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OPTIMIZING INVENTORY POLICY FOR TIME-DEPENDENT DEMAND WITH IMPERFECT ITEMS

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ABSTRACT. Through this paper, an inventory model is proposed for a manufacturing process which produces perfect and after some time imperfect items. It's been assumed that demand is time-dependent and production is greater than demand. The rate of production of items is directly affected by demand. A further assumption is made that the system starts producing imperfect items after some time of operation due to various factors. For imperfect items, collection and repair work has been considered which optimizes the inventory. Repair of the imperfect items starts when regular production stops. Using the concepts of differential calculus, the optimum inventory is obtained to capitalize on the profit and reduce the cost. An example is discussed to demonstrate the theory.

1. INTRODUCTION

In traditional manufacture inventory model it is understood that manufacturing systems are absolutely trustworthy. But for almost every actual system this assumption does not hold true. It's always possible for even the best production system to produce imperfect items. In this paper, the manufacturing process is taken to be flexible but flawed. During a production run, it is considered that after some time of operation the system could manufacture some imperfect items. Considering the present scenario of the market and the competition, it's very

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difficult to survive for a company without optimizing its production policy. So the companies have to adapt to different methods or strategies to increase profit by repairing the imperfect items rather than throwing the same. Further, the environmental issues caused by these forced governments around the globe to frame regulations that want manufacturers to reduce waste by repairing or recycling the imperfect items produced. Taking into consideration all the factors, it is enviable to study the significance of a flexible manufacturing system that can repair imperfect items and adjust itself to the demand.

S.R. Singh et al. (2014) obtained high profit using increased production uptime and reduced production rate in comparison with less production uptime and elevated production rates in the stochastic model. It was also concluded that the elevated production cost per unit reduced the anticipated profit. S.R. Singh et. al. (2013) studied the process of remanufacturing and its effects in an integrated manufacture inventory model which includes deteriorating products considering stock dependent demand facing shortages. They proved total cost function that can reduce total cost incurred. Himani Dem and Leena Prasher (2013), optimized the inventory with collection and rework of reusable items. Kung-Jeng Wang et. al. (2011) optimized inventory having multi-echelon deliver chain for products with deteriorating rate being time-sensitive. B.C. Giri and A. Chakraborty (2011) developed a model considering sole vendor and sole buyer. It was termed as supply chain coordination model. The demand by the buyer is taken as a linear function of the on-hand inventory, the buyer screens the products after every replenishment. A coordination policy for vendor-buyer was determined to reduce the cost of supply chain. Shib Sankar Sana (2010) proposed a model to calculate the most favorable product consistency and production rate that obtain the maximum total profit for a defective manufacturing practice. Kuo-Lung Hou (2006) derived an optimum stock model with shortages, fading items, stock-dependent consumption rates allowing for inflation for finite planning prospect. S. Rana et. al. (2004) proposed an optimal model taking demand as directly proportional to time with shortages, deterioration and finite production rate. S. Kar et. al. (2001) suggested an optimal model taking primary and secondary shops with detoration. Jinn-Tsair Teng et. al. (1999) proved with a mathematical formulation that a the flexibility in policy to beginning and/or end the planning horizon with shortages is found to be less expensive to operate in comparison with a policy without shortages at the start or end-stage. B.C. Giri and K.S. Chaudhuri (1998) worked on an extended EOQ -type inventory model for a perishable product. They reasoned that when controlling costs are kept nonlinear, demand rate is stock-dependent and with the end status of zero ending inventories, an optimal solution was obtained.

Assumptions

1) Production rate is in direct proportion with demand which, in turn, is time-dependent.
2) The relation of time-dependent demand and time is

$$f(q) = Dt^{\beta}, D > 0, 0 < \beta < 1, t \geq 0,$$

where β represents the sensitivity of demand.
3) The time scope of the inventory is taken to be t_5.

Notations:

Q : Maximum inventory of expected production uptime.
Q_1 : Perfect item inventory at time t_1.
f(q) : Demand rate, $f(q) = Dt^{\beta}, D > 0, 0 < \beta < 1, t \geq 0$
P : Production Rate
P_2 : production rate of the repaired item
K : Ordering cost per cycle
HC : Holding cost per cycle
DC : Deteriorating cost per cycle
TAC : Total average cost of inventory
cp : Item production cost
h : Holding cost of inventory per unit time
θ : Rate at which imperfect items are produced
Qc : Inventory of collective items
Qr : Inventory of repaired items
t_1 : Time when perfect and imperfect items produced and start of collection of imperfect item.
t_2 : When regular production, as well as the collection of imperfect items, stop and repair of collective item start
t_3 : When all the imperfect items have been repaired
t_5 : Duration of the complete cycle

Model Formulation

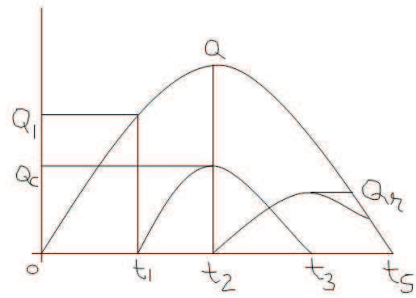

Fig.1: Inventory flow with collection & repair

It is assumed that from time 0 to t_1, the production is perfect. After time t_1, the production unit produces both perfect as well as defective items. These defective items are collected from time t_1 to t_2. Assuming inventory of imperfect items from t_1 to t_2 is q_2. Regular production stops at time t_2, and the repair of collective items starts. At time t_3, the imperfect item reduces to q3 i.e., from time t_2 to t_3 and repaired items are produced from t_2 to t_3. From t_3 to t_5, imperfect items that are perfect now, satisfy the demand.

The governing differential equations for perfect items are:

$$\frac{dq_1}{dt} = P - Dt^{\beta}, q_1(0) = 0, 0 \leq t \leq t_1$$
$$\frac{dq_1}{dt} = P - Dt^{\beta} - \theta P, q_1(t_2) = Q, t_1 \leq t \leq t_2$$

The governing differential equations for imperfect items collection and repair are:

$$\frac{dq_2}{dt} = \theta P, q_2(t_1) = 0, t_1 \leq t \leq t_2$$
$$\frac{dq_1}{dt} = -Dt^{\beta}, q_1(t_5) = 0, t_2 \leq t \leq t_5$$
$$\frac{dq_3}{dt} = P_2 - Dt^{\beta}, q_3(t_2) = 0, t_2 \leq t \leq t_3$$
$$\frac{dq_2}{dt} = -\theta P, q_2(t_3) = 0, t_2 \leq t \leq t_3$$
$$\frac{dq_3}{dt} = -Dt^{\beta}, q_3(t_5) = 0, t_3 \leq t \leq t_5$$

Solving the above differential equation and using the associated boundary conditions, we get the inventory level

$$q_1 = \frac{(l-1)Dt^{\beta+1}}{\beta+1}$$
$$q_1 = Q + \frac{(l-1-\theta D)(t^{\beta+1} - t_2^{\beta+1})}{\beta+1}$$
$$q_2 = \frac{l\theta D(t^{\beta+1} - t_1^{\beta+1})}{\beta+1}$$
$$q_1 = \frac{D(t_5^{\beta+1} - t^{\beta+1})}{\beta+1}$$

$$q_3 = \frac{(lD_2-D)(t^{\beta+1}-t_2^{\beta+1})}{\beta+1}$$
$$q_2 = -\frac{l\theta D(t^{\beta+1}-t_3^{\beta+1})}{\beta+1}$$
$$q_3 = \frac{D(t_5^{\beta+1}-t^{\beta+1})}{\beta+1}$$

Using the relations $q_1(t_2) = Q, q_2(t_2) = Q_c, q_3(t_3) = Q_r$ we get

$$t_1^{\beta+1} = \frac{Q_1(\beta+1)}{(l-1)D}$$
$$t_2^{\beta+1} = t_1^{\beta+1} - \frac{(Q_1-Q)(\beta+1)}{(l-1-\theta)D}$$
$$Q_c = \frac{l\theta D(Q-Q_1)}{(l-1-\theta)D}$$
$$t_5^{\beta+1} = t_2^{\beta+1} + \frac{Q(\beta+1)}{D}$$
$$t_3^{\beta+1} = t_2^{\beta+1} + \frac{Q_r(\beta+1)}{lD_2-D}$$
$$t_3^{\beta+1} = t_5^{\beta+1} - \frac{Q_r(\beta+1)}{D}$$
$$t_5^{\beta+1} = (\beta+1)(\frac{Q_1}{(l-1)D} - \frac{Q_1-Q}{(l-1-\theta)D} + \frac{Q}{D})$$

Holding cost from $0 \leq t \leq t_5$ = holding cost of perfect items + holding cost of collected items + holding cost of repaired items

$$=h[\frac{(Q-\frac{Q_1(\beta+1)}{(l-1)D}+\frac{(Q_1-Q)(\beta+1)}{(l-1-\theta)D)}(Q-Q_1)}{(l-1-\theta)D} - \frac{(Q_1-Q)}{\beta+1} + \frac{l\theta(Q-Q_1)(1-\frac{Q_1(\beta+1)}{(l-1)D})}{(l-1-\theta)D}$$
$$+\frac{D((\beta+1)(\frac{\frac{Q_1}{(l-1)D}-\frac{Q_1-Q}{(l-1-\theta)D}+\frac{Q}{D}Q}{D}-\frac{Q}{lD_2}}{\beta+1} + (lD_2-D)Q(1-\frac{Q_1(\beta+1)}{(l-1)D}$$
$$+\frac{(Q_1-Q)(\beta+1)}{l-1-\theta)D)}/(\beta+1)lD_2 + D[\frac{Q(lD_2-D)((\beta+1)\frac{Q_1}{(l-1)D}-\frac{Q_1-Q}{(l-1-\theta)D}+\frac{Q}{D}-1)}{(lDD_2(\beta+1)}$$
$$+\frac{Q_1}{\beta+1} - \frac{l\theta DQ_r(1-t_3^{\beta+1})}{(\beta+1)(lD_2-D)}]$$

$$Dc = \Phi cp\frac{(Q-\frac{Q_1(\beta+1)}{(l-1)D}+\frac{(Q_1-Q)(\beta+1)}{(l-1-\theta)D})(Q-Q_1)}{(l-1-\theta)D} - \frac{(Q_1-Q)}{\beta+1}$$
$$+\frac{l\theta}{(Q-Q_1)}(1-\frac{Q_1(\beta+1)}{(l-1)D}(l-1-\theta)D + D[\frac{(\beta+1)(\frac{Q_1}{(l-1)D}-\frac{Q_1-Q}{(l-1-\theta)D}+\frac{Q}{D})Q}{D}$$
$$-\frac{Q}{lD_2}\frac{1}{\beta+1} + \frac{Q\frac{(Q_1(\beta+1)}{(l-1)D}-\frac{(Q_1-Q)(\beta+1)}{(l-1-\theta)D}+\frac{(\beta+1)Q}{lD_2}-1}{\beta+1}]$$
$$+D[\frac{Q(lD_2-D)((\beta+1)(\frac{Q_1}{(l-1)D}-\frac{Q_1-Q}{(l-1-\theta)D}+\frac{Q}{D})-1)}{lDD_2(\beta+1)} + \frac{Q_1}{\beta+1} - \frac{l\theta DQ_r(1-t_3^{\beta+1})}{(\beta+1)(lD_2-D)}]$$

The total average cost per unit time is given by

$$TAC = \frac{K+HC+DC}{t_s}.$$

To find the expression for time when the production should stop, when q takes optimum value Q, we need to minimize TAC of the inventory system. The essential condition for TAC to be minimum is:

$$\frac{d}{dQ}(TAC) = 0.$$

This yields

$t_5[\frac{d(HC)}{dQ} + \frac{d(DC)}{dQ}] - (K + HC + DC)\frac{dt_5}{dQ} = 0$

$t_5 = (\frac{Q_1}{(l-1)D} - \frac{Q_1-Q}{(l-1-\theta)D} + \frac{Q}{D}$

If $Q_1 = NQ$

$\frac{dt_5}{dQ} = \frac{N}{(l-1)D} + \frac{1-N}{(l-1-\theta)D} + \frac{1}{D}$

Special case: as $\theta = 0, \Phi = 0, \beta = 0$ and l tends to infinity

$t_5 = \frac{Q}{D}$

$DC = 0$

$Hc = h(Q + \frac{2Q^2}{D}$

$Q = \sqrt{\frac{KD}{2h}}$

Thus as l increases production run size approaches to EOQ model.

Numerical example:

Here, we discuss one example using computational results which are obtained using Wolframe Mathematica7 giving insight about the response of optimal run size Q, production time t_5 and the total average cost TAC . The parametric values for the numerical example are taken as K = 200 , D = 2 , D_2 = 2 , N = 0.2 , c_h = 0.5 ,l =2 ,θ = 0.002 , c_p = 0.4.

Table 1: Effect of holding cost on the optimal value of Q, t_5 and TAC

$\beta = 0.2$					
h	0.2	0.4	0.6	0.8	1.0
Q	22.2221	15.7291	12.847	11.1277	9.96006
t_5	22.2399	15.74171	12.857	11.1366	9.96804
HC+DC	141.7351	142.5061	143.097	143.5963	144.2112
TAC	15.36586	21.75787	26.6856	30.85289	34.53148

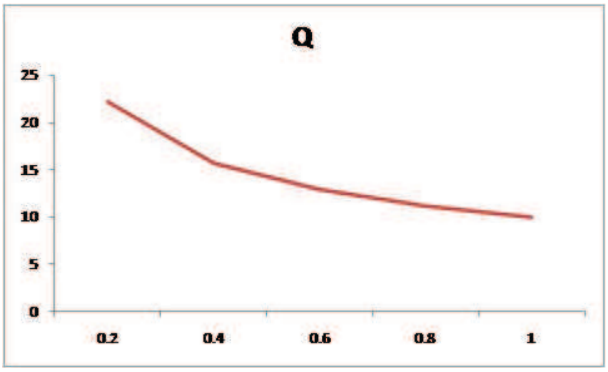

Fig.1: Variation of Q with HC

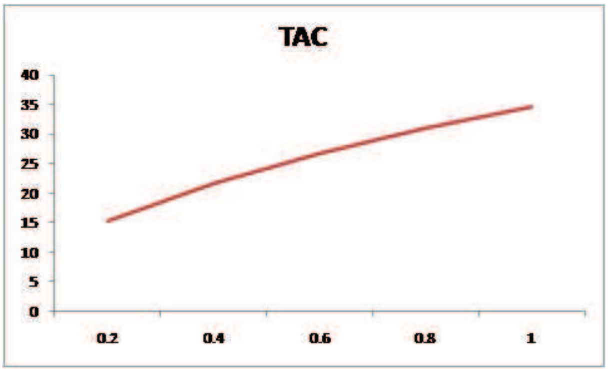

Fig.2: Variation of TAC with HC

It can be seen from the plots that when the holding cost increases inventory decreases and total average cost increases.

Table 2: Effect of β on the optimal value of Q, t_s and TAC

h=0.2				
β	0.2	0.4	0.6	0.8
Q	22.2221	22.4468	22.6784	22.9174
T_s	22.2399	22.46479	22.69658	22.93577
HC+DC	141.7351	144.5969	147.5765	150.6831
TAC	15.36586	15.33942	15.314047	15.28979

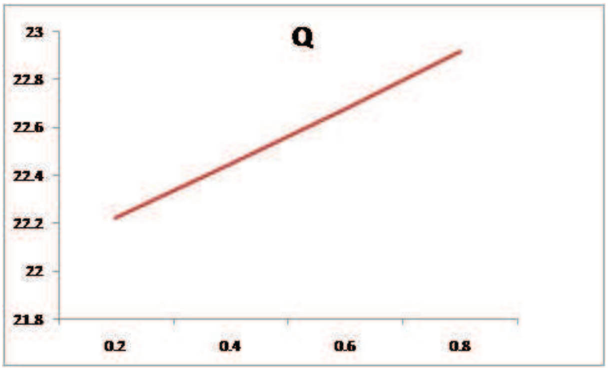

Fig.3: Variation of Q with β

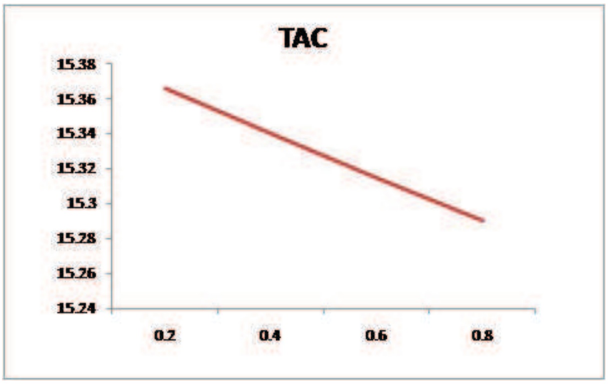

Fig.4: Variation of TAC with β

The plots show the variations of Q and TAC with β. We observe that as the responsiveness of demand increases inventory increases and corresponding TAC decreases.

Observation:

- Completion of time does not affect by perfect items or independent of perfect items.
- As β increases, Q decreases i.e. as respond of demand increases corresponding inventory automatically decreases.
- An increase in holding cost results in decrease in Q decreases i.e. inventory decreases.
- As the percentage of perfect items increases, inventory increases.

2. CONCLUSION

Time management should be in such a way that in which repaired articles that are perfect now and perfect articles meets the demand. We are finding the maximum inventory at which the production should stop. We observe that even if a collective rate is 0.2% per unit time, the repaired items are 50% of the inventory at a time when the production stops. As the holding cost increases, inventory decreases and the corresponding total average cost increases. This is true for the bulk inventory which leads to a decrease in cost. If the inventory is less, the cost will increase. Therefore, rather than to throw the imperfect items, if we collect and repair them with the same holding cost, the total inventory will increase and the corresponding cost will decrease. Total completion time of repair depends upon the inventory at which the production stops.

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