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ANNEALED GREY WOLF OPTIMIZATION

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ABSTRACT. A neoteric hybrid algorithm focused on Grey Wolf Optimizer (GWO) and Improved modified Simulated Annealing Algorithm called Annealed-Grey Wolf Optimizer is suggested here to gain advantages and overcome their disadvantages. Grey wolf optimizer is a metaheuristic swarm intelligence method and has been used widely due to its superb characteristics which take it one step ahead than other swarm intelligence methods as it efficiently solves various optimization problems. With various advantages it also has a disadvantage of bad local searching ability and here in this paper it is tried to overcome by simulated annealing as its major edge is to escape becoming captured in local minima.

1. INTRODUCTION

Actual-world optimizing challenges are always really difficult to fix, and countless systems have to struggle with NP-hard problems. Optimization techniques must be used to resolve these issues, but there is no assurance that the optimum solution will be attained. In reality there have been no efficient algorithms for NP problems. As a consequence, several problems need to be fixed by trials and error utilizing different techniques of optimizations. Therefore, novel algorithms have been introduced to see if these complex optimization problems can be tackled. Under many ways, nature has influenced many scholars, and is

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thus a rich source of inspiration. Many recent algorithms nowadays are inspired with nature. Those are metaheuristic techniques which do not depend upon gradient information about the objective function and the constraints, and can use probabilistic concepts of transformation. These are centered on stochastic inquiry strategies, making them successful and adaptable. Yet it is still pretty hard to detect a perfectly effective clarification for almost every question due to no free lunch rule (Wolpert, Macready, 1997, [1]). In this manner, researchers and engineers around the globe are still during the time spent finding more optimization algorithms and increasingly applicable techniques.

2. Simulated Annealing

Simulated annealing starts from a point which is selected arbitrarily, and here the core principle is that is move from point to point. So when it moves to the next point it can either be a good move or a bad move, good as in if the value of the objective function is getting minimized for a minimization function compared to the value of the objective function at the previous point, and thus we accept that new move and in that new point the free energy state is lesser than the previous point. But suppose the energy level in case of decreasing it increases i.e. if we move to the next point the value of the objective function is getting maximized we will still sometimes accept that bad move. In this not all bad moves are rejected and some of the bad moves are still accepted thus later on giving us a global optimum solution. Bad moves that gets accepted is decided a Boltzman probability distribution (P)

$$P = \exp\left(-\frac{\Delta E}{kT}\right),$$

ΔE Is the change of objective function by two consecutive iterations, k is the boltzman constant and T is the control parameter which is analogously known as temperature for annealing. However this probability is not equivalent in all circumstances. P actually depends on ΔE and T. If T is large, the probability is more and any point will be accepted for large value of T, and when T decreases the chances of an arbitrary point to get accepted becomes less. The Pseudo code of classical SA is as follows: Pseudo Code of Simulated Annealing:

(1) Select the initial solution val

(2) Initialize the parameters:
Initial Temperature $=$ Tinit Final Temperature $= \backslash$ Tfin Boltzman's Constant k Reduction factor c
(3) Set maximum number of perturbation at same temperature {iter }$\_\{\max\}$
(4) $\mathrm{T} =$ Tinit
(5) While $(\mathrm{T} >$ Tfin $)$
(6) for iter $= 1\backslash$ to $\backslash$ fiter $\}\_\{\max\}$
(7) Generate new value as $\backslash$ newval in the neighborhood of val
(8) if (newval) $\backslash \leq$ val) val = newval
else Calculate difference between newval and val, (ΔE) $Boltz\_Prob = \exp(-\Delta E/kT))$
$if(Boltz\_Prob > random(0,1))$
$val = newval$ end end end Decrease T by cooling schedule: $\mathrm{T} = \mathrm{c}\backslash$ ast T end
(9) Return the best solution found

3. Grey Wolf Optimizer

GWO is a newly discovered swarm intelligence algorithm presented by Mirjalili et. al, in 2014 [2], influenced by the grey wolves' social framework and attacking behavior. There are four levels: alpha (α), beta (β), delta (δ) and omega (ω) indexed by descending into force, in which the alpha wolf is the controller of the group, and ω wolves are the undermost among the unit ranking. The key foraging procedure for grey wolf is as follows:

a) Constantly monitoring, outrunning and inching closer to the prey.
b) The target is tracked, blockaded and intimidated until it stops moving.
c) Attempting to kill the prey.

The equations proposed for the mathematical model of encircling of grey wolves is as follows:

$$\vec{\{D\}} = \left|\vec{\{C\}}.\{\vec{\{X\}}\}_p\ (it) - \vec{\{X\}}(it)\right|$$

$$\vec{\{X\}}(it+1) = \{\ \vec{\{X\}}\}_p\,(it) - \vec{\{A\}}.\vec{\{D\}},$$

where it is the current iteration, $\vec{\{A\ \}}$ and $\vec{\{C\}}$ are coefficient vectors, $\vec{\{X\}}_p$ position vector of the victim $\vec{\{X\}}$ is the position vector of the grey wolf. Coefficient vectors are calculated as:

$$\vec{\{A\}} = 2\vec{\{a\}}.\{\vec{\{s\}}\}_1 - \vec{\{a\}}$$

$$V\vec{\{C\}} = 2.\{\vec{\{s\}}\}_2.$$

Here s_1 and s_2 are random vectors in [0,1] and $\vec{\{a\}}$ is linearly decreased from 2 to 0 during iterations. We select the initial three best solutions that we have up until now and update the position of other search agents as indicated by the situation of the elite search agent found. Mathematically it is proposed as follows:

$$\{\vec{\{}\ D\}\}_\alpha = \left|\{\vec{\{C\}}\}_1 \,.\, \{\vec{\{X\}}\}_\alpha - \vec{\{X\}}\right|;$$

$$\{\vec{\{D\}}\}_\beta = \left|\{\vec{\{C\}}\}_2 \,.\, \{\vec{\{X\}}\}_\beta - \vec{\{X\}}\right|;$$

$$\{\vec{\{D\}}\}_\delta = \left|\{\vec{\{C\}}\}_3 \,.\, \{\vec{\{X\}}\}_\delta - \vec{\{X\}}\right|;$$

$$\{\vec{\{X\}}\}_1 = \{\vec{\{X\}}\}_\alpha - \{\vec{\{A\}}\}_1.(\{\vec{\{D\}}\}_\alpha);$$

$$\{\vec{\{X\}}\}_2 = \{\vec{\{X\}}\}_\beta - \{\vec{\{A\}}\}_2.(\{\vec{\{D\}}\}_\beta);$$

$$\{\vec{\{X\}}\}_3 = \{\vec{\{X\}}\}_\delta - \{\vec{\{A\}}\}_3.(\{\vec{\{D\}}\}_\delta);$$

$$\vec{\{X\}}\,(it+1) = \frac{\{}{X_1} + X_2 + X_3\}\{3\}.$$

4. Proposed Method

Within this research a fresh hybrid is presented to improve the GWO calculations. Exploration is intended to assess the elite region of the search space, and exploitation is needed throughout this zone to come up with best solutions. There is a well acknowledge fact that exploration and exploitation are both necessary to demonstrate incredible performance in any algorithm based on the population.

Classical SA is a methodology adapted from statistical thermodynamics to simulate the activity of atomic arrangements during the annealing phase in liquid or solid structures. Its principal advantages over other methodologies of local search are its versatility and ability to achieve global optimality. Here the concept of Simulated Annealing is used to improve the values of first three best solutions i.e. α, β, δ because on that basis other search agents improve their positions. The condition which decides whether or not the worse solution will be approved is determined by a probability which is given by the:

$$P = \frac{1}{1 + 2exp\Delta E/kT},$$

Here in eq. (8), P the probability of accepting the new state. ΔE is the change in objective value of the current solution and the new solution. T is the system temperature, k is Boltzman Constant and e is the Euler number. The probability is defined in between the range of (0, 1/3). It lowers the acceptance range of the algorithm of the worst solutions. The pseudo code of the proposed method is as follows:

5. Pseudo Code of Annealed Grey Wolf Optimizer

(1) Create starting populace of search agent.
(2) Calculate objective function value for every search agent
(3) Select α candidate solution as Initial solution and initialize it as (val)
(4) Initialize maxiteration
(5) Set Initial Temperature Tinit
(6) Set Final Temperature Tfin
(7) Initialize Boltzman's Constant k
(8) Initialize Reduction factor c
(9) Set maximum number of perturbation at same temperature $iter_{max}$
(10) n=0
(11) While (n<maxiteration)
(12) T=Tinit
(13) While (T>Tfin)
(14) for iter=1 to iter$_{max}$
(15) Generate newval in the neighborhood of val
(16) if (newval $\leq$ val)
val=newval
else
Calculate difference between newval and val (ΔE)
$Boltz_Prob$ = 1/(1+2exp(ΔE/kT))
if($Boltz_Prob$>random(0,1/3))

val=newval
end
end
end
Decrease T by cooling schedule: T=c* T
n=n+1
end
Return the best solutions found and initialize it to the value of α candidate solution
end
Do the same process as above simultaneously for beta and delta candidate solution as an initial solution, and then continue as it is done for the solution from step (7) from the pseudo code of A-GWO.

6. Result

Benchmark problems are utilized in this segment to assess Annealed GWO's efficiency. Such problems can be split into three distinct classes: "unimodal", "multimodal" and "fixed-dimensional multimodal" functions. Annealed GWO was evaluated on 19 benchmark problems (Yao, Liu, & Lin, 1999, [3]), seven unimodal, six multimidal and six "fixed dimension multimodal" problems. The specific depictions of these testing problems are referenced in table 1 to table 3 in which we got better results.

Each function has been assessed 30 times with 30 search agents and the most extreme number of iterations as 500. The best values and the worst values have been reported in the table 2. Our key goal here is to convey the best reasonable optimal solution when contrasted with different metaheuristics.

7. Conclusion

A new hybrid version of Grey Wolf Optimizer with Improved Modified Simulated Annealing named Annealed Grey Wolf Optimizer (A-GWO) for optimizing global optimization problems has been introduced in this work. The main idea behind this paper was to lay more emphasis on exploration and local search ability in Grey Wolf Optimizer. Here in A-GWO, the values of alpha, beta, and delta operators are improved because based on these three operators other search agents improve their positions. These three operators are improved by introducing the concept of Simulated Annealing, as here the alpha, beta, and delta are considered as the starting point separately and the probability of accepting the worst solution is also reduced. Annealed-GWO, as appeared, is a truly steady algorithm, fit for accomplishing dependable, sensible execution. The algorithm seeks the global optimum in various functions as per robustness. Annealed-GWO can thus be regarded as a robust optimization algorithm. Annealed-GWO is a good optimization algorithm that can be applied to practical engineering related issues.

8. Tables

Table 1: Unimodal Benchmark Functions

Function	*Dim*	*Range*	f_{min}
$F_1(x)=\sum_{i=1}^{n} x_i^2$	30	$[-100,100]$	0
$F_2(x)=\sum_{i=1}^{n} \lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	30	$[-10,10]$	0
$F_3(x)=\sum_{i=1}^{n}\left(\sum_{j=1}^{i} x_j\right)^2$	30	$[-100,100]$	0
$F_4(x)=max_i\{\lvert x_i\rvert, 1\le i\le n\}$	30	$[-100,100]$	0
$F_5(x)=\sum_{i=1}^{n-1}\left[100\left(x_{i+1}-x_i^2\right)^2+(x_i-1)^2\right]$	30	$[-30,30]$	0
$F_6(x)=\sum_{i=1}^{n-1}(\lfloor x_i+0.5\rfloor)^2$	30	$[-30,30]$	0
$F_7(x)=\sum_{i=1}^{n} ix_i^4+rand[0,1)$	30	$[-1.28,1.28]$	0

Table 2: Multimodal Benchmark Functions

Function	*Dim*	*Range*	f_{min}
$F_8(x) = \sum_{i=1}^{n} -x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$	30	$[-500,500]$	-12569.5
$F_9(x) = \sum_{i=1}^{n}\left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	30	$[-5.12,5.12]$	0
$F_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=n}^{n} cos2\pi x_i\right)$	30	$[-32,32]$	0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	$[-600,600]$	0
$F_{12}(x) = \frac{\pi}{n}\left\{10sin(\pi y_i) + \sum_{i=1}^{n-1}(y_i-1)^2[1+10sin^2(\pi y_{i+1}) + (y_{n-1})^2]\right\} + \sum_{i=1}^{n} u(x_i,10,100,4)$ $y_i = 1 + \frac{x_i+1}{4}$ $u(x_i,a,k,m) = \begin{cases} k(x_i-a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i-a)^m & x_i < -a \end{cases}$	30	$[-50,50]$	0
$F_{13}(x) = 0.1\left\{sin^2(3\pi x_1) + \sum_{i=1}^{n-1}(x_i-1)^2[1+sin^2(3\pi x_{i+1})] + (x_n-1)^2[1+sin^2(2\pi x_n)]\right\} + \sum_{i=1}^{n} u(x_i,5,100,4)$	30	$[-50,50]$	0

Table 3: fixed dimension multimodal benchmark problems

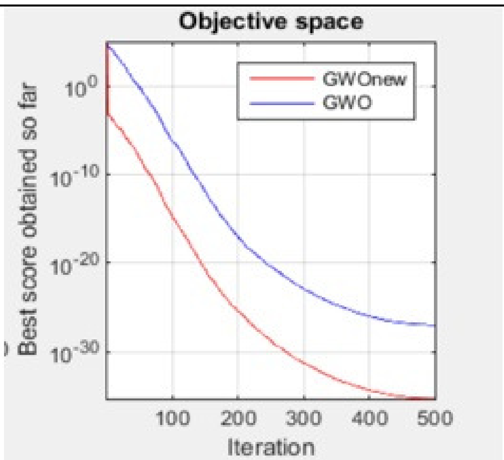
Function	*Dim*	*Range*	f_{min}
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} (x_i - a_{ij})^6} \right)^{-1}$	2	[−65.536,65.536,]	1
$F_{15}(x) = -\sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{4} a_{ij} (x_j - p_{ij})^2 \right]$	3	[0,1]	−3.86
$F_{16}(x) = -\sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{6} a_{ij} (x_j - p_{ij})^2 \right]$	6	[0,1]	−3.32
$F_{17}(x) = -\sum_{i=1}^{5} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0,10]	−10.
$F_{18}(x) = -\sum_{i=1}^{7} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0,10]	−10
$F_{19}(x) = -\sum_{i=1}^{10} [(x - a_i)(x - a_i)^T + c_i]^{-1}$	4	[0,10]	−10"

Table 4 shows the performance of Annealed Grey Wolf Optimizer, and is compared to the original GWO.

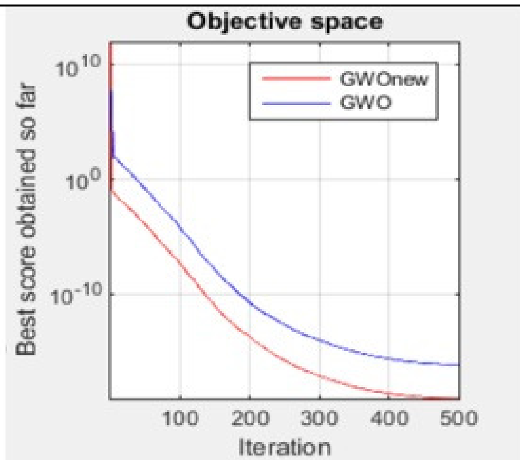
Table 4

Function	GWO		A − GWO	
	Best	Worse	Best	Worse
F_1 (a)	$1.182e^{-28}$	$7.694e^{-27}$	$9.092e^{-37}$	$5.975e^{-35}$
F_2 (b)	$1.298e^{-17}$	$4.346e^{-16}$	$5.452e^{-20}$	$2.372e^{-19}$
F_3 (c)	$9.782e^{-8}$	$1.502e^{-5}$	$1.581e^{-16}$	$6.523e^{-14}$
F_4 (d)	$3.802e^{-7}$	$4.627e^{-6}$	$1.322e^{-11}$	$2.789e^{-10}$
F_5 (e)	26.136	28.019	25.5728	26.165

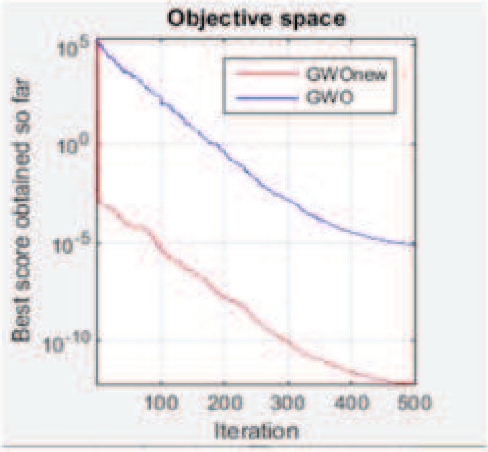
F_6 (f)	0.251	1.251	0.201	1.766
F_7 (g)	0.000495	0.00350	$1.599e^{-6}$	$4.705e^{-4}$
F_8 (h)	−7029.366	−3395.181	−7625.465	−3589.212
F_9 (i)	$1.1368e^{-13}$	9.066	0	$5.684e^{-14}$
F_{10} (j)	$6.838e^{-14}$	$1.572e^{-13}$	$3.386e^{-14}$	$4.352e^{-14}$
F_{11} (k)	0	0.04027	0	0.0257
F_{12} (l)	0.0162	0.0847	0.0127	0.0777
F_{13} (m)	0.3896	1.321	0.3186	0.8265



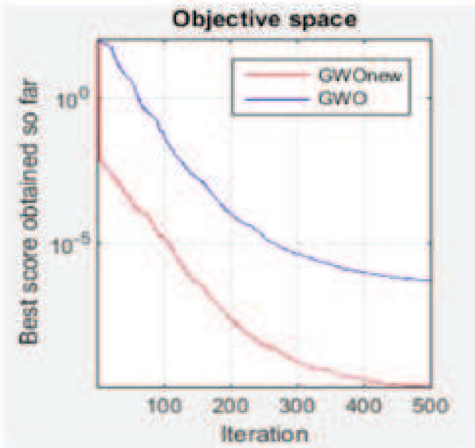
(a)



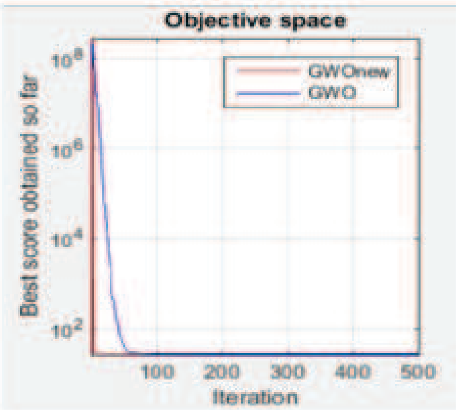
(b)


Objective space
GWOnew
GWO
Best score obtained so far
10^{5} 10^{0} 10^{-5} 10^{-10}
100 200 300 400 500
Iteration

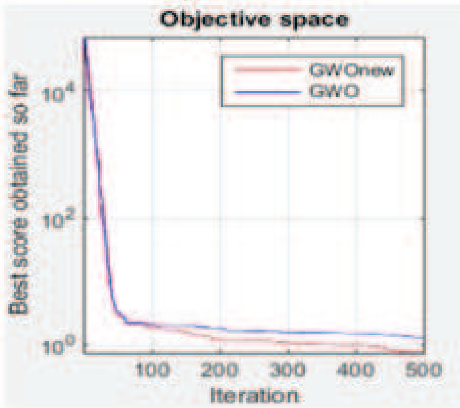
(c)


Objective space
GWOnew
GWO
Best score obtained so far
10^{0} 10^{-5}
100 200 300 400 500
Iteration

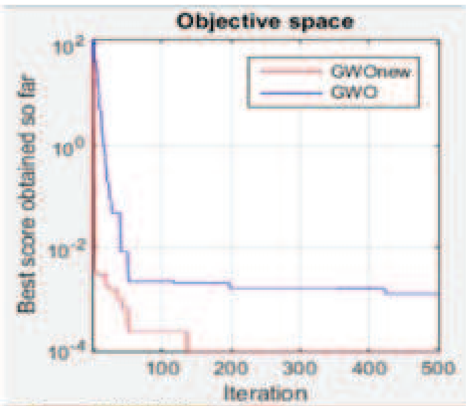
(d)


Objective space
GWOnew
GWO
Best score obtained so far
10^{8} 10^{6} 10^{4} 10^{2}
100 200 300 400 500
Iteration

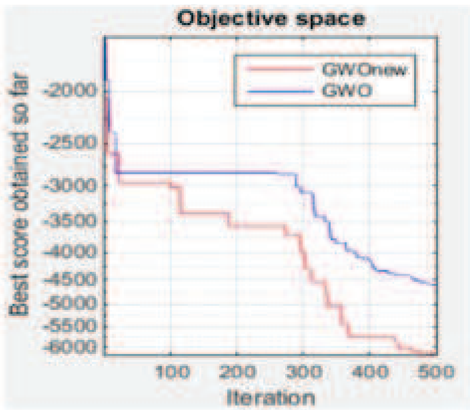
(e)


Objective space
GWOnew
GWO
Best score obtained so far
10^{4} 10^{2} 10^{0}
100 200 300 400 500
Iteration

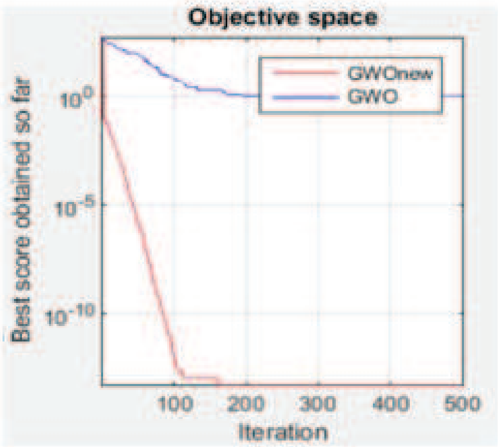
(f)


Objective space
GWOnew
GWO
Best score obtained so far
10^{2} 10^{0} 10^{-2} 10^{-4}
100 200 300 400 500
Iteration

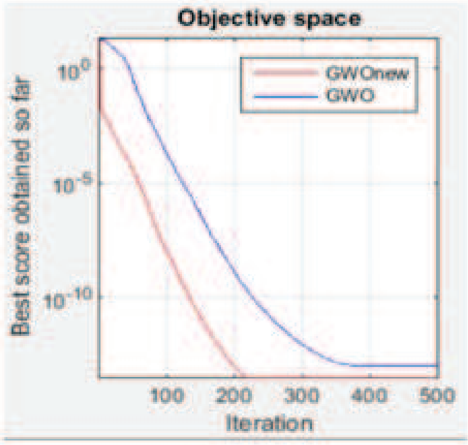
(g)


Objective space
GWOnew
GWO
Best score obtained so far
-2000 -2500 -3000 -3500 -4000 -4500 -5000 -5500 -6000
100 200 300 400 500
Iteration

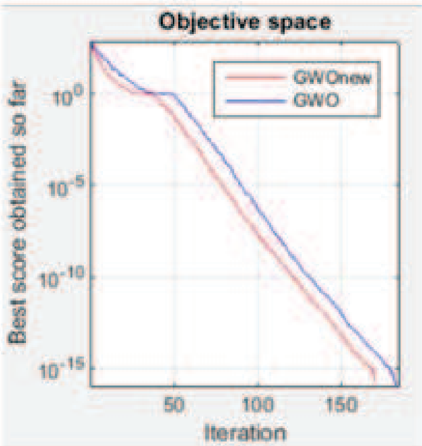
(h)


Objective space
GWOnew
GWO
Best score obtained so far
10^0
10^{-5}
10^{-10}
100
200
300
400
500
Iteration

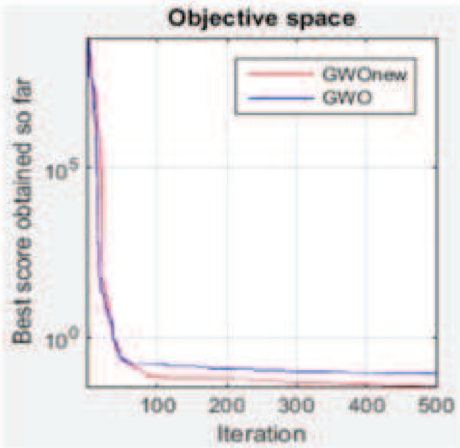
(i)


Objective space
GWOnew
GWO
Best score obtained so far
10^0
10^{-5}
10^{-10}
100
200
300
400
500
Iteration

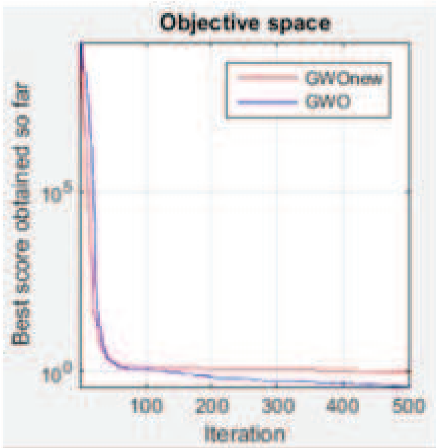
(j)


Objective space
GWOnew
GWO
Best score obtained so far
10^0
10^{-5}
10^{-10}
10^{-15}
50
100
150
Iteration

(k)


Objective space
GWOnew
GWO
Best score obtained so far
10^5
10^0
100
200
300
400
500
Iteration

(l)


Objective space
GWOnew
GWO
Best score obtained so far
10^5
10^0
100
200
300
400
500
Iteration

(m)

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