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KANNAN-TYPE FIXED POINT RESULTS IN EXTENDED RECTANGULAR B-METRIC SPACE

KANIKA RANA[1] AND ARUN KUMAR GARG

ABSTRACT. Present study extends the work done by M. Asim et al in extended rectangular b-metric space andto prove the actuality and individuality of fixed point theorem connected with Kannan contraction. An example will also be provided to examine the authenticityof our result.

1. INTRODUCTION

With the celebration of well-known result in fixed point theory acknowledged as BCP-banach contraction principle, there is an extensive demand of fixed point theory, due to its multiple applications in other classifications highlighted as digital topology , biology , computer applications , engineering and many more.Over the past years, BCP [1] have been generalized in numerous directions in different metric spaces, see [2,3,4]. These generalizations have been obtained either by extending the domain of the mapping or by considering a more general contractive conditions on the mappings. Kannan contraction [5] is also one of the important result in literature. A great number of applications and extensions of these results have performed in the literature and plays an essential part in nonlinear analysis. Over the past few periods there are many extensions of metric space highlighted as b-metric space, extended b-metric space

[1]*corresponding author*

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[6], rectangular metric space [7], rectangular b metric space [4]. Also in 2019, a new class of these generalisations of metric space known as extended rectangular b-metric space [8] have been familiarised in which the author proved an analogue of BCP with an application.

Motivated by the research done in [8], we broaden our work to prove the Kannan type fixed point results followed by an example.

2. Preliminaries

Below are some appropriate definitions that are favourable to prove our result. Throughout our study, we will consider extended rectangular b-metric space as ERbAS.

Definition 2.1. *([9]) An ordered pair (Z, γ) is a metric space , where set Z is not empty and $\gamma : Z \times Z \to R$ such that for any $g, h, t \in Z$, the following fulfilled:*

$(\mathrm{M1})$ $\gamma(g, h) \geq 0$*;*

$(\mathrm{M2})$ $\gamma(g, h) = 0$ *iff* $g = h$*;*

$(\mathrm{M3})$ $\gamma(g, h) = \gamma(g, h)$*;*

$(\mathrm{M4})$ $\gamma(g, h) \leq \gamma(g, t) + \gamma(i, h)$.

Definition 2.2. *([3]) let Z be a set and let $d \geq 1$ be a certain real number. A function $\gamma : Z \times Z \to [0, \infty)$ is Icnown to be b-metric space if for all $g, h, i \in Z$, below conditions fulfilled:*

$(\mathrm{M1})$ $\gamma(g, h) \geq 0, \gamma(g, h) = 0$ *if and only if* $g = h$*;*

$(\mathrm{M2})$ $\gamma(g, h) = \gamma(g, h)$*;*

$(\mathrm{M3})$ $\gamma(g, h) \leq d[\gamma(g, i) + \gamma(i, h)]$.

The pair (Z, γ) is called a b-metric space.

If $\mathrm{d} = \underline{1}$, then without exception metric space is a b-metric space.

Definition 2.3. *([6]) let Z be a set which is not empty and $\theta : Z \times Z \to [1, \infty)$ be a mapping. We describe the extended b-metric to the function $\gamma_\theta : Z \times Z \to [0, \infty)$ that satisfies the following conditions;*

$(\mathrm{M1})$ $\gamma_\theta(g, h) = 0$ *if and only if* $\mathrm{g} = \mathrm{h}$*;*

$(\mathrm{M2})$ $\gamma_\theta(g, h) = \gamma_\theta(h, g)$*;*

$(\mathrm{M3})$ $\gamma_\theta(g, h) \leq \theta(g, h)\,[\gamma_\theta(g, i) + \gamma_\theta(i, h)]$.

The space (Z, γ_θ) is labelled as an extended b-metric space.

Without exception, consider b -metric space as an extended b-metric space by taking $\theta(a, c) = d \geq 1$ to be a constant function.

Definition 2.4. *([7]) Let Z be a set which is not empty and the mapping $\Delta : Z \times Z \to [0, \infty)$ satisfies:*

(M1) $\Delta(g, h) = 0$ *iff* $g = h$ *for all* $g, h \in Z$*;*

(M2) $\Delta(g, h) = \Delta(h, g)$*;*

(M3) $\Delta(g, h) \leq \Delta(g, i) + \Delta(i, j) + \Delta(j, h)$ *for all* $i, j \in Z$.

Then Δ is labelled as rectangular metric on Z and (Z, Δ) is labelled as rectangular metric space.

Definition 2.5. *([4]) Let Z be a set which is not empty and the mapping $\Delta_a : Z \times Z \to [0, \infty)$ satisfies:*

(M1) $\Delta_\alpha(g, h) = 0$ *iff* $g = h$ *for all* $g, h \in Z$*;*

(M2) $\Delta_a(g, h) = \Delta_\alpha(h, g)$*;*

(M3) $\Delta_\alpha(g, h) \leq d\,[\Delta_a(g, i) + \Delta_a(i, j) + \Delta_a(j, h)]$ *for all* $g, h \in Z; d \geq 1$.

Then Δ_α is labelled as rectangular b-metric on Z and (Z, Δ_α) is labelled as rectangular b-metric space.

Every metric space is labelled as rectangular metric space and every rectangular metric space is known to be a rectangular b-metric space (with esefficient $d = 1$), without exception. Though, the contrary of the above implication is not certainly sersect.

Definition 2.6. *([8]) let Z be a set which is not empty and $\theta : Z \times Z \to [1, \infty)$. A mapping $\Delta_\theta : Z \times Z \to R^+$ is termedas an extended rectangular b-metric on Z if, Δ_θ satisfies the following (for all $g, h \in Z$ and all distinct $i, j \in Z \backslash \{g, h\}$*

(M1) $\Delta_\theta(g, h) = 0$ *iff* $g = h$*;*

(M2) $\Delta_\theta(g, h) = \Delta_\theta(h, g)$*;*

(M3) $\Delta_\theta(g, h) \leq \theta(g, h)\,[\Delta_\theta(g, i) + \Delta_\theta(i, j) + \Delta_\theta(j, h)$

Then the pair (Z, Δ_θ) is said to be an ERbMS.

Example 1. *([8]) Consider, $= \{1, 2, 3, 4, 5\}$. Define $\theta : Z \times Z \to [1, \infty)$ by $\theta(g, h) = g + h + 2$, for all $g, h \in Z$ is an example of Definition 2.6 and Definition 2.7 ([8]). A sequence $\{g_p\}$ in (Z, Δ_θ) is known to be Cauchy if $\lim_{p,q \to \infty} \Delta_\theta\,(g_p, h_q) = 0$.*

Definition 2.7. *([8]) A sequence $\{g_p\}$ in (Z, Δ_θ) is lonown to be convergent if $\lim_{p \to \infty} \Delta_\theta\,(g_p, g) = 0$.*

Definition 2.8. *([8]) If, every Cauchy in Z is convergent to some point in Z then ERbMS (Z, Δ_θ) is complete.*

Lemma 2.1. *([8]) Let (Z, Δ_θ) be an ERbMS and sequence $\{g_p\}$ is Cauchy in Z such that $g_p \neq h_9$ whenever $\mathrm{p} \neq q$. Then $\{a_p\}$ converges to atmost one point.*

Definition 2.9 (10). *Let (Z, Δ_θ) be an ERbMS. Then for a mapping, $: Z \to Z$, define $(g \in Z$ and $p \in \mathbb{N}$,*

$$\Theta(g : p) = \{g, Sg, \ldots, S^p g\} \quad \textit{and} \quad \Theta(g; \infty) = \{g, Sg, \ldots, S^p g, \ldots\}.$$

Then $\Theta(g; \infty)$ is called an orbit of S.

Definition 2.10. *([8]) Let (Z, Δ_θ) be an ERbMS. A mapping $S : Z \to Z$ is called orbitally continuos if $\lim_{k\to\infty} S^{p_k} g = g$ for some $p \in Z$ implies $\lim_{k\to\infty} S\left(S^{p_k} g\right) = Sg$. Besides, (Z, Δ_θ) is labelled as S-orbitally complete, if every Cauchy sequence which found in $\{g, Sg, \ldots, S^p g, \ldots\}$ for some $g \in Z$ converges to Z.*

3. Main Result

Our main theorem extends Kannan type contraction in ERbMS. All through this we will consider $\Theta\ (g; \infty) = \{g, Sg, \ldots, S^p g, \ldots\}$ an orbit of a self-map S.

Theorem 3.1. *let (Z, Δ_θ) be an ERbMS and $S : Z \to Z$ be a self map. Assume that following restrictions satisfied:*

- (i) *For all $g, h \in Z$, we have $\Delta_\theta\left(S_g, S_h\right) \leq \tau\left[\Delta_\theta\left(g, S_g\right) + \Delta_\theta\left(h, S_h\right)\right]$ Where $\tau \in \left(0, \frac{1}{2}\right)$;*
- (ii) $\lim_{p,q\to\infty} \theta\left(g_p, h_q\right) < \frac{1}{r'}$;
- (iii) *(Z, Δ_θ) is S-grbitally complete;*
- (iv) *S is orbitally continuous.*

Then a unique fixed point found in S.

Proof. let $g_0 \in Z$ be an arbitrary point, construct an iterative sequence $\{g_p\}$ such that:

$$g_1 = Sg_0, g_2 = Sg_1 = S^2 g_0, \ldots, g_p = Sg_{p-1} = S^p g_0, \ldots \tag{3.1}$$

Now, we define $\lim_{p\to\infty}\Delta_\theta\left(g_p,g_{p+1}\right)=0$ and using (i), we get

$$\begin{aligned}\Delta_\theta\left(S^p g_0,S^{p+1}g_0\right)&=\Delta_\theta\left(Sg_p,Sg_{p+1}\right)=\Delta_\theta\left(g_{p+1},g_{p+2}\right)\\&\leq\tau\left[\Delta_\theta\left(g_p,Sg_p\right)+\Delta_\theta\left(g_{p+1},Sg_{p+1}\right)\right]\\&\leq\tau\left[\Delta_\theta\left(g_p,g_{p+1}\right)+\Delta_\theta\left(g_{p+1},g_{p+2}\right)\right]\end{aligned}$$

which implies that

$$\begin{aligned}\Delta_\theta\left(g_{p+1},g_{p+2}\right)-\tau\Delta_\theta\left(g_{p+1},g_{p+2}\right)&\leq\tau\Delta_\theta\left(g_p,g_{p+1}\right)\\(1-\tau)\Delta_\theta\left(g_{p+1},g_{p+2}\right)&\leq\tau\Delta_\theta\left(g_p,g_{p+1}\right)\\\Delta_\theta\left(g_{p+1},g_{p+2}\right)&\leq\frac{\tau}{(1-\tau)}\left(g_p,g_{p+1}\right).\end{aligned}$$

Continuing in this way we get,

$$\Delta_\theta\left(g_{p+1},g_{p+2}\right)\leq\left(\frac{\tau}{(1-\tau)}\right)^p\left(g_0,g_1\right),$$

i.e.,

$$\rightarrow_\theta\left(g_{p+1},g_{p+2}\right)\leq\vartheta^p\left(g_0,g_1\right);\text{ where }\frac{\tau}{(1-\tau)}=\vartheta<1.$$

Applying limit $\to\infty$, gives us

$$\lim_{p\to\infty}\Delta_\theta\left(S^p g_0,S^{p+1}g_0\right)=0\quad\text{ and }\lim_{p\to\infty}.\Delta_\theta\left(S^{p+1}g_0,S^{p+2}g_0\right)=0$$

Next step is to show the Cauchy sequence $\{g_p\}$ in (Z,Δ_θ). For this consider two cases.

Case 1: In the first step, let, g is odd, then$g=2q+1$ for any $q\geq 1$. Now using $(\mathrm{M3})$ of def. (2.6) for any $p\in\mathbb{N}$, we have

$$\begin{aligned}&\Delta_\theta\left(g_p,g_{p+2q+1}\right)\\&\quad\leq\sigma\left(g_p,g_{p+2q+1}\right)\left[\Delta_\theta\left(g_p,g_{p+1}\right)+\Delta_\theta\left(g_{p+1},g_{p+2}\right)+\Delta_\theta\left(g_{p+2},g_{p+2q+1}\right)\right]\\&\quad\leq\sigma\left(g_p,g_{p+2q+1}\right)\left[\theta^p\left(g_0,g_1\right)+\vartheta^{p+1}\Delta_\theta\left(g_0,g_1\right)\right]+\sigma\left(g_p,g_{p+2q+1}\right)\\&\qquad\times\Delta_\theta\left(g_{p+2},g_{p+2q+1}\right)\\&\quad=\sigma\left(g_{p,g_{p+2q+1}}\right)\left(\vartheta^p+\vartheta^{p+1}\right)\Delta_\theta\left(g_0,g_1\right)+\sigma\left(g_p,g_{p+2q+1}\right)\\&\qquad\times\Delta_\theta\left(g_{p+2},g_{p+2q+1}\right)\end{aligned}$$

$$\begin{aligned}
&\leq \sigma\left(g_p, g_{p+2q+1}\right)\left(\vartheta^p+\vartheta^{p+1}\right) \Delta_\theta\left(g_0, g_1\right)+\sigma\left(g_{p+2}, g_{p+2q+1}\right) \\
&\quad \times \Delta_\theta\left(g_{p+2}, g_{p+2q+1}\right)\left(\vartheta^{p+2}+\vartheta^{p+3}\right) \Delta_\theta\left(g_0, g_1\right)+\cdots \sigma\left(g_p, g_{p+2q+1}\right) \\
&\quad\ \sigma\left(g_{p+2q-2, g_{p+2q+1}}\right)\left(\vartheta^{p+2q-2}+\vartheta^{p+2q-1}\right) \Delta_\theta\left(g_0, g_1\right)+ \\
&\quad\ \sigma\left(g_{p, g_{p+2q+1}}\right) \ldots \sigma\left(g_{p+2q-2, g_{p+2q+1}}\right) \vartheta^{p+2q} \Delta_\theta\left(g_0, g_1\right) \\
&=\vartheta^p(1+\vartheta) \Delta_\theta\left(g_0, g_1\right) \sum_{i=0}^{q-1} \vartheta^{2i} \prod_{j=0}^{i} \sigma\left(g_{p+2j}, g_{p+2q+1}\right) \\
&\quad +\vartheta^{p+2q} \prod_{j=0}^{q-1} \sigma\left(g_{p+2j}, g_{p+2q+1}\right) \Delta_\theta\left(g_0, g_1\right)
\end{aligned}$$

Continue the process, we get

$$\sum_{i=0}^{q-1} \vartheta^{2i} \prod_{j=0}^{i} \sigma\left(g_{p+2j}, g_{p+2q+1}\right) \leq \sum_{i=0}^{q-1} \vartheta^{2i} \prod_{j=0}^{i} \sigma\left(g_{2j}, g_{p+2q+1}\right).$$

According to(ii), we have $\lim_{p,q\to\infty} \theta\left(g_p, h_q\right)\tau < 1$, we conclude that

$$\sum_{i=0}^{q-1} \vartheta^{2i} \prod_{j=0}^{i} \sigma\left(g_{2j}, g_{p+2q+1}\right)$$

is a convergent series for each $q \in \mathbb{N}$:

$$A=\sum_{i=0}^{q-1} \vartheta^{2i} \prod_{j=0}^{i} \sigma\left(g_{2j}, g_{p+2q+1}\right), A_p=\Sigma_{i=0}^{p} \vartheta^{2i} \prod_{j=0}^{i} \sigma\left(g_{2j}, g_{p+2q+1}\right).$$

Therefore, from the above result

$$\begin{aligned}
\Delta_\theta\left(g_{p, g_{p+2q+1}}\right) &\leq \vartheta^p(1+\vartheta) \Delta_\theta\left(g_0, g_1\right)\left[A_{q-1}-A_{p-1}\right] \\
&=\vartheta^{p+2q} \prod_{j=0}^{q-1} \sigma\left(g_{p+2j}, g_{p+2q+1}\right) \Delta_\theta\left(g_0, g_1\right).
\end{aligned}$$

Putting the limits $p \to \infty$ in equation (3.1), we get to the conclusion that $\Delta_\theta\left(g_p, g_{p+2q+1}\right) \to 0$.

Case 2: in the second step, let gis even, then $g = 2q$ for any $q \geq 1$. Now using $(\mathrm{M3})$ of def. (2.6) for any $\in \mathbb{N}$, we have

$$\begin{aligned}\Delta_\theta(g_p, g_{p+2q}) &\leq \sigma(g_p, g_{p+2q})[\Delta_\theta(g_p, a_{p+1}) + \Delta_\theta(g_{p+1}, g_{p+2}) + \Delta_\theta g_{p+2}, g_{p+2q}] \\ &\leq \sigma(g_p, g_{p+2q})[\vartheta^p\,(g_0, g_1) + \vartheta^{p+1}\Delta_\theta g_0, g_1] + \ \sigma\,(g_p, g_{p+2q}) \times \Delta_\theta g_{p+2}, g_{p+2q} \\ &= \sigma(g_p, g_{p+2q})(\vartheta^p + \vartheta^{p+1})\Delta_\theta g_0, g_1 + \sigma\,(g_p, g_{p+2q}) \times \Delta_\theta gp+2, gp+2q \\ &\leq \sigma(g_p, g_{p+2q})(\vartheta^p + \vartheta^{p+1})\Delta_\theta g0, g1 + \sigma\,(g_{p+2}, g_{p+2q}) \\ &\quad \times \Delta_\theta(g_{p+2}, g_{p+2q}\vartheta^{p+2} + \vartheta^{p+3}\Delta_\theta(g_0, g_1) + \ldots + \sigma(g_p, g_{p+2q}) \ldots \\ &\quad\ \sigma\,(g_{p+2q-2}, g_{p+2q})\left(\vartheta^{p+2q-2} + \vartheta^{p+2q-1}\right)\Delta_\theta(g_0, g_1) \\ &\quad + \sigma\,(g_p, g_{p+2q}) \ldots \sigma\,(g_{p+2q-2}, g_{p+2q})\,\vartheta^{p+2q}\Delta_\theta g0, g1 \\ &= \vartheta^p\,(1+\vartheta)\,\Delta_\theta(g_0, g_1)\sum_{i=0} q - 1\vartheta^{2i}\prod_{j=0}^{i}\sigma(g_{p+2j}, g_{p+2q} \\ &\quad + \vartheta^{p+2q-2}\prod_{j=0}^{q-1}\sigma\,(g_{p+2j}, g_{p+2q})\,\Delta_\theta(g_0, g_1).\end{aligned}$$

Suppose, $= \sum_{i=0}^{q-1}\vartheta^{2i}\prod_{j=0}^{i}\sigma\,(g_{2j}, g_{p+2q+1})\,,\ A_p = \sum_{i=0}^{p}\vartheta^{2i}\prod_{j=0}^{i}\sigma\,(g_{2j}, g_{p+2q+1})\,.$ Therefore, from the above result

$$\begin{aligned}\Delta_\theta\left(g_{p, g_{p+2q+1}}\right) &\leq \vartheta^p(1+\vartheta)\Delta_\theta\,(g_0, g_1)\,[A_{q-1} - A_{p-1}] \\ &+ \quad \vartheta^{p+2q-2}\prod_{j=0}^{q-1}\sigma\,(g_{p+2j}g_{p+2q})\,\Delta_\theta\,(g_0, g_1)\,.\end{aligned}$$

Putting the limits $p \to \infty$ in equation (3.1), we get to the conclusion that $\Delta_\theta\,(g_p, g_{p+2q}) \to 0$ From the above cases we get that, $\Delta_\theta\,(g_p, g_{p+2q}) = 0$.

Hence, $\{g_p\}$ is Cauchy in Z. Since, (Z, Δ_θ) is S-orbitally complete (iii).

Therefore, there exists $g \in Z$ such that $g_p \to g$. Also S is orbitally continuous. We have

$$\begin{aligned}\Delta_\theta(Sg, g) &\leq \sigma(Sg, g)\,[\Delta_\alpha\,(Sg, g_p) + \Delta_\theta\,(g_{p, g_{p+1}}) + \Delta_\theta\,(g_{p+1, g})] \\ &\leq (Sg, g)\,[\Delta_\alpha\,(Sg, g_p) + \Delta_\theta\,(g_{p-1, g}) + \Delta_\theta\,(g_{p+1, g})] \\ &= (Sg, g)\left[\Delta_\alpha\,(Sg, g_p) + \Delta_\theta\left(g_{p, g_{p+1}}\right) + \Delta_\theta\,(g_{p+1, g})\right]\end{aligned}$$

making the limits $p \to \infty$ gives $\Delta_\theta(Sg, g) \to 0$.

So, we conclude that $\Delta_\theta(Sg, g) = 0$ which implies that $= g$. Hence, g is the fixed point of S and from Lemma 2.1, we can say that $^*\mathrm{g}'$ is a unique fixed point of S. □

Example 2. *Let $Z = [0.1]$. Define $\Delta_\theta(g, h) = |g - h|^2$ and $\theta(g, h) = g + h + 2$ for all $a, b \in$ Z. Then, (Z, Δ_θ) is a complete ERbMS. Define a mapping $S : Z \to Z$ by $Sg = \frac{g}{2}$. Clearly, we can say that all the restrictions of the theorem above proved are satisfied and $g = 0$ is the unique fixed point of S.*

Below are some new results that can be derived from our main Theorem 3.2.

Corollary 3.1. *let (Z, Δ_θ) be an ER $6\mathrm{MS}$ and $S : Z \to Z$ be a self map. Suppose that resulting restrictions hold:*

(i) For all $g, h \in Z$, we have

$$\Delta_\theta (S_g, S_h) \leq \tau \left[\Delta_\theta (g, S_g) + \Delta_\theta (h, S_h)\right].$$

(ii) $\lim_{p,q\to\infty} \theta (g_p, h_q) < \frac{1}{\tau}$.
(iii) (Z, Δ_θ) is S is complete.
(iv) S is continuous. Then a unique fixed point is found in S.

Corollary 3.2. *let (Z, Δ_θ) be an ERbMS with $u \geq 1$ and $S : Z \to Z$ be a self map. Suppose that following conditions holds:*

(i) For all $g, h \in Z$, we have

$$\Delta_\theta (S_g, S_h) \leq \tau \left[\Delta_\theta (g, S_g) + \Delta_\theta (h, S_h)\right]$$

where $\tau \in \left(0, \frac{1}{u}\right)$
(Z, Δ_θ) is $\mathrm{S}$is orbitally complete.
(iii) S is orbitally continuous. Then a unique fixed point is found in S.

Corollary 3.3. *Let (Z, Δ_θ) be an ERbMS with $u \geq 1$ and $S : Z \to Z$ be a self map. Suppose that following conditions fulfilled:*

(i) For all $g, h \in Z$, we have

$$\Delta_\theta (S_g, S_h) \leq \tau \left[\Delta_\theta (g, S_g) + \Delta_\theta (h, S_h)\right]$$

where $\tau \in \left(0, \frac{1}{2}\right)$.
(ii) (Z, Δ_θ) is $\mathrm{S}$is orbitally complete.
(iii) S is orbitally continuous. Then a unique fixed point is found in S.

4. Conclusion

ERbMS is an extension of several metric spaces labelled as rectangular, b, rectangular b, extended b and ERbMS is new to this literature so, firstly we discussed some important definitions that are required to justify our result and then we extend our work in this class of metric space. The result of the present study is an extended version of Kannan type contraction followed by an example which will be considered to do further study in this literature.

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Department of Mathematics,
Chandigarh University,
Gharuan, Punjab-140413, India.

Department of Mathematics,
Chandigarh University,
Gharuan, Punjab-140413, India.