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POSITIVITY AND MONOTONICITY SHAPE PRESERVING USING RATIONAL QUINTIC FRACTAL INTERPOLATION FUNCTIONS

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ABSTRACT. The traditional interpolation schemes has great limitations for irregular shape kind of data. So for this we establish FIF for describe the irregular shape data or derivatives. In this paper we developed rational quintic fractal interpolation function of the form $\frac{p(\theta)}{q(\theta)}$ having three shape parameters and a vertical scaling factor in each sub-interval. The sufficient conditions for positive preservation and monotonicity preservation are presented in this paper.

1. INTRODUCTION

The practical examples such as information representation, data science, PC illustrations, data is produced by complex functions or some scientific methods, as a rule need to create a smooth function, a set of data is interpolate and hold some geometric traits, we for the most part call it shape preserving function, the spline interpolation method called classical cubic spline interpolation, generally refused shape preserving function. In ongoing 30 years, rational cubic spline has become a problem area in modern structure and logical information perception on account of its less oscillation and preferred properties over common polynomial interpolation. The theory of fractal interpolation function was presented by Barnsley [1, 2] for the first time by using the iterated function

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system(IFS). Barnsley and Harrington [3] developed a smooth FIF to estimated the unknown function value. Later many research scholars presented the FIF theory [5, 6, 18]. More and more authors have been contributed in the research field of fractal theory which has been used in various field of complex phenomenon. In practical theory fractal theory is very useful and easy to calculate. We have examined the properties of rational fractal interpolation functions in recent years.For instance, SQ Deng et al. [7] introduced the shape control conditions of rational cubic interpolating spline. Karim S A A et al. [8] studied the shape preserving interpolation using C2 rational cubic spline and obtained the monotonicity conditions. Abbas M et al. [9] studied the monotonicity preserving C2 bivariate rationalcubic spline for monotone data.

In literature [10-12] the research on the positive preservation of rational fractal interpolation functions were discussed for all kind of systems. In [13] convexity preservation through rational cubic spline fractal interpolation functions were introduced. In [15] the positivity preserving rational cubic Ball constrained interpolation was presented. In [16] the construction of rational cubic trigonometric fractal interpolation functions was introduced and discussed its positive preserving aspects.Based on the article [17] the further properties of shape preserving are discussed in this paper and the corresponding data constraints are obtained in terms of positivity and monotonicity preserving. The objective of this paper is to present the monotonicity shape preserving conditions and positive shape preserving conditions of the given set of data for C^2-RQFIF with three free parameters. In section 2, the general introduction about FIF is reviewed. In section 3, the construction of C^2-RQFIF is discussed. In section 4, we establish the sufficient conditions for positivity preservation. In section 5, the sufficient conditions for monotonicity preservation are established.

2. Fractal interpolation function(FIF)

Suppose we have a set of data points $\{(x_i, t_i) \in I \times K : j = 1, 2, 3..., n\}$, where K is compact set in R, set $I_i = [x_i, x_{i+i}]$ and $I = [x_1, x_n]$, $i \in \wedge$. Let $L_I : I \to I_i$, be the contraction homeomorphism such that

$$L_i(x_1) = t_i, L_i(x_m) = x_{i+1}$$
$$|L_i(x) - L_i(x')| \le l_i|x - x'| \tag{2.1}$$

Here $0 < l_i < 1$, Denote C = I $\times$ K, and Define n-1 continuous mappings

$F_i : C \to T$ satisfying the

$$F_i(x_1, t_1) = t_i, F_i(x_n, t_n), i \in \wedge, \tag{2.2}$$

$$|F_i(x, y) - F_i(x, z)| \leq |\sigma_i||y - z| \forall y, z \in K, 0 \leq |\sigma_i| < 1. \tag{2.3}$$

Define $\omega_i : C \to I_i \times K$ such that $\omega_i(x, t) = (L_i(x), F_i(x, t)) \forall i \in \wedge$.

Then a unique attractor is generated by an IFS which is a graph G of a continuous function $f' : I \to K$ satisfying $f'(x_i) = t_i$, i = 1,2,3...n.. This defined function f' is called fractal interpolation function (FIF) corresponding to an IFS. Also FIF is given by an IFS in the following form:

$$\{C; \omega_i \equiv L_i(x) = a_i x + b_i, F_i(x, t) = \sigma_i t + r_i(x)\}. \tag{2.4}$$

Here $r_i : I \to R$ is a continuous function which satisfying above descriptions. we take $r_i(x)$ as a quintic rational function in this paper which is continuous as well as satisfying above descriptions.

3. Construction of RQFIF

Let we have a $\{(x_j, t_j), j \in \wedge'\}$ given set of data s.t $x_1 < x_2 < .... < x_n$. Consider the IFS $\{I \times K; \omega_i(x, t) = (L_i(x), F_i(x, t)) : i = 1, ..., n - 1\}$ where $L_i(x) = a_i x + b_i i \in J$ and $F_i(x, t) = \sigma_i t + r_i(x)$, $r_i(x) = \frac{p_i(x)}{q_i(x)}$ where $p_i(x)$ is a quintic polynomial and $q_i(x)$ is a quadratic polynomial, $q_i(x) \neq 0 \forall x \in [x_1, x_n]$. By [3] integer $|\sigma_i| < a_i^p, i = 1, 2..., n - 1$. Let $F_i^{(1)}(x, d) = \frac{\sigma_i d + r_i^{(1)}(x)}{a_i^1}$ and $F_i^{(2)}(x, D) = \frac{\sigma_i d + r_i^{(2)}(x)}{a_i^2}$ where $r_i^{(1)}(x)$ and $r_i^{(2)}(x)$ are the first and second derivatives of $r_i(x)$ respectively. $F_i(x, \sigma)$ satisfying the following C^2-interpolatory conditions:

$$F_i(x_1, \sigma_1) = t_i,\ F_i(x_n, \sigma_n) = t_{i+1},\ F_i^{(1)}(x_1, d_1) = d_i,\ F_i^{(1)}(x_n, d_n) = d_{i+1},$$
$$F_i^{(2)}(x_1, D_1) = D_i, F_i^{(2)}(x_n, D_n) = D_{i+1}, \tag{3.1}$$

where d_i represent the first order derivative of σ w.r.t x at knot x_i.

In this section, the rational quintic fractal interpolation function is constructed.Let we have a $\{(x_j, t_j), j \in \wedge'\}$ given set of data s.t $x_1 < x_2 < .... < x_n$. Suppose d_i represent the first order derivative of σ w.r.t x at knot x_i. By considering the proposition 2.1 with

$$\psi(L_i(x)) = \sigma_i \psi(x) + r_i(x) \tag{3.2}$$

$$r_i(x) = \frac{p_i(\theta)}{q_i(\theta)}, \theta = \frac{x - x_1}{l}, l = x_n - x_1, x \in I \tag{3.3}$$

$$p_i(\theta) = \sum_{i=0}^{5} (1 - \theta)^{5-i} \theta^i U_i$$

$$p_i(\theta) = (1-\theta)^5 U_0 + (1-\theta)^4\theta U_1 + (1-\theta)^3\theta^2 U_2 + (1-\theta)^2\theta^3 U_3$$
$$+(1-\theta)^1\theta^4 U_4 + \theta^5 U_5 \quad (3.4)$$
$$q_i(\theta) = u_i(1-\theta)^2 + v_i(1-\theta)\theta + w_i\theta^2 \quad (3.5)$$

and u_i, v_i and w_i are the positive shape parameters. To shield that the fractal function ψ is C^2-interpolant, we impose some interpolation properties:

$$\psi(L_i(x_1)) = t_i, \psi(L_i(x_n)) = t_{i+1}, \psi^{(1)}(L_i(x_1)) = d_i,$$
$$\psi^{(1)}(L_i(x_n)) = d_{i+1}, \psi^{(2)}(L_i(x_1)) = D_i, \psi^{(2)}(L_i(x_n)) = D_{i+1}, i \in \wedge.$$

Put $x \equiv x_1$ in equation (3.2) and(3.3) then we get

$$U_0 = u_i(t_i - \sigma_i t_1)$$
$$U_0 = u_i t^*_{i,1} \quad (3.6).$$

Put $x = x_n$ in equation (3.3) then we get

$$U_5 = w_i(t_{i+1} - \sigma_i t_n)$$
$$U_5 = w_i t^*_{i+1,n} \quad (3.7).$$

Differentiate (3.2) to (3.5) w.r.t x then we get

$$\psi'(L_i(x))L_i'(x) = \sigma_i\psi'(x) + r_i'(x) \quad (3.8)$$

and

$$r_i'(x) = \frac{q_i(\theta)p_i'(\theta) - q_i'(\theta)p_i(\theta)}{l(q_i(\theta))^2}) \quad (3.9)$$
$$p_i'(\theta) = -5(1-\theta)^4U_0 - 4(1-\theta)^3\theta U_1 + (1-\theta)^4U_1 - 3(1-\theta)^2\theta^2U_2$$
$$+2(1-\theta)^3\theta^1U_2 + 3(1-\theta)^2\theta^2U_3 + 2(1-\theta)^1\theta^3U_3$$
$$+4(1-\theta)^1\theta^3U_4 - \theta^4U_4 + 5\theta^4U_5 \quad (3.10)$$
$$q_i'(\theta) = -2u_i(1-\theta)^1 + v_i(1-\theta) - v_i\theta + 2w_i\theta \quad (3.11).$$

Put $x = x_1$ in equation (3.2) and (3.8) then we get

$$d_i a_i l u_i^2 = \sigma_i d_1 l u_i^2 - 3U_0u_i + U_1u_i - v_iU_0.$$

Put U_0 from (3.6)

$$U_1 = a_il(u_id_i - \sigma_id_1) + (3u_i + v_i)(t_i - \sigma_it_1)$$
$$U_1 = lu_id^*_{i,1} + (3u_i + v_i)t^*_{i,1}. \quad (3.12)$$

Put $x = x_n$ in equation (3.3) and (3.8)then we get

$$a_ild_{i+1}w_i^2 = \sigma_id_nlw_i^2 + (-U_4w_i + 5U_5w_i + v_iU_5 - 2w_iU_5).$$

Put U_5 from (3.7)

$$U_4 = w_il(-a_id_{i+1} + \sigma_id_n) + (3w_i + v_i)(t_{i+1} - \sigma_it_n)$$
$$U_4 = -d^*_{i+1,n}w_il + (3w_i + v_i)t^*_{i+1,n}. \quad (3.13)$$

Again differentiate (3.8) to (3.11) then we get

$$\sigma''(L_i(x))L_i'(x) = \sigma_i\psi''(x) + r_i''(x) \quad (3.14)$$

and

$$r_i''(x) = \frac{p_i''(\theta)(q_i(\theta))^2 - q_i''(\theta)p_i(\theta)q_i(\theta) + 2p_i(\theta)(q_i'(\theta))^2 - 2p_i'(\theta)q_i'(\theta)q_i(\theta)}{l^2(q_i(\theta))^3)} \quad (3.15)$$

$$p_i''(\theta) = 20(1-\theta)^3U_0 + 12(1-\theta)^2\theta U_1 - 8(1-\theta)^3U_1 - 12(1-\theta)^2\theta^1U_2$$
$$+6(1-\theta)^1\theta^2U_2 + 2(1-\theta)^3U_2 + 6(1-\theta)^2\theta^1U_3 - 12(1-\theta)^1\theta^2U_3 + 2\theta^3U_3$$
$$+12(1-\theta)^1\theta^2U_4 - 8\theta^3U_4 + 20\theta^3U_5 \quad (3.16)$$

$$q_i''(\theta) = 2(u_i - v_i + w_i). \quad (3.17)$$

Put $x \equiv x_1$ in (3.14) to (3.17) then we have

$$a_i^2D_il^2u_i^3 = \sigma_iD_1l^2\alpha_i^3 + (20U_0 - 8U_1 + 2U_2)u_i^2$$
$$-(2u_i - 2v_i + 2w_i)U_0u_i + 2U_0(4u_i^2 + v_i^2 - 4u_iv_i)$$
$$-2(-5U_0 + U_1)(-2u_i + v_i)w_i.$$

Put U_0 and U_1 from (3.6) and (3.12) then we get

$$U_2 = 0.5u_il^2D_{i,1}^* + (3u_i + 3v_i + w_i)t_{i,1}^* + ld_{i,1}^*(2u_i + v_i) \quad (3.18).$$

Put $x = x_n$ in (3.14) to (3.17) implies

$a_i^2D_{i+1}l^2w_i^3 = \sigma_iD_1l^2u_i^3 + \{(2U_3 - 8U_4 + 20U_5)w_i^2 - (2u_i - 2v_i + 2w_i)U_5w_i + 2U_5(4w_i^2 + v_i^2 - 4w_iv_i) - 2(5U_5 - U_4)(-2w_i + v_i)w_i\}$.

Put U_4 and U_5 from (3.7) and (3.13) then we get

$$U_3 = 0.5w_il^2D_{i+1,n}^* + t_{i+1,n}^*(u_i + 3v_i + 3w_i) - l(2w_i + v_i)d_{i+1,n}^*. \quad (3.19)$$

From (3.6), (3.7), (3.12), (3.13), (3.18) and (3.19) we get

$$U_0 = u_it_{i,1}^*$$
$$U_1 = lu_id_{i,1}^* + (3u_i + v_i)t_{i,1}^*$$
$$U_2 = 0.5u_il^2D_{i,1}^* + (3u_i + 3v_i + w_i)t_{i,1}^* + ld_{i,1}^*(2u_i + v_i)$$
$$U_3 = 0.5w_il^2D_{i+1,n}^* + t_{i+1,n}^*(u_i + 3v_i + 3w_i) - l(2w_i + v_i)d_{i+1,n}^*$$
$$U_4 = -d_{i+1,n}^*w_il + (3w_i + v_i)f_{i+1,n}^*$$
$$U_5 = w_it_{i+1,n}^*.$$

Put the values of B_0, B_1, B_2, B_3, B_4 and B_5 in (3.4) then we get

$$p_i(\theta) = (1-\theta)^5u_it_{i,1}^* + (1-\theta)^4\theta lu_id_{i,1}^* + (3u_i + v_i)t_{i,1}^*$$
$$+(1-\theta)^3\theta^2 0.5u_il^2D_{i,1}^* + (3u_i + 3v_i + w_i)t_{i,1}^* + ld_{i,1}^*(2u_i + v_i)$$
$$+(1-\theta)^2\theta^3 0.5w_il^2D_{i+1,n}^* + t_{i+1,n}^*(u_i + 3v_i + 3w_i) - l(2w_i + v_i)d_{i+1,n}^*$$
$$+(1-\theta)^1\theta^4 - d_{i+1,n}^*w_il + (3w_i + v_i)f_{i+1,n}^* + \theta^5w_it_{i+1,n}^*.$$

Remark 3.1. *If the scaling factor $\sigma_i = 0$, the rational quintic fractal interpolation function becomes the classical rational quintic function.* $\Phi(x) = \frac{p_i(\alpha)}{q_i(\alpha)}$ *where*

$$p_i(\alpha) = (1-\alpha)^5U_0' + (1-\alpha)^4\theta U_1' + (1-\alpha)^3\alpha^2U_2' + (1-\alpha)^2\alpha^3U_3'$$
$$+(1-\alpha)^1\alpha^4U_4' + \alpha^5U_5'$$
$$q_i(\alpha) = u_i(1-\alpha)^2 + v_i(1-\alpha)\alpha + w_i\alpha^2,$$
$$\alpha = \frac{x - x_i}{x_{i+1} - x_i}, x \in [x_i, x_{i+1}]$$

with

$$U_0' = u_i t_i,$$
$$U_1' = a_i l u_i d_i + (3u_i + v_i)t_i$$
$$U_2' = 0.5u_i l^2 D_i a_i^2 + t_i(3u_i + 3v_i + w_i) + l d_i a_i(2u_i + v_i)$$
$$U_3' = 0.5w_i l^2 D_{i+1} a_i^2 + t_{i+1}(u_i + 3v_i + 3w_i) - l(2w_i + v_i)d_{i+1}a_i$$
$$U_4' = -a_i d_{i+1} w_i l + (3w_i + v_i)t_{i+1}$$
$$U_5' = w_i t_{i+1}.$$

4. Positivity of RQFIF

In this section, we obtained the sufficient conditionsfor the C^2- rational quintic fractal interpolation functions to preserve the positivity of the given set of data.

Suppose $(x_i, t_i), i = 1, 2, 3..., n$ be a given set of positive data. Let d_i be the derivative value at the knot x_i. The RQFIF is positive if $\psi(x) > 0$ for all $x \in [x_1, x_n]$.

Let $\sigma_i > 0 \forall i = 1, 2, 3...n$. Also $u_i > 0, v_i > 0$ and $w_i > 0$ gives $q_i(\theta) > 0$. So $\psi(L_i(x)) > 0 \forall$i=1,2,3...n, if $p_i(\theta) > 0$. Now we have,

$$p_i(\theta) = (1-\theta)^5 U_0 + (1-\theta)^4 \theta U_1 + (1-\theta)^3 \theta^2 U_2 + (1-\theta)^2 \theta^3 U_3 + (1-\theta)^1 \theta^4 U_4 + \theta^5 U_5$$

$p_i(\theta) > 0$ if $U_0 > 0, U_1 > 0, U_2 > 0, U_3 > 0, U_4 > 0, U_5 > 0$

we get

$U_0 > 0$ if $\sigma_i < \frac{t_i}{t_1}$ and $U_5 > 0$ if $\sigma_i < \frac{t_{i+1}}{t_n}$,

$U_1 > 0$ if $v_i > \frac{-u_i l d^*_{i,1}}{t^*_{i,1}}$,

For $3t^*_{i,1} + l d^*_{i,1} > 0, U_2 > 0$ if $v_i > \frac{-u_i l(4d^*_{i,1} + lD^*_{i,1})}{2(3t^*_{i,1} + l d^*_{i,1})}$,

For $3t^*_{i,1} + l d^*_{i,1} < 0, U_2 > 0$ if $v_i < \frac{-u_i l(4d^*_{i,1} + lD^*_{i,1})}{2(3t^*_{i,1} + l d^*_{i,1})}$,

For $3t^*_{i+1,n} - l d^*_{i+1,n} > 0, U_3 > 0$ if $v_i > \frac{w_i l(4d^*_{i+1,n} - lD^*_{i+1,n})}{2(3t^*_{i+1,n} - l d^*_{i+1,n})}$,

For $3t^*_{i+1,n} - l d^*_{i+1,n} < 0, U_3 > 0$ if $v_i < \frac{w_i l(4d^*_{i+1,n} - lD^*_{i+1,n})}{2(3t^*_{i+1,n} - l d^*_{i+1,n})}$, $U_4 > 0$ if $v_i > \frac{w_i l d^*_{i+1,n}}{t^*_{i+1,n}}$.

From the above results, it is clear that $\psi(L_i(x)) > 0$ for all $i = 1, 2, 3, \ldots, n$.

The above result discussion is summarized in the following theorem.

Theorem 4.1. *The sufficient conditions for the C^2 rational quintic fractal interpolation function to preserve the positive shape of data if in each subinterval shape parameters u_i, v_i, w_i satisfy the following constraints:*

$0 \leq \sigma_i < min\{a_i, \frac{t_i}{t_1}, \frac{t_{i+1}}{t_n}\}$
$u_i > 0, w_i > 0,$
$v_i > max\{0, v_{1,i}, v_{2,i}, v_{3,i}, v_{4,i}\}$

where

$$v_{1,i} = \frac{-u_i l d^*_{i,1}}{t^*_{i,1}}, \quad v_{2,i} = \frac{-u_i l(4d^*_{i,1} + lD^*_{i,1})}{2(3t^*_{i,1} + ld^*_{i,1})},$$

$$v_{3,i} = \frac{w_i l(4d^*_{i+1,n} - lD^*_{i+1,n})}{2(3t^*_{i+1,n} - ld^*_{i+1,n})}, \quad v_{4,i} = \frac{w_i l d^*_{i+1,n}}{t^*_{i+1,n}},$$

for all $i = 1, 2, 3...n.$

5. MONOTONICITY OF RQFIF

In this section, we have presented the monotonicity conditions for the C^2-rational quintic fractal interpolation functions. Suppose $\{(x_i, t_i), i = 1, 2, 3..., n\}$ be a given set of positive data. Let d_i be the derivative value at the knot x_i. Without loss of generality let the data be the monotonically increasing i.e, $t_1 \leq t_2 \leq ...t_n$. Then $\frac{t_{i+1}-t_i}{x_{i+1}-x_i} \geq 0, \forall i = 1, 2...n$. ψ is monotonically increasing in $[x_1, x_n]$ if $\psi'(x) \geq 0 \forall x \in [x_1, x_n]$. We have,

$$\psi'(L_i(x)) = \sigma_i/a_i\psi'(x) + \frac{\sum_{j=1}^{7} F_{j,i}\theta^{j-1}(1-\theta)^{7-j}}{l(q_i(\theta))^2}, \tag{5.1}$$

where

$F_{1,i} = a_iU_1 - (3u_i + v_i)U_0$
$F_{2,i} = 2u_iU_2 - 2v_iU_1 - (4v_i + 2w_i)U_0$
$F_{3,i} = (v_i - u_i)U_2 + 3u_iU_3 - 5w_iU_0 - (3v_i + w_i)U_1$
$F_{4,i} = 2v_iU_3 + 4u_iU_4 - 4v_iU_2 - 4w_iU_1$
$F_{5,i} = (u_i + 3v_i)U_4 + 5u_iU_5 - 3w_iU_2 + (-v_i + w_i)U_3$
$F_{6,i} = 2w_iU_4 + (4v_i + 2u_i)U_5 - 2w_iU_3$
$F_{7,i} = (v_i + 3w_i)U_5 - w_iU_4.$

Let $\sigma_i > 0 \forall$ i=1,2,3...n. It can be seen that $(q_i(\theta))^2 > 0$. Therefore $\psi'(L_i(x)) \geq 0, if \sum_{j=1}^{7} F_{j,i}\theta^{j-1}(1-\theta)^{7-j} \geq 0,$
$\sum_{j=1}^{7} F_{j,i}\theta^{j-1}(1-\theta)^{7-j} \geq 0 if F_{j,i} \geq 0 \forall j = 1, 2, 3, 4, 5, 6, 7.$

We have

$F_{1,i} \geq 0 if \sigma_i \leq \frac{a_i d_i}{d_1}, F_{7,i} \geq 0 if \sigma_i \leq \frac{a_i d_{i+1}}{d_n}.$

Now consider

$F_{2,i} = F_{1,i} + u_i l(d^*_{i,1}(u_i + 2v_i) + u_i lD^*_{i,1}).$

If $D^*_{i,1} = a_i^2 D_i - \sigma_i D_1 \geq 0$, then it is clear that $F_{2,i} \geq 0$. Otherwise choose the parameter $v_i \geq \frac{-lu_i D^*_{i,1}}{2d^*_{i,1}}$, and so, $F_{2,i} \geq 0$, provided that $d^*_{i,1} \geq 0$.

Similarly, consider

$$F_{6,i} = F_{7,i} + w_i l(d^*_{i+1,n}(w_i + 2v_i) - w_i l D^*_{i+1,n}).$$

If $D^*_{i+1,n} = a_i^2 D_{i+1} - \sigma_i D_n \geq 0$, then it is clear that $F_{6,i} \geq 0$. Otherwise choose the parameter $v_i \geq \frac{-lw_i D^*_{i+1,n}}{2d^*_{i+1,n}} \Rightarrow F_{6,i} \geq 0$, provided that $d^*_{i+1,n} \geq 0$. Now take

$$F_{3,i} = (9w_iu_i + 9v_iu_i + 3u_i^2)(t^*_{i+1,n} - t^*_{i,1}) + 3u_i l(0.5w_i l D^*_{i+1,n} - (2w_i + v_i)d^*_{i+1,n})$$
$$+ l(v_i - u_i)(0.5u_i l D^*_{i,1} + (2u_i + v_i)d^*_{i,1}) - lu_i(3v_i + w_i)d^*_{i,1})$$
$$F_{3,i} \geq 0 if \sigma_i \leq (t_{i+1} - t_1)/(t_n - t_1),$$

and

$$v_i \geq \max\{(-w_i \frac{lD^*_{i+1,n} - 4d^*_{i+1,n})}{2d^*_{i+1,n}}, \frac{-u_i(lD^*_{i,1} + 4d^*_{i,1})}{2d^*_{i,1}}\}.$$

Now take

$$F_{5,i} = (9w_iu_i + 9v_iw_i + 3w_i^2)(t^*_{i+1,n} - t^*_{i,1}) + 3w_i l(-0.5u_i l D^*_{i,1} - (2u_i + v_i)d^*_{i,1})$$
$$+ l(w_i - v_i)(0.5w_i l D^*_{i+1,n} - (2w_i + v_i)d^*_{i+1,n}) - lw_i(3v_i + u_i)d^*_{i+1,n})$$
$$F_{5,i} \geq 0 if \sigma_i \leq (t_{i+1} - t_1)/(t_n - t_1),$$

and

$$v_i \geq \max\{\frac{-w_i(lD^*_{i+1,n} - 4d^*_{i+1,n})}{2d^*_{i+1,n}}, (u_i \frac{lD^*_{i,1} + 4d^*_{i+1,n})}{2d^*_{i+1,n}}\}.$$

Now take

$$F_{4,i} = (12w_iu_i + 6v_iw_i + 6u_iv_i + 3v_i^2)(t^*_{i+1,n} - t^*_{i,1})$$
$$+ v_i l(-u_i l D^*_{i,1} - (2u_i + v_i)d^*_{i,1}) + lv_i(w_i l D^*_{i+1,n} - (2w_i + v_i)d^*_{i+1,n})$$
$$-4lw_iu_i(d^*_{i+1,n} + d^*_{i,1})$$
$$F_{4,i} \geq 0 if \sigma_i \leq (t_{i+1} - t_1)/(t_n - t_1),$$

and

$$v_i \geq \max\{(-w_i \frac{lD^*_{i+1,n} - 4d^*_{i+1,n})}{2d^*_{i+1,n}}, \frac{-u_i(lD^*_{i,1} + 4d^*_{i,1})}{2d^*_{i,1}}\}.$$

From the above results, it is clear that $\psi'(x) \geq 0$ for all $i = 1, 2, 3, \ldots, n$.

The above result discussion is summarized in the following theorem.

Theorem 5.1. *The sufficient conditions for the C^2 rational quintic fractal interpolation function to preserve the monotonicity of data if in each subinterval shape parameters u_i, v_i, w_i satisfy the following constraints:*

$$0 \leq \sigma_i < \min\{a_i, \frac{a_i d_i}{d_1}, \frac{a_i d_{i+1}}{d_n}, \frac{t_{i+1} - t_1}{t_n - t_1}\}$$
$$u_i \geq 0, w_i \geq 0,$$

and $v_i > \max\{0, v_(i_2), v_(i_3,)v_(i_4), v_(i_5), v_{i_6}\}$*, where*

$$v_(i_2) = \frac{-lu_i D^*_{i,1}}{2d^*_{i,1}},\ v_(i_3) = \max\{\frac{-w_i(lD^*_{i+1,n} - 4d^*_{i+1,n})}{2d^*_{i+1,n}}, \frac{-u_i(lD^*_{i,1} + 4d^*_{i,1})}{2d^*_{i,1}}\},$$

$$v_(i_4) = max\{\frac{-w_i(lD^*_{i+1,n}-4d^*_{i+1,n})}{2d^*_{i+1,n}}, \frac{-u_i(lD^*_{i,1}+4d^*_{i,1})}{2d^*_{i,1}}\},$$
$$v_(i_5) = max\{\frac{-w_i(lD^*_{i+1,n}-4d^*_{i+1,n})}{2d^*_{i+1,n}}, \frac{u_i(lD^*_{i,1}+4d^*_{i+1,n})}{2d^*_{i+1,n}}\},\ v_{i_6} = \frac{-lw_iD^*_{i+1,n}}{2d^*_{i+1,n}},$$
$i = 1, 2, 3...n.$

6. Conclusion

In this paper, the C^2- rational quintic fractal interpolation functions having the three shape parameters is constructed. The constructed RQFIF can be used to preserve the shapes of the given data. The sufficient positive shape preservation conditions for RQFIF are developed in this paper. Further the sufficient monotonicity preserving conditions for RQFIF are developed. The RQFIF becomes the classical rational quantic interpolant if we take scaling factor is zero. This developed RQFIF can be used in various fields of visualization of data.

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