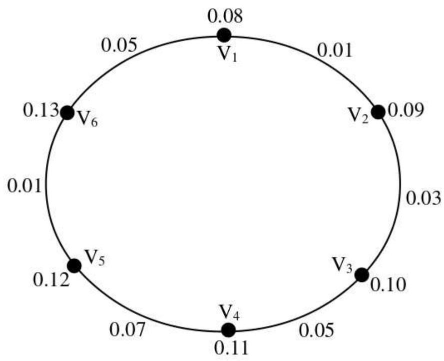

FIGURE 2. Felicitous fuzzy graph for cycle graph

Algorithm 3.1.
Step 1: Fix any $(s,t)-$ *graph* $G,(n\geq 3)$*, where* n *is odd.*
Step 2: Fix the membership function of the node, $\eta(v_i) = t+1, t \geq n$ *where* $i=1,2,\dots,s$.
Step 3: Fix the membership function of the line, $\beta(v_i,v_{i+1}) = (\eta(v_i)+\eta(v_{i+1}))$ $(mod(s.h))$*, where* $i=1,2,\dots,t$*, h=0.01 if n*≤ 3*, h=0.001 if n*≥ 51.
Step 4 : Every membership function of nodes and lines must satisfies the fuzzy graph condition, $\mu(u,v)\leq \eta(u)\wedge\eta(v)$*. Otherwise it fails.*

Algorithm 3.2.
Step 1: Fix any cycle related $(s,t)-$ *graph* G *with* $n\geq 3$.
Step 2: Fix the membership function of the node, $\eta(v_i) = t+1, t \geq n$ *where* $i=1,2,\dots,s,$.
Step 3: Fix the membership function of the line, $\beta(v_i,v_{i+1}) = (\eta(v_i)+\eta(v_{i+1}))$ $(mod(s.h))$*, where* $i=1,2,\dots,t$*, h=0.01 if n*≤ 3*, h=0.001 if n*≥ 50.
Step 4: Every membership function of nodes and lines must satisfies the fuzzy graph condition, $\mu(u,v)\leq \eta(u)\wedge\eta(v)$*. Otherwise it fails.*

Remark 3.1. *Suppose that* $(s,t)-$ *fuzzy graph* G *with odd number of lines and* $V(G)=\{t+1,t+2,\dots,s+t\}$ *be a even number of nodes of* G*. Then* G *is felicitous fuzzy graph.*

Proof. Let $V(G)=\{t+1,t+2,\dots,s+t\}$ be a even number of nodes of a fuzzy graph G, since s is even. After taking mod(s), $E(G)=\{0.01,0.03,\dots,t\}$ if the length of the graph is great than or equal to 3. Therefore G is felicitous fuzzy graph. □

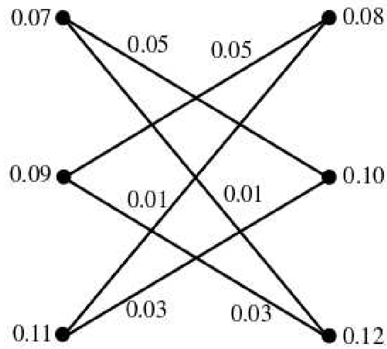

FIGURE 3. Felicitous fuzzy graph for crown graph

Remark 3.2. *Suppose that* $(s,t)-$ *fuzzy graph* G *with odd number of lines and* $V(G) = \{t+1, t+2, \ldots, s+t\}$ *be a odd number of nodes of* G. *Then* G *is not felicitous fuzzy graph.*

Proof. Let $V(G) = \{t+1, t+2, \ldots, s+t\}$ be a odd number of nodes of a fuzzy graph G, since s is odd. After taking mod(s), $E(G)$ ={even numbers and zero} so by the definition of fuzzy graph, G is not felicitous fuzzy graph. □

Remark 3.3. *Suppose that* $(s,t)-$ *fuzzy graph* G *with even number of lines and* $V(G) = \{t+1, t+2, \ldots, s+t\}$ *be a even number of nodes of* G. *Then* G *is felicitous fuzzy graph.*

Proof. Let $V(G) = \{t+1, t+2, \ldots, s+t\}$ be a even number of nodes of a fuzzy graph G, since s is even. After taking mod(s), $E(G) = \{0.01, 0.03, \ldots, t\}$ if the length of the graph is great than or equal to 3. Therefore G is felicitous fuzzy graph. □

Remark 3.4. *Suppose that* $(s,t)-$ *fuzzy graph* G *with even number of lines and* $V(G) = \{t+1, t+2, \ldots, s+t\}$ *be an odd number of nodes of* G. *Then* G *is felicitous fuzzy graph.*

Proof. Let $V(G) = \{t+1, t+2, \ldots, s+t\}$ be a odd number of nodes of a fuzzy graph G, since s is even. After taking mod(s), $E(G) =$ {even number and zero} so by the definition of fuzzy graph, G is not felicitous fuzzy graph. □

4. Conclusion

In this paper, the new concept of Felicitous Fuzzy Graph has been discussed. When there is an even number of lines, the results have been analyzed for odd number of nodes and even number of nodes. similarly, for odd number of lines, the results have been analyzed for odd number of nodes and even number of nodes. Finally, for the following graphs (i.e) line, path, cycle, ladder, crown, sunlet, quadilateral snake graph, the concept of felicitous fuzzy graph admits strictly.

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