

## **DERIVING SHAPE FUNCTIONS FOR 20 NODDED CUBIC TETRAHEDRON ELEMENT USING POLYNOMIAL IN NATURAL COORDINATE SYSTEM AND VERIFIED**

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**ABSTRACT.** In this paper I derived shape functions for 20 noded Cubic tetrahedron element using polynomial in natural coordinate system and I verified two shape function verification conditions. First shape function verification condition is sum of all the shape functions is equal to one at each nodal value and second one each shape function has a value of one at its own nodal value and zero at the remaining nodal values. For Computation Purpose I used Mathematica 9 software

### **1. INTRODUCTION**

The feature of the Shape function to compute accurate results for problems by including terms in the basis functions that are good approximations of the problem solution. The Property of shape functions is shape functions are linearly independent and significance of shape functions is it ensures the shape functions have Delta function properties.

### **2. GEOMETRICAL INTERPRETATION**

The 20-noded tetrahedron element is a cubic element shown in Figure.1. A tetrahedral element is a volume with four faces and is analogous to a triangle

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in two dimensions. In tetrahedron element there are four sides along any side therefore displacement varies in the cubic form. In figure.1 total number of nodes 20. Total number of edges(sides) six namely they are 1-2,1-3,1-4,2-3,2-4,3-4. Two additional nodes are added evenly on each edge of the element. Total number of nodes on these edges are 16. On side 1-2 additional nodes are 7 and 8. Now on side 1-2 nodes are 1,7,8 and 2. On edge 1-3 additional nodes are 5 and 6, now on edge 13 nodes are 1,5,6 and 3. On edge 1-4 additional nodes are 11 and 12, now on edge 1-4 nodes are 1,11,12 and 4. On edge 2-3 additional nodes are 9 and 10, now on edge 2-3 nodes are 2,9,10 and 3. On edge 2-4 additional nodes are 13 and 14, now on edge 2-4 nodes are 2,13,14 and 4. On edge 34 additional nodes are 15 and 16, now on edge 34 nodes are 3,15,16 and 4. Four-node central-face nodes are taken at the geometry centre of each triangular surface of the element. 17 node is taken on triangular surface of the element 2-3-4. 18 node is taken on triangular surface of the element 1-2-3. 19 node is taken on triangular surface of the element 1-3-4. 20 node is taken on triangular surface of the element 1-2-4. 20 nodal points in natural coordinate system are 1(1,0,0,0) , 2(0,1,0,0), 3(0,0,1,0), 4(0,0,0,1), 5(3/4,0,1/4,0), 6(1/4,0,3/4,0) 7(3/4,1/4,0,0), 8(1/4,3/4,0,0), 9(0,3/4,1/4,0), 10(0,1/4,3/4,0), 11(3/4,0,0,1/4), 12(1/4,0,0,3/4), 13(0,3/4,0,1/4), 14(0,1/4,0,3/4), 15(0,0,3/4,1/4), 16(0,0,1/4,3/4) 17(0,1/3,1/3,1/3), 18(1/3,1/3,1/3,0), 19(1/3,0,1/3,1/3), 20(1/3,1/3,0,1/3)

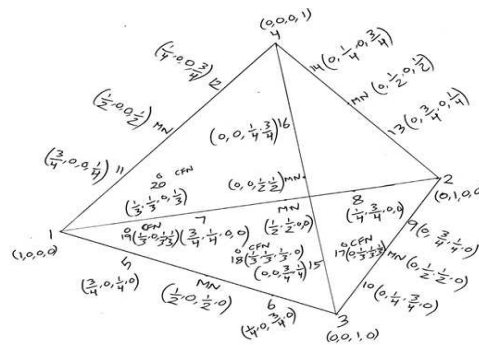


FIGURE 1. 20 Noded Cubic Tetrahedron Element



$$(3.3) \quad \{\alpha\} = \frac{1}{[A]} \{u\}_e, \{\alpha\} = [A]^{-1} \{u\}_e,$$

where  $\{u\}_e$  is the vector of nodal displacements,  $[A]$  is the matrix shown in (3.3) and  $\{\alpha\}$  is the vector of generalized coordinates (constants in Polynomials)

$$A := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{27}{64} & 0 & \frac{1}{64} & 0 & \frac{9}{64} & \frac{3}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{64} & 0 & \frac{27}{64} & 0 & \frac{3}{64} & \frac{9}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{27}{64} & \frac{1}{64} & 0 & 0 & 0 & 0 & \frac{9}{64} & \frac{3}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{64} & \frac{27}{64} & 0 & 0 & 0 & 0 & \frac{3}{64} & \frac{9}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{27}{64} & \frac{1}{64} & 0 & 0 & 0 & 0 & 0 & \frac{9}{64} & \frac{3}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{64} & \frac{27}{64} & 0 & 0 & 0 & 0 & 0 & \frac{3}{64} & \frac{9}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{27}{64} & 0 & 0 & \frac{1}{64} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{64} & \frac{3}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{64} & 0 & 0 & \frac{27}{64} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{64} & \frac{9}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{27}{64} & 0 & \frac{1}{64} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{64} & \frac{3}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{64} & 0 & \frac{27}{64} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{64} & \frac{9}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{27}{64} & \frac{1}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{9}{64} & \frac{3}{64} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{64} & \frac{27}{64} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{64} & \frac{9}{64} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & 0 & 0 & 0 & 0 & \frac{1}{27} & \frac{1}{27} & 0 & 0 & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & 0 & 0 & 0 \\ \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & 0 & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{27} & 0 & 0 \\ \frac{1}{27} & 0 & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & 0 & 0 & 0 & 0 & \frac{1}{27} & \frac{1}{27} & 0 & 0 & \frac{1}{27} & \frac{1}{27} & 0 & 0 & \frac{1}{27} \\ \frac{1}{27} & \frac{1}{27} & 0 & \frac{1}{27} & 0 & 0 & \frac{1}{27} & \frac{1}{27} & 0 & 0 & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & \frac{1}{27} & 0 & 0 & 0 & 0 & 0 & \frac{1}{27} \end{pmatrix}$$

By using Mathematica Software calculate inverse of A.  
Inverse of A (Matrix Form):

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{10}{3} & 0 & 1 & 0 & 8 & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{10}{3} & 0 & -\frac{8}{3} & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{10}{3} & 1 & 0 & 0 & 0 & 0 & 8 & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -\frac{10}{3} & 0 & 0 & 0 & 0 & -\frac{8}{3} & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{10}{3} & 1 & 0 & 0 & 0 & 0 & 0 & 8 & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{10}{3} & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{10}{3} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -\frac{10}{3} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & 8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{10}{3} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{10}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{10}{3} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & -\frac{8}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{10}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & -\frac{8}{3} & 0 & 0 & 0 \\ 0 & \frac{11}{3} & \frac{11}{3} & \frac{11}{3} & 0 & 0 & 0 & 0 & -\frac{16}{3} & -\frac{16}{3} & 0 & 0 & -\frac{16}{3} & -\frac{16}{3} & -\frac{16}{3} & -\frac{16}{3} & 27 & 0 & 0 & 0 \\ \frac{11}{3} & \frac{11}{3} & \frac{11}{3} & 0 & -\frac{16}{3} & -\frac{16}{3} & -\frac{16}{3} & -\frac{16}{3} & -\frac{16}{3} & -\frac{16}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 27 & 0 & 0 \\ \frac{11}{3} & 0 & \frac{11}{3} & \frac{11}{3} & -\frac{16}{3} & -\frac{16}{3} & 0 & 0 & 0 & 0 & -\frac{16}{3} & -\frac{16}{3} & 0 & 0 & -\frac{16}{3} & -\frac{16}{3} & 0 & 0 & 27 & 0 \\ \frac{11}{3} & \frac{11}{3} & 0 & \frac{11}{3} & 0 & 0 & -\frac{16}{3} & -\frac{16}{3} & 0 & 0 & -\frac{16}{3} & -\frac{16}{3} & -\frac{16}{3} & -\frac{16}{3} & 0 & 0 & 0 & 0 & 0 & 27 \end{pmatrix}$$

But

$$u = \begin{Bmatrix} \xi_1^3 & \xi_2^3 & \xi_3^3 & \xi_4^3 & \xi_1^2 \xi_3 & \xi_1 \xi_3^2 & \xi_1^2 \xi_2 & \xi_1 \xi_2^2 \\ \xi_2^2 \xi_3 & \xi_2 \xi_3^2 & \xi_1^2 \xi_4 & \xi_1 \xi_4^2 & \xi_2^2 \xi_4 & \xi_2 \xi_4^2 & \xi_3^2 \xi_4 & \xi_3 \xi_4^2 \\ \xi_3 \xi_4^2 & \xi_3 \xi_4^2 & \xi_2 \xi_3 \xi_4 & \xi_1 \xi_2 \xi_3 & \xi_1 \xi_3 \xi_4 & \xi_1 \xi_2 \xi_4 \end{Bmatrix} \{\alpha\}$$

$$u = \begin{Bmatrix} \xi_1^3 & \xi_2^3 & \xi_3^3 & \xi_4^3 & \xi_1^2 \xi_3 & \xi_1 \xi_3^2 & \xi_1^2 \xi_2 & \xi_1 \xi_2^2 & \xi_2^2 \xi_3 \\ \xi_2 \xi_3^2 & \xi_1^2 \xi_4 & \xi_1 \xi_4^2 & \xi_2^2 \xi_4 & \xi_2 \xi_4^2 & \xi_3^2 \xi_4 & \xi_3 \xi_4^2 & \xi_2 \xi_3 \xi_4 & \xi_1 \xi_2 \xi_3 \\ \xi_1 \xi_3 \xi_4 & \xi_1 \xi_2 \xi_4 \end{Bmatrix} [A]^{-1} \{u\}_e$$

After multiplying Coefficients and Inverse matrix we get

$$\begin{aligned}
u = & \left\{ \left\{ \xi_1^3 - \frac{10}{3} \xi_1^2 \xi_2 + \xi_1 \xi_2^2 - \frac{10}{3} \xi_1^2 \xi_3 + \frac{11}{3} \xi_1 \xi_2 \xi_3 + \xi_1 \xi_3^2 - \frac{10}{3} \xi_1^2 \xi_4 + \frac{11}{3} \xi_1 \xi_2 \xi_4 \right. \right. \\
& + \frac{11}{3} \xi_1 \xi_3 \xi_4 + \xi_1 \xi_4^2, \xi_1^2 \xi_2 - \frac{10}{3} \xi_1 \xi_2^2 + \xi_2^3 + \frac{11}{3} \xi_1 \xi_2 \xi_3 - \frac{10}{3} \xi_2^2 \xi_3 + \xi_2 \xi_3^2 + \frac{11}{3} \xi_1 \xi_2 \xi_4 \\
& - \frac{10}{3} \xi_2^2 \xi_4 + \frac{11}{3} \xi_2 \xi_3 \xi_4 + \xi_2 \xi_4^2, \xi_1^2 \xi_3 + \frac{11}{3} \xi_1 \xi_2 \xi_3 + \xi_2^2 \xi_3 - \frac{10}{3} \xi_1 \xi_3^2 - \frac{10}{3} \xi_1 \xi_3^2 \\
& - \frac{10}{3} \xi_2 \xi_3^2 + \xi_3^3 + \frac{11}{3} \xi_1 \xi_3 \xi_4 + \frac{11}{3} \xi_2 \xi_3 \xi_4 - \frac{10}{3} \xi_2^2 \xi_4 + \xi_3 \xi_4^2, \xi_1^2 \xi_4 + \frac{11}{3} \xi_1 \xi_2 \xi_4 \\
& + \xi_2 \xi_4^2 + \frac{11}{3} \xi_1 \xi_3 \xi_4 + \frac{11}{3} \xi_2 \xi_3 \xi_4 + \xi_3^2 \xi_4 - \frac{10}{3} \xi_1 \xi_4^2 - \frac{10}{3} \xi_2 \xi_4^2 - \frac{10}{3} \xi_3 \xi_4^2 + \xi_4^3, 8 \xi_1^2 \xi_3 \\
& - \frac{16}{3} \xi_1 \xi_2 + \xi_3 - \frac{8}{3} \xi_1 \xi_3^2 - \frac{16}{3} \xi_1 \xi_3 \xi_4, -\frac{8}{3} \xi_1^2 \xi_3 - \frac{16}{3} \xi_1 \xi_2 \xi_3 + 8 \xi_1 \xi_3^2 \\
& - \frac{16}{3} \xi_1 \xi_3 \xi_4, \xi_1^2 \xi_2 - \frac{8}{3} \xi_1 \xi_2^2 - \frac{16}{3} \xi_1 \xi_2 \xi_3 - \frac{16}{3} \xi_1 \xi_2 \xi_4, -\frac{8}{3} \xi_1^2 \xi_2 + 8 \xi_1 \xi_2^2 \\
& - \frac{16}{3} \xi_1 \xi_2 \xi_3 - \frac{16}{3} \xi_1 \xi_2 \xi_4, -\frac{16}{3} \xi_1 \xi_2 \xi_3 + 8 \xi_2^2 \xi_3 - \frac{8}{3} \xi_2 \xi_3^2 - \frac{16}{3} \xi_2 \xi_3 \xi_4, \\
& - \frac{16}{3} \xi_1 \xi_2 \xi_3 - \frac{8}{3} \xi_2^2 \xi_3 + 8 \xi_2^2 \xi_3 - \frac{16}{3} \xi_2 \xi_3 \xi_4, 8 \xi_1^2 \xi_4 - \frac{16}{3} \xi_1 \xi_2 \xi_4 - \frac{16}{3} \xi_1 - \frac{8}{3} \xi_1 \xi_4^2, \\
& - \frac{8}{3} \xi_1^2 \xi_4 - \frac{16}{3} \xi_1 \xi_2 \xi_4 - \frac{16}{3} \xi_1 \xi_3 \xi_4 + 8 \xi_1 \xi_4^2, -\frac{16}{3} \xi_1 \xi_2 \xi_4 + 8 \xi_2^2 \xi_4 - \frac{16}{3} \xi_2 \xi_3 \xi_4 \\
& - \frac{8}{3} \xi_2 \xi_4^2, -\frac{16}{3} \xi_1 \xi_2 \xi_4 - \frac{8}{3} \xi_2^2 \xi_4 - \frac{16}{3} \xi_2 \xi_3 \xi_4 + 8 \xi_2 \xi_4^2, -\frac{16}{3} \xi_1 \xi_3 \xi_4 - \frac{16}{3} \xi_2 \xi_3 \xi_4 \\
& + 8 \xi_3^2 \xi_4 + \frac{-8}{3} \xi_3 \xi_4^2, -\frac{16}{3} \xi_1 \xi_3 \xi_4 + \frac{-16}{3} \xi_2 \xi_3 \xi_4 + \frac{-8}{3} \xi_3^2 \xi_4 + 8 \xi_3 \xi_4^2, \\
& \left. \left. 27 \xi_2 \xi_3 \xi_4, 27 \xi_1 \xi_2 \xi_3, 27 \xi_1 \xi_3 \xi_4, 27 \xi_1 \xi_2 \xi_4 \right\} \right\} \{u\}_e
\end{aligned}$$

$$\begin{aligned}
u = & [N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, \\
& N_{14}, N_{15}, N_{16}, N_{17}, N_{18}, N_{19}, N_{20}] \{u\}_e
\end{aligned}$$

Here:

$$\begin{aligned}
N_1 &:= \xi_1^3 - \frac{10}{3} \xi_1^2 \xi_2 + \xi_1 \xi_2^2 - \frac{10}{3} \xi_1^2 \xi_3 + \frac{11}{3} \xi_1 \xi_2 \xi_3 + \xi_1 \xi_3^2 - \frac{10}{3} \xi_1^2 \xi_4 + \frac{11}{3} \xi_1 \xi_2 \xi_4 + \frac{11}{3} \xi_1 \xi_3 \xi_4 + \xi_1 \xi_4^2 \\
N_2 &:= \xi_1^2 \xi_2 - \frac{10}{3} \xi_1 \xi_2^2 + \xi_2^3 + \frac{11}{3} \xi_1 \xi_2 \xi_3 - \frac{10}{3} \xi_2^2 \xi_3 + \xi_2 \xi_3^2 + \frac{11}{3} \xi_1 \xi_2 \xi_4 - \frac{10}{3} \xi_2^2 \xi_4 + \frac{11}{3} \xi_2 \xi_3 \xi_4 + \xi_2 \xi_4^2 \\
N_3 &:= \xi_1^2 \xi_3 + \frac{11}{3} \xi_1 \xi_2 \xi_3 + \xi_2^2 \xi_3 - \frac{10}{3} \xi_1 \xi_3^2 - \frac{10}{3} \xi_1 \xi_3^2 - \frac{10}{3} \xi_2 \xi_3^2 + \xi_3^3 + \frac{11}{3} \xi_1 \xi_3 \xi_4 + \frac{11}{3} \xi_2 \xi_3 \xi_4 \\
& - \frac{10}{3} \xi_2^2 \xi_4 + \xi_3 \xi_4^2 \\
N_4 &:= \xi_1^2 \xi_4 + \frac{11}{3} \xi_1 \xi_2 \xi_4 + \xi_2 \xi_4^2 + \frac{11}{3} \xi_1 \xi_3 \xi_4 + \frac{11}{3} \xi_2 \xi_3 \xi_4 + \xi_3^2 \xi_4 - \frac{10}{3} \xi_1 \xi_4^2 - \frac{10}{3} \xi_2 \xi_4^2 - \frac{10}{3} \xi_3 \xi_4^2 + \xi_4^3 \\
N_5 &:= 8 \xi_1^2 \xi_3 - \frac{16}{3} \xi_1 \xi_2 + \xi_3 - \frac{8}{3} \xi_1 \xi_3^2 - \frac{16}{3} \xi_1 \xi_3 \xi_4 \\
N_6 &:= -\frac{8}{3} \xi_1^2 \xi_3 - \frac{16}{3} \xi_1 \xi_2 \xi_3 + 8 \xi_1 \xi_3^2 - \frac{16}{3} \xi_1 \xi_3 \xi_4 \\
N_7 &:= \xi_1^2 \xi_2 - \frac{8}{3} \xi_1 \xi_2^2 - \frac{16}{3} \xi_1 \xi_2 \xi_3 - \frac{16}{3} \xi_1 \xi_2 \xi_4
\end{aligned}$$

$$N_{17} := 27\xi_2\xi_3\xi_4 \quad N_{18} := 27\xi_1\xi_2\xi_3 \quad N_{19} := 27\xi_1\xi_3\xi_4 \quad N_{20} := 27\xi_1\xi_2\xi_4$$

**Where  $N_1, N_2, N_3, N_4, N_5, N_6, N_7, N_8, N_9, N_{10}, N_{11}, N_{12}, N_{13}, N_{14}, N_{15}, N_{16}, N_{17}, N_{18}, N_{19}, N_{20}$  are Shape Functions**

Output at Node 5  $(\frac{3}{4}, 0, \frac{1}{4}, 0)$       Where  $\xi_1 = \frac{3}{4}, \xi_2 = 0, \xi_3 = \frac{1}{4}, \xi_4 = 0$

|                                                                |                                                                                  |
|----------------------------------------------------------------|----------------------------------------------------------------------------------|
| I. 1                                                           | II. 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0                                      |
| Output at Node 6 $(\frac{1}{4}, 0, \frac{3}{4}, 0)$            | Where $\xi_1 = \frac{1}{4}, \xi_2 = 0, \xi_3 = \frac{3}{4}, \xi_4 = 0$           |
| I. 1                                                           | II. 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0                                      |
| Output at Node 7 $(\frac{3}{4}, \frac{1}{4}, 0, 0)$            | Where $\xi_1 = \frac{3}{4}, \xi_2 = \frac{1}{4}, \xi_3 = 0, \xi_4 = 0$           |
| I. 1                                                           | II. 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0                                      |
| Output at Node 8 $(\frac{1}{4}, \frac{3}{4}, 0, 0)$            | Where $\xi_1 = \frac{1}{4}, \xi_2 = \frac{3}{4}, \xi_3 = 0, \xi_4 = 0$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0                                      |
| Output at Node 9 $(0, \frac{3}{4}, \frac{1}{4}, 0)$            | Where $\xi_1 = 0, \xi_2 = \frac{3}{4}, \xi_3 = \frac{1}{4}, \xi_4 = 0$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0                                      |
| Output at Node 10 $(0, \frac{1}{4}, \frac{3}{4}, 0)$           | Where $\xi_1 = 0, \xi_2 = \frac{1}{4}, \xi_3 = \frac{3}{4}, \xi_4 = 0$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0                                      |
| Output at Node 11 $(\frac{3}{4}, 0, 0, \frac{1}{4})$           | Where $\xi_1 = \frac{3}{4}, \xi_2 = 0, \xi_3 = 0, \xi_4 = \frac{1}{4}$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0                                      |
| Output at Node 12 $(\frac{1}{4}, 0, 0, \frac{3}{4})$           | Where $\xi_1 = \frac{1}{4}, \xi_2 = 0, \xi_3 = 0, \xi_4 = \frac{3}{4}$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0                                      |
| Output at Node 13 $(0, \frac{3}{4}, 0, \frac{1}{4})$           | Where $\xi_1 = 0, \xi_2 = \frac{3}{4}, \xi_3 = 0, \xi_4 = \frac{1}{4}$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0                                      |
| Output at Node 14 $(0, \frac{1}{4}, 0, \frac{3}{4})$           | Where $\xi_1 = 0, \xi_2 = \frac{1}{4}, \xi_3 = 0, \xi_4 = \frac{3}{4}$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0                                      |
| Output at Node 15 $(0, 0, \frac{3}{4}, \frac{1}{4})$           | Where $\xi_1 = 0, \xi_2 = 0, \xi_3 = \frac{3}{4}, \xi_4 = \frac{1}{4}$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0                                      |
| Output at Node 16 $(0, 0, \frac{1}{4}, \frac{3}{4})$           | Where $\xi_1 = 0, \xi_2 = 0, \xi_3 = \frac{1}{4}, \xi_4 = \frac{3}{4}$           |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0                                      |
| Output at Node 17 $(0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ | Where $\xi_1 = 0, \xi_2 = \frac{1}{3}, \xi_3 = \frac{1}{3}, \xi_4 = \frac{1}{3}$ |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0                                      |
| Output at Node 18 $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0)$ | Where $\xi_1 = \frac{1}{3}, \xi_2 = \frac{1}{3}, \xi_3 = \frac{1}{3}, \xi_4 = 0$ |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0                                      |
| Output at Node 19 $(\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3})$ | Where $\xi_1 = \frac{1}{3}, \xi_2 = 0, \xi_3 = \frac{1}{3}, \xi_4 = \frac{1}{3}$ |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1                                      |
| Output at Node 20 $(\frac{1}{3}, \frac{1}{3}, 0, \frac{1}{3})$ | Where $\xi_1 = \frac{1}{3}, \xi_2 = \frac{1}{3}, \xi_3 = 0, \xi_4 = \frac{1}{3}$ |
| I. 1                                                           | II. 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1                                      |

From this output we observe that:

- (i) Sum of all the shape functions is equal to one at all nodal values.

- (ii) Each Shape function value is equal to one at that nodal value and zero at the other remaining nodes.

## 5. CONCLUSIONS

- 1) In this paper I derived shape functions for 20 noded cubic tetrahedron element.
- 2) In this paper I verified shape function two verification conditions.

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