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AN ASSESSMENT ON MAGNETO HYDRODYNAMIC FLOW OF NON-NEWTONIAN FLUID IN POROUS MEDIUM

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ABSTRACT. In this review paper, the results of researches of a number 0f researchers on magneto hydrodynamic flow in porous medium, of non-Newtonian fluids have been studied in detail with reference to basic concepts, importance and applications in various spheres/fields. They have examined situations using different kinds of objects. Effects of various forces acting in the process, noted. The equations of motion and heat equations solved by using different methods like Shooting, fourth order Runge-Kutta and Perturbation methods, gone through. Examined profiles of temperature and velocity, computed and graphically represented terms. Non-dimensional physical parameters like skin-friction coefficient, Eckert number, Prandlt number, non-Darcy number, Nusselt number have been applied. Changes brought about by above factors have been noticed.
Objective: For further research/study on the topic, it is necessary to review the research paper systematically keeping in view the importance and application of MHD flow in porous medium, of non-Newtonian fluids.

1. INTRODUCTION

The mechanics of fluid is the subject of physics and fluid flow is dealt with by fluid mechanics. Fluids are broadly divided into Ideal and Viscous fluids. Fluids not obeying the law of viscosity are called viscous. Plastics, Slurries, Banana

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puree, industrial polymer fluids come in the category of non-Newtonian viscous fluids. Several mathematical models like Micro-polar model and Bingum model are very useful in describing the flow of fluids in biological systems and fluid machinery used in industry. The word Magneto hydrodynamics is constituted of magneto which stands for magnetic, hydro stands for water and dynamic stands for movement of an object by force. A great scholar of sweden, a physicist, Hannes Alfven who got Nobel prize for his work in physics in 1977 did fundamental work on hydrodynamics, he found various useful applications of MHD in different fields.

Magneto hydrodynamic flow of fluids gained a great importance due to its vast applications in a number of fields. It has its application in the field of Engineering, Chemical Engineering, Food processing, wire and fibre coating, Polymer technology, purification of molten metals, magneto hydro power generators, aerodynamic heating, petroleum industry.

Basic Equations

Equation of continuity (Law of Conservation of Mass)

Mass enclosed in a given volume 0f fluid remains the same throughout the motion. This endorses that matter is neither created nor destroyed but it is conserved. In mathematics it is represented as

$$\frac{\partial \rho}{\partial t} + [\nabla . \left(\rho \vec{V}\right)] = 0.$$

Here density of fluid is represented by ρ and velocity of the fluid by $\vec{V}$, t is the time, ∇ is the vector operator.

For viscous incompressible fluid, $\nabla . \vec{V} = 0$.

Law of Conservation of Momentum (Navier-Stokes Equations)

Rate at which the momentum of a flowing fluid changes in a given direction is the net force in that direction. Mathematically represented for incompressible viscous fluid,

$$\rho \left[\frac{\partial \overset{V}{V \rightarrow}}{\partial t} + (\vec{V} . \nabla) \vec{V} \right] = -\nabla p + \mu \nabla^2 \overset{V}{V \rightarrow} + \rho \vec{F}.$$

Here pressure is represented by p and coefficient of viscosity as μ. This linear equation is called Stock's equation.

Law of Conservation of Energy (The Equation of Energy)

A system isolated from its surroundings where in the energy remains conserved. Since energy can neither be created nor destroyed it remains conserved, it only changes its form. Mathematically for incompressible fluid,

$$\rho C_p \left(\frac{\partial}{\partial t} + \vec{V}.\nabla \right) T = \kappa \nabla^2 T + \varphi.$$

The definitions of terms used

Mathematical representation,

$$\left(\frac{\partial V}{\partial t} \right)_{(x_0,y_0,z_0)} = 0, \left(\frac{\partial p}{\partial t} \right)_{(x_0,y_0,z_0)} = 0, \left(\frac{\partial \rho}{\partial t} \right)_{(x_0,y_0,z_0)} = 0 (x_0, y_0, z_0)$$

is a fixed point in fluid field.

Unsteady Flow

If at a point, time changes the velocity, pressure or density, the flow is unsteady. Pumping of water at an increasing rate through a fixed system is an example of this type of flow.

Mathematically,

$$\left(\frac{\partial V}{\partial t} \right)_{(x_0,y_0,z_0)} \neq 0, \left(\frac{\partial p}{\partial t} \right)_{(x_0,y_0,z_0)} \neq 0, \left(\frac{\partial \rho}{\partial t} \right)_{(x_0,y_0,z_0)} \neq 0.$$

Uniform Flow

Velocity at a given time remains unchanged with respect to length or direction in a uniform flow. Example of such flow is flow through long pipe lines with constant diameter, under pressure.

Mathematically,

$$\left(\frac{\partial V}{\partial s} \right)_{t=\text{constant}} = 0.$$

Here, ∂V is the change of velocity, and ∂s is the length of flow in the direction s.

Non-uniform Flow

If at a given time, in a flowing fluid, the velocity of flow changes at different points the flow is said to be non-uniform. A liquid being pumped through a curved line is an example of such flow.

Mathematically,

$$\left(\frac{\partial V}{\partial s}\right)_{t=\text{constant}} \neq 0.$$

2. Non dimensional physical parameters

Reynold Number (Re)

Ratio between inertial force and viscous force.

Mathematically,

$$Re=\frac{\rho VL}{\mu} = \frac{VL}{\nu}.$$

Here, V is the velocity of object, ρ is the density of the fluid, L is the length, and μ is viscosity of the fluid.

Darcy Number (Da)

It represents the effect of permeability of medium with relation to its area of cross-section:

$$Da = \frac{K}{L^2}.$$

Local Skin Friction Coefficient ($\mathcal{C}_{\{}$)

A dimensionless shearing stress caused on the surface of a body as a result of motion of fluid:

$$C_f = \frac{\tau_w}{\rho V^2/2}.$$

Prandtl number (Pr)

It denotes the ratio between the kinematics viscosity (v) and thermal diffusivity (α) of the fluid,

$$Pr=\left(\frac{\nu}{\alpha}\right) = \frac{\nu}{K/(\rho C_p)} = \frac{\mu C_p}{K}$$

$(K/(\rho C_p) = \alpha$ is the thermal diffusivity of the fluid.

Its value for low viscous fluids is quite small while it may be large for highly viscous fluid.

GrashofNumber (Gr)

Ratio between, product of buoyancy and inertial force and square of viscous force.

$$Gr = \text{buoyancyforce} \times \frac{\text{inertiaforce}}{(\text{viscousforce})^2}$$
$$= (g\beta(T - T_\infty)L^3\rho)\, \frac{\rho V^2 L^2}{(\mu V L)^2} = \left(\frac{g\beta(T - T_\infty)L^3\rho^2}{\mu^2}\right),$$

where μ is viscosity of the fluid, L is characteristic length, T_∞ is bulk temperature, T represents surface temperature, volumetric thermal expansion coefficient is $\beta, g(msec^{-2})$ is gravitational force.

Nusselt Number (coefficient of heat transfer)

Coefficient of heat transfer between fluid and the body. Mathematically, Nu is represented,

$$Nu = \left(\frac{hl}{\kappa}\right),$$

where Nu establishes relation between Convective film Coefficient = h, thermal conductivity of the fluid and length parameter = l.

An energy balance at the surface of a heated plate ensures that energy transport by conduction must equal the heat transfer into the fluid flowing past the plate. Thus

$$Q = -\kappa A \left(\frac{\partial T}{\partial y}\right)_{y=0} = hA\left(T_w - T_\infty\right)$$

$$h = \frac{-\kappa \left(\frac{\partial T}{\partial y}\right)_{y=0}}{\left(T_w - T_\infty\right)}, \quad \frac{hl}{\kappa} = \frac{-\left(\frac{\partial T}{\partial y}\right)_{y=0}}{\left(T_w - T_\infty\right)/l}.$$

Large value of Nusselt number tells that the convection is more effective.

Hartman Number (Ha)

It denotes ratio between electromagnetic force and viscous force.

$$\mathcal{H}_{\dashv}^{\mathbf{2}} = \frac{\sigma \mathcal{B}_{\mathbf{0}}^{\mathbf{2}} \mathcal{V}}{\mu \mathcal{V}/\mathcal{L}^{\mathbf{2}}}, \quad \mathcal{H}_{\dashv} = \mathcal{B}_{\mathbf{0}} \mathcal{L} \sqrt{\frac{\sigma}{\mu}}.$$

Here, magnetic field strength denoted by $\mathcal{B}_{\mathbf{0}}$, characteristic length by L and electrical conductivity of the fluid by ρ.

3. Review Literature

S.M.M. EL-Kabeir et.al. [1] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form.

Topic of research in this paper is heat along with mass transfer taken together for group method analysis of non-Newtonian, non-Darcy MHD natural convection in a porous medium taken to be saturated, adjacent to horizontal cylinder. One parameter group is applied and the governing equations are converted to ordinary differential equations,R.k. method of fourth order is used to numerically solve these equations. Velocity increases by increasing Radiation Parameter and reduces when magnetic parameter, Ergun and Prandtlnumbersincrease. An increase in Radiation, magnetic field and Ergun number increases the temperature whereas it decreases when there is increase in prandtl number.

Abeer A Shaaban et.al. [2] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of the research paper concerns the study of effects of heat and mass transfer on peristaltic MHD flow between two coaxial cylinders, of a non-Newtonian fluid in porous medium. In the present investigation technique of finite indifference has been used to solve the problem numerically. Decrease in Darcy number results in increase of velocity but it decreases when there is increase in magnetic field, pressure gradient and Erying parameter. Increase in temperature is noticed when Eckert, prandtl and Darcy numbers increase but it decreases if there is increase inPressure gradient, Eyring Powell parameter Reaction rate parameter, Schmidt and soret numbers.

Gamal M. Abdel-Rahman [3]A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of the research concerns study of the effects of viscosity as well as thermal conductivity on MHD flow given to be unsteady pertaining to non-Newtonian fluid over a stretching porous sheet. The present problem has been considered by assigning numerical values to the involved parameters using graphs and tables. Increasing porous medium parameters Temperature increases whereas the velocity decreases. When the prandtl number increases, there is increase in velocity profile but decrease in temperature profile.

Dhiman Bose et al. [4] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of the research concerns investigation of fluctuating MHD flow pertaining to non-Newtonian fluid contained in infinite porous plate through porous medium.In this problem, governing equations have been solved analytically. Porosity of medium poses resistance to velocity due to the acting magnetic field and permeability.It has its application on underground water resources.

Dhiman Bose et al. [5] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of the research concerns analysis of MHD convective flow through a porous medium related to non-Newtonian fluid over a porous oscillating plate with suction.In the present problem, It is observed that when increase occurs in permeability, velocity also increases but it faces decrease when heat source parameter, suction parameter, magnetic field and prandtl number increase. Decrease is noted in temperature when there is increase in prandtl number, suction and heat source parameters.

Gamel M. Abdel-Rahman Rashed et al. [6] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of this research is related to the study of effects of Radiation and Reaction on non-Newtonian (Walter's B) unsteady MHD fluid flow in porous medium. The present problem has been solved numerically. It is noted that there occurs a decrease in velocity as well as temperature profiles as a result of increase in porosity parameter and magnetic field. When there is increase in thermal radiation and heating sourceparameters there is decrease in velocity whereas temperature increases.

Monika Srivastava et al. [7] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of the research concerns study of a non-Newtonian fluid flow through a vertical channel with porous medium. problem has been solved numerically using tables and graphs. It is found that an increase in magnetic field decreases the velocity for every value of Newtonian factor. Starting from a fixed value of velocity the form of each curve remains the same. Curvature is highly effected by these factors, the curve has more curvature for greater values 0f these factors. It is observed that non-Newtonian factor is an important component in controlling the velocity. Hyperbolic functions of various parameters, too, play important role in controlling the velocity.

Lazarus Rundora et al. [8] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The concern ofthe present research is to examine MHD unsteady flow pertaining to non-Newtonian fluid in a channel filled with porous medium in a saturated condition with asymmetric Navier slip and convective heating.The problem has been solved numerically using graphs. It is noted that Hartmann number and porous medium parameter cause reduction in velocity and temperature profiles. Increase in Fluent velocity profile is noticed under the effect of lower wall slip parameter but upper wall slip parameter reduces it. But fluid temperature is increased.

Funmilayo H. Oyelami et al. [9] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of the research is investigation of non-Newtonian fluid flow given to be unsteady MHD with slip, through porous medium. The problem has been solved mathematically and the resultant equations obtained have been solved using Kranknikolson scheme. The distribution for both prandtl-Eyring and Eyringpowell non-Newtonian models are studied for velocity as well as temperature. Comparing the two diverse models, it is noted that the velocity is higher for prandtl-Eyring model with regard to Eyring-powell model but temperature variation is a little lower in Eyringpowell model as compared toprandtl-Eyring model.

D. DastagiriBabu et.al. [10] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of the research is the study of heat and mass transfer of non-Newtonian fluid MHD flow over porous plate, inverted and vertical with Hall Effect. Numerical method has been used to solve the present problem with the help of tables and graphs. Increase in scalar constant increases the velocity. The velocity decreases when dimensionless heat absorption coefficient and parameter of dimensionless visco-elasticity increase. Resultant velocity increases when there is increase in Hall parameter but it decreases but it decreases when Hartmann number increases. When heat absorption co-efficient increases the temperature decreases but in the presence of a constant the position is reverse.

B. Lakshmanna et.al. [11] A review of this research paper has been made after a thorough study and results and conclusions have been given as obtained by the researcher, using technical words, statements, definitions and mathematical symbols in their original form. The topic of research is to examine transfer of heat on MHD Non-Newtonian rotating fluid flow through parallel porous channel. Laplace technique has been used to analytically solve the problem. It is observed that when Eckeman number and Visco elastic fluid parameter are increased the resultant velocity increases. When there is increase in Eckmann number there is increase in rate of heat transfer. WhenHartmann Number is increased there is increase in fluid temperature.

4. Conclusion

A review of eleven Research papers under consideration using several methods under different parameter reveals the following results.

Effect of Buoyancy increases the velocity. Chemical reaction, Radiation, Reduced heat transfer, Permeability, Scaler constant, Hall parameter, and Darcy number.

Porous medium, magnetic parameter, Eyring Powell, Heat source, suction, Porosity parameter Pressure gradient, Ergun number, Hartmann number, thermal radiation, and prandtl number decrease the velocity.

Temperature of the fluid increases under the effect of magnetic, Radiation, Porous medium, Temperature buoyancy, Heat source, Chemical reaction, Ergun and other numbers.

The Temperature of the fluid decreases under the effect of Pressure gradient, Surface mass transfer parameter, Heat source parameter, Porosity parameter, Suction parameter.

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