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A GENERLIZATION OF GINI SIMPSON INDEX UNDER FUZZY ENVIRONMENT

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ABSTRACT. The current paper,logically based on Gini Simpson index of diversity, in the setting of fuzzy set theory we introduced the measure of fuzzy entropy. The fuzzy entropy axiomatic requirements are satisfied in a mathematical view point for the new fuzzy entropy.To show the successfulness of the proposed entropy we compare it with the some existing entropies.

1. INTRODUCTION

The idea of the FS created by Zadeh [33] to demonstrate and process uncertain data in a much successful manner. By allocating the membership degree between 0 and 1 to the elements of a set, the FS can depict the state between "belong to" and " not belong to". So, numerous types of uncertainties that can't be delineated by classical sets can be portrayed by FS. Initially, FSs has been applied in numerous territories, for example, decision-making,automatic control, pattern recognition and so on. The Zadeh [34] was the first to propose the entropy of FS to portray the fuzziness. De Luca and Termini [7] extend the Zadeh's work by proposing a probabilistic entropy measures for FSs.Yager [32] characterized an entropy proportion of a FS in regards in absence of capability between FS and its complement. They proposed some axiomatic characteristics

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to measure the fuzzy entropy by which we can define the fuzzy entropy. Yager's idea was reached out by Higashi and klir [12] to a progressively broad sort of fuzzy complementation. As a result of its importance in delineating a fuzzy set, the entropy measure of FS has been creating to a functioning subject in FS theory. In the fuzzy environment,many authors have described various theories on entropy measures, such as the papers (Kosko [19], Hwang and Yung [11], Pal and Pal [24], Verma and Sharma [28], Joshi and Kumar [14, 16, 20], Hooda [9], Li and Lu [21], Bhandari and Pal [5], Pal and Pal [25]).

The proposed measure is Gini Simpson's index which is neither additive nor non additive entropy measure from probabilistic setting to fuzzy set theory.Some researcher studied the generalized entropy [5,14,16,17,20,31,23].

The major targets of bringing out this theory are: (1) To introduce a parametric fuzzy information measure based on the Gini Simpson diversity (2) Comparative analysis is given with existing entropies in the literature based on linguistic variables.

The following paper is structured as Section[2] consists of some definitions and basic concept related to FSs. In Section[3], we proposed and proved the basic properties of new fuzzy information measure.Section[4] with the help of examples we will analyze the comparison of proposed measure with some existing fuzzy entropy measures. In the last section[5], the paper is summarised with "Conclusions" in Section 5.

Throughout this paper, it is assumed that $k \in I^+$ (set of positive integers) and base of all logarithms are 2 Corresponding to Shannon entropy, De Luca and Termini [7] defined a fuzzy entropy for a fuzzy set K as :

$$J(K) = -L \sum_{i=1}^{k} \left[\lambda_k (r_i) \log (\lambda_K (r_i)) + (1 - \lambda_K (r_i) \log (1 - \lambda_K (r_i))\right] . \tag{1.1}$$

For some constant $\mathrm{L} = 1/\mathrm{k}$ Several generalized fuzzy entropies were defined by authors e.g., In $1993,$ Bhandari and Pal suggested new fuzzy entropy measures based on the Ren [26] as follows:

$$J_{xy^a}(K) = \frac{1}{1-\alpha} \sum_{i=1}^{k} \log \left[\lambda_x (r_l)^a + (1 - \lambda_k (r_l))^a\right], \tag{1.2}$$

where $\alpha > 0 (\neq 1)$.

In section [3], we proposed a new entropy of FSs. To prove the effectiveness of the proposed entropy we made some comparison with the existing one.

2. A NEW FUZZY INFORMATION MEASURE

For this section we briefly review the theoretical concept of information theory and then introduce a generalization of Gini- Simpson entropy, along with studying with their properties.

Let

$$\Gamma_k = \left\{ S = (s_1, s_2, \ldots, s_k) : s_i \geq 0; \sum_{i=1}^{k} s_i = 1 \right\}, k \geq 2$$

be set of k -complete probability distributions. For any probability distributions $S = (s_1, s_2, \ldots, s_k) \in \Gamma_k$, Shannon [27] defined an entropy as:

$$J_{\mathrm{s}_r}(S) = -\sum_{i=1}^{k} (s_i) \log (s_i). \tag{2.1}$$

Tsallis [29] introduced a generalized form of Shannon entropy, Tsallis entropy is defined by

$$J_{\pi}^{\alpha}(S) = \frac{1}{\alpha - 1} \left[1 - \sum_{i=1}^{k} s_j^a \right]; \alpha \in (0,1) \cup (1, \infty), \tag{2.2}$$

since, $\lim_{a \to 1} H_{\pi s}^{a}(S) = H_{st}(S)$. In particular, Tsallis [29] and Ren entropy [26] having a close relationship between them as follows:

$$J_{\infty}(S) = \frac{1}{\alpha - 1} \log_D (1 - (1 - \alpha) J_{\alpha}(S)) = \frac{1}{1 - \alpha} \log_D \sum_{i=1}^{k} s_i^{\alpha}, \tag{2.3}$$

where $J_{Ra}^{a}(S)$ is the Renyi entropy. The major difference between them is, the Renyi [26] and Shannon [27] entropy are additive whereas the Tsallis entropy [29] is non-additive, i.e.,

$$J_{TS}^{\omega}(S,T) = J_{TS}^{\infty}(S) + J_{TS}^{\infty}(T) + (1 - \alpha) J_{\pi s}^{\alpha}(S) J_{TS}^{\alpha}(T), \tag{2.4}$$

where $S, T \in \Gamma_k$.

However, in the literature of information theory, there exists various generalizations of Shannon's entropy [27], we introduced a new information measure

$J_2^{\infty\pi} : \Gamma_L \to \Re^+($ set of positive real numbers$)$; $k \geq 2$ as follows:

$$J_2^{\infty \sim rw}(S) = \sum_{r=1}^{k} \left(\sqrt{s_i} - s_i^2\right) \alpha \in (0,1) \cup (1,\infty), \tag{2.5}$$

which is known as a generalization of Gini Simpson index of diversity.

2.1. **Properties of proposed entropy.**

Theorem 2.1. *The parametric entropy* $J_2^{meren}(S), S \in \Gamma_k$, *satisfied below said properties:*

1) *Symmetry:* $J_2^{\min}(s_1, s_2, \ldots, s_k)$ *is a symmetric function of* $(s_1, s_2, \ldots, s_k)$.
2) *Non-Negative:* $J_2^{w-v}(S) \geq 0$.
3) *Expansible:* $J_2^{\min}(s_1, s_2, \ldots, s_k, 0) = J_2^{m,m}(s_1, s_2, \ldots, s_k)$.
4) *Decisive:* $J_2^{new}(0,1) = 0 = J_2^{\infty-v}(1,0)$.
5) *Maximility:* $H_2^{new}(s_1, s_2, \ldots, s_k) \leq H_2^{new}\left(\frac{1}{k}, \frac{1}{k}, \ldots, \frac{1}{k}\right)$.
6) *Concavity:* $J_2^{new}(tS_1 + (1-t)S_2) \geq tJ_2^{new}(S_1) + (1-t)J_2^{new}(S_2)$.
7) *Continuity:* $J_2^{new}(s_1, s_2, \ldots, s_k)$ *is continuous in the region* $s_i \geq 0$ *for all* $i = 1, 2, \ldots, k$.

Proof. The proof of the above theorem are trivial and omitted. □

Definition 2.1. *Corresponding to* (3.5), *we proposed the following fuzzy information measure:*

$$J_2^{new}(K) = \frac{1}{k}\sum_{i=1}^{k}\left[(\lambda_K(r_i))^{1/2} + (1-\lambda_L(r_i))^{1/2} - (\lambda_K(r_i))^2 + (1-\lambda_K(r_i))^2\right] \tag{2.6}$$

We show that the new proposed fuzzy entropy measure satisfy all the entropy properties which are given in the preceding theorem.

Theorem 2.2. *The fuzzy entropy measure (2.6) satisfied four fuzzy entropy axiomatic requirements.*

Proof. For validity the measure defined by (2.6), we should fulfills the axiomatic requirements $(\mathrm{A1}) - (\mathrm{A4})$.

A_1 **(Sharpness)**: From (2.5), we have

$$J_2^{n+v}(K) = \frac{1}{k}\sum_{i=1}^{k}\left[(\lambda_K(r_i))^{1/2} + (1-\lambda_K(r_i))^{1/2} - (\lambda_K(r_i))^2 + (1-\lambda_K(r_i))^2\right] \tag{2.7}$$

If $J_2^{new}(M) = 0$ in (2.5), then

$$(\lambda_K(r_i))^{1/2} + (1-\lambda_K(r_i))^{1/2} - (\lambda_K(r_i))^2 + (1-\lambda_K(r_i))^2 = 0, \tag{2.8}$$

for all$i = 1, 2, \ldots, k$. Clearly (2.8) will be satisfied if $\lambda_K(r_i) = 0$ or 1, for all $i = 1, 2, \ldots, k$.

Conversely, let K be a non FS, i.e., crisp set, then either $\lambda_X(r_i) = 0$ or 1. This implies that

$$(\lambda_K(r_i))^{1/2} + (1-\lambda_K(r_i))^{1/2} - (\lambda_K(r_i))^2 + (1-\lambda_K(r_i))^2 = 0 \tag{2.9}$$

for all $i = 1, 2, \ldots, k$. Hence, $J_2^{new}(K) = 0$ iff K is a crisp set , i.e., $\lambda_K(r_i) = 0$ or 1 for all $i = 1, 2, \ldots, k$.

A_2 **(Maximality)**: Differentiating (2.6) with respect to $\lambda_K(r_i)$, we get

$$\begin{aligned}\frac{\partial J_2^{nn(}K\big)}{\partial \lambda_K(r_i)} =& \frac{1}{k}\left[\frac{1}{2}\left\{\lambda_K(r_i)^{\frac{-1}{2}} - (1-\lambda_K(r_i))^{\frac{-1}{2}}\right\}\right] \\ & - \left[2\left\{2_K(r_i) - (1-\lambda_K(r_i))\right\}\right].\end{aligned} \tag{2.10}$$

Differentiating (2.13) with respect to $\lambda_K(r_i)$, again, we get

$$\frac{\partial^2 J_2^{nmv}(K)}{\partial \lambda_K(r_i)^2} = \frac{1}{k}\left[\left(-\frac{1}{4}\right)\left(\lambda_K(r_i)^{\frac{-3}{2}} + \left((1-\lambda_K(r_i))^{\frac{-3}{2}}\right)\right]\right]. \tag{2.11}$$

We can prove that $\frac{\partial^2 J_2^{new}(K)}{\partial \lambda_K(r_i)^2} < 0$. It is evident that

$$\frac{\partial J_2^{\text{ner}}(K)}{\partial \lambda_K(r_i)} = 0, \text{ when } \lambda_K(r_i) = 0.5.$$

This proves that $J_2^{n\text{ of }(K)}$ is a concave function and has a global maximum at $\lambda_K(r_i) = 0.5\ldots$ It proves that $J_2^{new}\cdots(K)$ is maximum iff K is the most fuzzy set ,i.e., $\lambda_K(r_i) = 0.5$ for all r_i.

This proves that $J_2^{\text{ner}}(K)$ is maximum iff K is the most fuzzy set, i.e., $\lambda_K(r_i) = 0.5$, for all r_i.

A_3 **(Resolution)**: $\mathrm{In}(2.10)$, we have

$$\frac{\partial J_2^{\min}(K)}{\partial \lambda_K(r_i)} > 0 \text{ in } [0, 0.5),$$

$$\frac{\partial J_2^{n\text{ sw}}(K)}{\partial \lambda_x(r_i)} < 0 \text{ in } (0.5, 1]$$

and $\frac{\partial J_2^{nm\prime}(K)}{\partial \lambda_K(r_i)} = 0$ at $\lambda_K(r_i) = .5$.

Therefore, $J_2^{new}(K)$ is an increasing function of $\lambda_K(r_i)$ in [0,0.5) and decreasing function of

$$\lambda_K(r_i) \text{ in } (0.5, 1].$$

Now, let K^* be crisper than K. This implies

$$0 \leq \lambda_{K^*}(r_i) \leq \lambda_K(r_i) < 0.5 \Rightarrow J_2^{nm}(K^*) \leq J_2^{nm''}(K) \tag{2.12}$$

and

$$0.5 < \lambda_K(r_i) \leq \lambda_{K^*}(r_i) \leq 1 \Rightarrow J_2^{n+m}(K^*) \leq J_2^{nrv}(K). \tag{2.13}$$

From (2.12) and (2.13), we get $J_2^{new}\left(K^{*^*}\right) \leq J_2^{new}(K)$, where K^* is crisper than K.

A_4 **(Symmetry)**: This is straightforward by the definition of $J_2^{\text{new}}(K)$ and $\lambda_K(r_i) = 1 - \lambda_K(r_i)$. Hence, $J_2^{\text{new}}(K)$ satisfies all properties in the axiomatic definition of fuzzy measure. Therefore, $J_2^{\text{new}}(K)$ is a fuzzy measure of FSs.

Now, we have a property of the proposed information measure. □

Theorem 2.3. *If $K, L \in FSs(R)$, then $J_2^{new}(K \cup L) + J_2^{new}(K \cap L) = J_2^{new}(K) + J_2^{new}(L)$.*

Proof. Let

$$R_1 = \{r \in R \mid \lambda_K(r) \geq \lambda_L(r)\} \tag{2.14}$$

and

$$R_2 = \{r \in R \mid \lambda_K(r) < \lambda_L(r)\}, \tag{2.15}$$

where $\lambda_K(r)$ and $\lambda_L(r)$ are the membership functions of K and L, respectively.

If $r \in R_1$, then $\lambda_{K\cup L}(r) = \max\{\lambda_K(r), \lambda_L(r)\} = \lambda_K(r)$ and

$$\lambda_{K\sim L}(r) = \min\{\lambda_K(r), \lambda_L(r)\} = \lambda_L(r).$$

If $r \in R_2$, then $\lambda_{K\cup L}(r) = \max\{\lambda_K(r), \lambda_L(r)\} = \lambda_L(r)$ and

$$\lambda_{K\sim L}(r) = \min\{\lambda_K(r), \lambda_L(r)\} = \lambda_K(r).$$

Now, consider

$$\begin{aligned}
&J_2^{\log w}(K \cup L) + J_2^{\log(K\cap L)} \\
&\quad = \tfrac{1}{k}\left[\textstyle\sum_{i=1}^R \left\{(\lambda_K(r_i))^{\frac{1}{2}} + (1 - \lambda_L(r_i))^{\frac{1}{2}} - (\lambda_K(r_i))^2 + (1 - \lambda_K(r_i))^2\right\}\right] \\
&\quad + \textstyle\sum_{i=1}^R \left[\left\{(\lambda_L(r_i))^{\frac{1}{2}} + (1 - \lambda_L(r_i))^{\frac{1}{2}} - (\lambda_L(r_i))^2 + (1 - \lambda_L(r_i))^2\right\}\right].
\end{aligned}$$

On simplifying, we get

$$J_2^{new}(K \cup L) + J_2^{new}(K \cap L) = J_2^{new}(K) + H_2^{new}(L).$$

□

3. Numerical Examples

In this part, to check the validity and the effectiveness of the new fuzzy measure $J_2^{\text{never}}(K)$ we will compare it with the existing fuzzy measures which are widely used for FSs with the help of some examples. Some of the accepted fuzzy measures for FSs are proposed by:

Yager [32]:

$$J_{\mathrm{I}_1}(K) = 1 - \frac{d_p\left(K, K^c\right)}{n^{\frac{1}{p}}}$$

Kosko [19]:

$$J_{KOS}(K) = \frac{d_p\left(K, K_{\text{pear}}\right)}{d_p\left(K, K_{fy}\right)}$$

Pal and Pal [25]:

$$J_{Pa}(K) = \frac{1}{k}\sum_{i=1}^{k}\left[\lambda_K\left(r_i\right) e^{1-i_L(\gamma_i)} + \left(1-\lambda_K\left(r_i\right)\right) e^{i_K(\tau_i)}\right]$$

Li and Liu [21]:

$$J_{LL}(K) = \sum_{i=1}^{k} S\left(cr\left(\xi_P = r_i\right)\right)$$

Hwang and Yung [11]:

$$J_{HY}(K) = \frac{1}{1-e^{\frac{-1}{2}}}\sum_{i=1}^{k}\left[\left(1-e^{-\lambda_{K^c}(r_i)}\right) I_{\left[\lambda_K(r_i)\geq\frac{1}{2}\right]} + \left(1-e^{-\lambda_K(r_i)}\right) I_{\left[\lambda_K(r_i)<\frac{1}{2}\right]}\right].$$

Joshi and Satish [18]:

$$J_\alpha^\beta(K) = \frac{\alpha\times\beta}{k(\alpha-\beta)}\left[\sum_{i=1}^{k}\left\{\left[\lambda_K\left(r_i\right)^\rho + \left(1-\lambda_K\left(r_i\right)\right)^\rho\right)^{\frac{1}{\beta}} - \left(\lambda_K\left(r_i\right)^\alpha + \left(1-\lambda_K\left(r_i\right)\right)^\alpha\right)^{\frac{1}{\alpha}}\right\}\right]$$

Example 1: Consider a FS K_1 of $R = \{3,4,5,6,7\}$. The FS is defined as:

$$K_1 = \{(3,0.1),(4,0.3),(5,0.4),(6,0.9),(7,1)\}.$$

Then the modifier for the fuzzy set

$$K = \{(r_? (\lambda_K(r)) \mid r \in R\}$$

in R is given by

$$K^n = \left\{ (r, \lambda_K(r))^k \right) \mid r \in R \right\}. \tag{3.1}$$

Based on the operations, Hwang and Yang [11] and Hung and Yang [13] and in equation we have:

$$\begin{aligned}
K_1^{\frac{1}{2}} &= \{(3, 0.316), (4, 0.548), (5, 0.632), (6, 0.949), (7, 1)\} \\
K_1^2 &= \{(3, 0.01), (4, 0.09), (5, 0.16), (6, 0.81), (7, 1)\} \\
K_1^3 &= \{(3, 0.001), (4, 0.027), (5, 0.064), (6, 0.729), (7, 1)\}, \\
K_1^4 &= \{(3, 0), (4, 0.008), (5, 0.026), (6, 0.656), (7, 1)\}.
\end{aligned}$$

We can regard the FS K_1 is "LARGE" on R by considering the characterization of linguistics variables. Correspondingly, to FSs $K_1^{\frac{1}{2}}, K_1^2, K_1^3$ and K_1^4 may be behaved as "More or Less Large", "Very LARGE", "Quite Very LARGE", "Very Very LARGE", respectively. The idea of Shannon's entropy has been utilized for simple weighting calculation method [31, 32]. If the value of the information entropy is greater then the entropy weight information will be lesser [22], The specific attribute will provide the lesser information as the smaller the different alternatives in this specific attribute and in decision making process this specific attribute importance will be less [31]. Intuitively, hidden loss of information will become less from $K_1^{\frac{1}{2}}$ to $K_1^{\frac{1}{2}}$ to K_1^4 and the entropy provided by them increasing. So the following order will holds [11, 13, 18]:

$$J(K^{\frac{1}{2}}) > J(K) > J(K^2) > J(K^3) > J(K^4). \tag{3.2}$$

To make a comparison, entropy measures $J_{HY}(K_1), J_\alpha^\beta(K_1), J_2^{new}(K_1)$ are placed to find the analysis. We will compare the different results obtained from the different measures as shown in Table 1.

From Table 1 we observed that when we apply the entropy measure $J_{Y_1}(K_1)$, $J_{KOS}(K_1)$ and $J_{LL}(K_1)$ FS $K_1^{\frac{1}{2}}$ will be assigned less entropy than K. Based on these measures we will obtained the order of ranking as follows:

Table 1: Fuzzziness values corresponding to distinct information measures

Fuzzy Sets	$J_{Y_1}(K_1)$	$J_{KOS}(K_1)$	$J_{Pal}(K_1)$	$J_{LL}(K_1)$	$J_{HY}(K_1)$	$J_\alpha^\beta(K_1)$	$J_2^{new}(K_1)$
$K_1^{\frac{1}{2}}$	0.397	0.220	1.389	0.810	0.505	0.4672	0.3857
K_1	0.360	0.311	1.331	0.723	0.397	0.4672	0.3442
K_1^2	0.167	0.099	1.202	0.378	0.212	0.2834	0.2349
K_1^3	0.145	0.078	1.151	0.870	0.167	0.2202	0.1795
K_1^4	0.151	0.082	1.136	0.692	0.165	0.1906	0.1531

$$J_{Y_1}\left(K_1^{\frac{1}{2}}\right) > J_{Y_1}\left(K_1\right) > J_{Y_1}\left(K_1^2\right) > J_{Y_1}\left(K_1^4\right) > J_{Y_1}\left(K_1^3\right)$$
$$J_{\text{Kos}}\left(K_1\right) > J_{\text{Kos}}\left(K_1^{\frac{2}{2}}\right) > J_{\text{Kos}}\left(K_1^2\right) > J_{\text{Kos}}\left(m_1^4\right) > J_{\text{Kos}}\left(K_1^3\right)$$
$$J_{LL}\left(K_1^3\right) > J_{LL}\left(K_1^{\overline{2}}\right) > J_{LL}\left(K_1\right) > J_{LL}\left(K_1^4\right) > J_{LL}\left(K_1^2\right)$$

From the above order it is clear that this order of entropy measures is not satisfying the intuitive analysis equation (4.2), while rest of the entropy measures are providing the desirable results. $J_{Pal}\left(K_1\right), J_{HY}\left(K_1\right), J_\alpha^\beta\left(K_1\right)$ and $J_2^{new}\left(K_1\right)$ are doing well in this example. This shows that these entropy measures are not sufficient enough to differentiate the uncertainty of FSs with linguistic information.

Example 2: Take another FSK_2 defines on R as:

$$K_2 = \{(3, 0.2), (4, 0.3), (5, 0.4), (6, 0.7), (7, 0.8)\}.$$

We calculate $K_2^{\frac{1}{2}}, K_2^2, K_2^3$ and K_2^4. Now we compare only $J_{Pal}\left(K_2\right)$, $J_{HY}\left(K_2\right)$, $J_a^\beta\left(K_2\right)$ and $J_2^{\text{new}}\left(K_2\right)$.

Table 2: Fuzziness values with $J_{Pal}\left(K_2\right), J_{HY}\left(K_2\right), J_a^\beta\left(K_2\right)$ and $J_2^{\text{new}}\ \left(K_2\right)$

Fuzzy Sets	$J_{Y_1}(K_1)$	$J_{KOS}(K_1)$	$J_{Pal}(K_1)$	$J_{LL}(K_1)$	$J_{HY}(K_1)$	$J_\alpha^\beta(K_1)$	$J_2^{new}(K_1)$
$K_1^{\frac{1}{2}}$	0.397	0.220	1.389	0.810	0.505	0.4672	0.3857
K_1	0.360	0.311	1.331	0.723	0.397	0.4672	0.3442
K_1^2	0.167	0.099	1.202	0.378	0.212	0.2834	0.2349
K_1^3	0.145	0.078	1.151	0.870	0.167	0.2202	0.1795
K_1^4	0.151	0.082	1.136	0.692	0.165	0.1906	0.1531

Moreover, the results obtained from entropy measures $J_{Pal}(K_2), J_\alpha^\beta(K_2)$ are also not reasonable, which we can see from the following equations:

$$J_{Pal}(K_2) > J_{Pal}(K_2^{\frac{1}{2}}) > J_{Pal}(K_2^2) > J_{Pal}(K_2^4) > J_{Pal}(K_2^3),$$
$$J_\alpha^\beta(K_2) < J_\alpha^\beta(K^{\frac{1}{2}}) > J_\alpha^\beta(K_2^2) > J_\alpha^\beta(K_2^3) > J_\alpha^\beta(K_2^4).$$

Therefore, the entropy measures $J_{Pal}(K_2), J_{\alpha}^{\beta}(K_2)$ are not suitable for differentiating the information conveyed by FSs. But $J_{HY}(K_2)$ and $J_2^{new}(K_2)$ are also satisfy the ranking order in equation (4.2). The effectiveness of proposed fuzzy measure $J_2^{new}(K_2)$ and $J_{HY}(K_2)$ is indicated by this example once again. Hence, the proposed measure consider one parameter which increase the flexibility due to the parameter α whereas J_{HY} does not due to the absence of parameters. Therefore, the proposed measure is encouraging. So the presence of parameter in an information measure makes it flexible from application point of view.

4. Conclusions

In this paper, a new information measure which is generalization of Gini Simpson index of diversity has been effectively introduced. We found that the four axiomatic requirements properties are satisfied with the new fuzzy entropy. The proposed information measure has been compared with existing entropies. Some numerical examples based on linguistic terms have been offered to show the successful applicability of the proposed information measure. Another possible topic for future research is to use proposed entropy in the fuzzy setting for multi criteria decision making problems which can be applied in evaluating mobile services. The proposed entropy can further be applied to the concept of the parametric directed divergence measure, similarity and dissimilarity measure for fuzzy sets, interval valued intuitionistic, pythagorian and picture fuzzy sets,coding theory etc.

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