

DEVELOPMENTS IN THEORY OF THERMO ELASTICITY

MANDEEP SINGH¹ AND SANGEETA KUMARI

ABSTRACT. The Present review article is about the developments in the theory of thermoelasticity. The main aim of this paper to go through the Preliminaries of this theory and various progress made in this area. It includes: a) uncoupled thermoelasticity (Duhamel, 1837); b) Coupled thermoelasticity (Biot, 1956); c) Generalized thermoelasticity. The term Generalized thermoelasticity referred to as hyperbolic thermoelasticity that include L-S model (Lord-Shulman, 1967), G-L model (Green-Lindsay, 1972), G-N model (Green-Lindsay, 1993), Dual Phase Lag model (Tzou, 1995), H-I model (Hetnarski-Ignaczak, 1996), C-T model (Chandrasekharaiah and Tzou, 1998), Three Phase Lag model (Roy, 2007). The focus is on theoretical significance of these model.

1. INTRODUCTION

Elasticity is one of vital branches of continuum mechanics which deals with the study of every aspect of response in elastic bodies under the action of external forces. These responses are portrayed by strain and stress distributions that developed in elastic bodies when these are subjected to external forces or difference in temperature. If forces causing deformation in the body removed and body returns to its original shape, then it is called elastic body. Each body has particular characteristics of its material limitation beyond which deformations cannot be exceeded. There are certain mathematical relationships that associate forces and deformations in the body that determines the elastic property of the

¹*corresponding author*

2010 *Mathematics Subject Classification.* 74B99.

Key words and phrases. Hyperbolic Equations, Parabolic Equations, Dual Phase Lag, Three Phase Lag.

body. A linear elastic solid undergoes infinite deformation under the effect of external forces and obeys linear material law.

The theory of linear elasticity developed by Robert Hook (1678), who studied the relationship between deformation and applied forces known as Hooke's law. This Law states that applied forces(F) is directly proportional to displacement or change in length x . Mathematically it is written as $F = kx$. Here the Constant of proportionality not only depends upon dimensions and shape of material but also on the elastic property of the material. In other words ,it also give the relationship between stress and strain. The theory proposed by Robert Hook immensely influence the scientist to solve the problems like beams structure, column stability, bending and vibrations of plates. This classical theory of elasticity provides a fine model for indepth study of material property of various elastic solids that used mechanical and civil engineering design.

In 1821, Navier made first attempt to propose the differential equations for the displacement's components inside isotropic elastic solids. Navier Stokes's equation in modern notation is:

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{\nabla P}{\rho} + \nu \nabla^2 u,$$

where u represents fluid velocity vector, P represents fluid pressure, ρ is density of fluid, ν is kinematic viscosity and ∇^2 is Laplacian operator.

With the help of virtual work given by Lagrange's principle, he developed equations for its equilibrium position on solid's surface.

Thermoelasticity: Thermoelasticity is the fusion of theory of elasticity and thermal effects. It deals with influence of thermal disturbances on strain distribution of an elastic solids and gives rise to association between deformation and thermal fields. Due to heat flow in elastic body, stress and strain are developed in body. Stress produced by temperature field can be calculated by using theory of thermoelasticity.

The body shows change in temperature not only thorough external and internal heat sources but also during the process of deformations. In coupled thermo elastic equations, equation of motion comprised inertia terms are employed. In most of elasto-dynamic problems following assumptions have been made:

- (a) It is assumed that all elastic properties of materials are identical in all directions and remain same for most of the time.

- (b) The influence of coupling of temperature and strain fields are considered in determining temperature field.
- (c) The deformation in the body is considered negligible.

The theory of Thermoelasticity has been adopted by various engineering sciences. With the developments of aircraft and machine structures various problems of thermal stresses have been arisen.

During the last Two decades, Various theories on Thermoelasticity have been formulated and developments of these theories have been discussed in this topic.

Uncoupled Thermoelasticity: In the nineteenth Century, Duhamel and Neumann formulated the theory of uncoupled thermoelasticity which states that with the change of mechanical state of elastic body, there is no effect on temperature of the body which was not in accordance with the physical experiments. The major drawback of this theory is that the temperature governing equation is of parabolic form which predicts the thermal signals of infinite speed and the thermal disturbances can be felt far away from its source.

Coupled Thermoelasticity: To overcome the drawbacks of uncoupled thermoelasticity. In 1956, Biot [1, 2] formulated the theory of coupled thermoelasticity with coupling equations of elasticity and heat conduction. It includes the theory of heat conduction, thermal strain and stress arises due to heat flow in elastic body and deformation causes the reverse effect of temperature distribution that results in thermoelastic dissipation. It also introduces the concept of thermoelastic potential which represents the elastic and thermodynamic properties of the medium. In this theory concept of thermal force is introduced in which generalized force is considered as the product of temperature and virtual entropy displacement. In spite of formulation of coupled thermoelasticity, this theory shares the drawback of infinite speed of thermal disturbances felt at infinite distance from the source that already exist in uncoupled thermoelasticity. In Boley [3] studied the mechanical and thermal disturbances by solving the boundary value problems in half space using coupled thermoelasticity. The problems include the variations in step time strain and stress is uniformly distributed over the free surface. Other Contributions in this theory are attributed to Nowacki [4].

Generalized Thermoelasticity: The major drawback thermal signals of infinite speed was removed in non-classical theories. These theories employed the modified form of Fourier law of heat conduction and in result hyperbolic type heat transport equations are obtained that emit thermal signals of finite speed. All these non-classical theories are termed as Generalized theories.

1. Lord-Shulman(L-S) model of Thermoelasticity: Lord-Shulman (L-S) [5] model is the extension of coupled thermoelasticity which used coupling of temperature strain rate and heat conduction Fourier law is taken over by law of Maxwell-Cattaneo that uses the generalised form of Fourier law with the induction of single relaxation time.

Linear form of governing equations: Consider a elastic solid is having thermal conduction is subjected to negligible strain and small change in temperature. Therefore the energy equation given as

$$\sigma_{ij}(e_{ij}) + \rho T \dot{S} = \rho \dot{e} \quad (1)$$

$$-q_{i,i} = \rho T \dot{S} \quad (2)$$

Here σ_{ij} represents stress tensor, e_{ij} represents strain tensor, ρ represents mass density, e represents internal energy density, T is the absolute temperature, S represents entropy density, q_i represents heat flux vector if τ_0 be the relaxation time or time lag that represents the time needed to accelerate the heat flow, the relationship between q_i and $T_{,i}$ is obtained as follows

$$q_i + \tau_0 \dot{q}_i = -k T_{,i}. \quad (3)$$

it represents the modified form of heat conduction law. The Constitutive equation taken of the form:

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - (3\lambda + 2\mu)\alpha(T - T_0)\delta_{ij}. \quad (4)$$

Assuming the specific heat c_E

$$k T_{,ii} = \rho c_E (\dot{T} + \tau_0 \ddot{T}) + (3\lambda + 2\mu)\alpha T_0 (e_{kk} + \tau_0 \ddot{e}_{kk}). \quad (5)$$

The equation of motion of linear elastic continuum

$$\rho \ddot{u}_i = (\lambda + \mu) u_{i,ij} + \mu u_{i,ij} - (3\lambda + 2\mu)\alpha T_{,i} \quad (6)$$

Here u_i is the displacement vector.

2. Green-Lindsay(G-L) model of Thermoelasticity: In Green -Lindsay[6] formulated a new theory thermoelasticity with the introduction of concept of entropy production inequality (as proposed by Green and Laws[7]) in the constitutive relations. The theory provides the theoretical proof of uniqueness theorem.

The expressions for stress tensor, heat conduction vector and entropy in the context of two scalar function is obtained.

As Green and Laws suggest the entropy production inequality

$$\frac{d}{dt} \int \rho_0 \eta dV + \int \left(\frac{Q_A}{\emptyset} + \lambda_A \right) N_A dA - \int \left(\frac{\rho_0 r}{\emptyset} - \mu \right) dV \geq 0 \quad (7)$$

for all volume P in the reference configuration. Here η is specific entropy, $\emptyset$ is scalar function, μ and λ_A scalar and vector function subject to the condition with the help of entropy inequality in

$$\rho_0 r - Q_{k,k} - \rho_0 \epsilon + \dot{S}_{AB} \dot{e}_{AB} = 0 \quad (8)$$

we can obtain

$$-\rho_0(\psi + \eta\varphi) + \varphi(\mu + \lambda_{A,A}) + s_{AB} e_{AB} - \frac{Q_A \emptyset_{,A}}{\emptyset} \geq 0. \quad (9)$$

where $\psi = \epsilon - \eta\varphi$, ρ_0 represent reference mass density, $Q_{k,k}$ heat flux vector per unit of e_k -plane, r is the external supplied specific heat, ϵ is specific internal energy.

If x_i represent the point in reference configuration of the body, x_i is set as $x_i = X_i + u_i$ at the temperature $\theta_0 + \theta$ where θ_0 is constant. Then $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ representor the stress tensor and has zer heat flux, we have represent stress tensor and has zero flux. we have

$$\sigma = \sigma_0 - a\theta - b\dot{\theta} - \frac{1}{2}d\theta^2 - e\dot{\theta}\theta - \frac{1}{2}f\dot{\theta}^2 + a_i\theta_{,i} + \alpha b_i\dot{\theta}\theta_{,i} + a_{ij}e_{jk}\theta_{jk} \quad (10)$$

$$+ a_{ik}e_{ik}\theta + b_{ik}e_{ik}\dot{\theta} + \frac{1}{2}\alpha k_{rj}\theta_{,r}\theta_{,j} + \frac{1}{2}k_{ikrj}e_{ik}e_{rj}. \quad (11)$$

All the constants in above equation have symmetries in the tensor indices and non-isotropic body have constants

$$t_{ik} = K_{ikrj}e_{rj} + a_{ik}\theta + b_{ik}\dot{\theta} + a_{ikr}\theta_{,r} \quad (12)$$

$$q_i = \frac{-\theta_0}{\alpha} [a_i\theta + \alpha b_i\dot{\theta} + a_{rs}e_{rs} + \alpha K_{ij}\theta_{,j}] \quad (13)$$

t_{ik} represent stress at heat flux components denote by q_i measured per unit area of coordinate plane in reference body.

$$\rho_0 \eta = \frac{1}{\alpha} [b + (e - \frac{b\beta}{\alpha})\theta + (f - \frac{b\gamma}{\alpha})\dot{\theta} - \alpha b_i\theta_{,i} - b_{ij}e_{ij}]. \quad (14)$$

Using the entropy inequality, then the coefficients can be obtained as follows:

$$a_i = 0, a_{ij}k = 0 \quad (15)$$

$$b = \alpha a, b_{ij} = \alpha a_{ij}, e - a\beta = d\alpha, (d\alpha - h)\dot{\theta}^2 + 2b_i\dot{\theta}\theta_{,i} + K_{ij}\theta_{,i}\theta_{,j} \geq 0. \quad (16)$$

Here $h\alpha = f - \frac{b\gamma}{\alpha}$.

3. Green Nagadhi (G-N) Theory of Thermoelasticity: Green-Nagadhi formulated three theories of thermoelasticity known as GN-I,GN-II,GN-III theory of thermoelasticity.

Green-Nagadhi-I(GN-I) : GN-I [8] theory refers to classical theory of thermoelasticity in its linearised version. In this theory the basic postulates and governing equations of thermomechanics are re-examined and procedure of developing the main postulates in the context of heat flow in solid is studied thoroughly.

Green-Nagadhi-II(GN-II): GN-II [9] theory is commonly known as GN model of thermoelasticity without energy dissipation. In this theory fourier law in heat conduction is replaced by the relation

$$\dot{q} = -K_1^* \nabla T, \quad (17)$$

where q and T represent heat flux vector and temperature change fields respectively and K_1^* represent the positive definite tensor having dimension $[K_1^*] = [K_1 T_0^{-1}]$ and T_0 is unit of time and K_1 as heat conduction.

Therefore thermoelastic wave propagating in G-N model may be defined in terms of (V, T) satisfy the conditions:

$$\nabla \cdot C[\nabla V] - \rho(V + \nabla \cdot (\ddot{T}M)) = -b \text{ on } U \times [0, \infty) \quad (18)$$

$$\nabla \cdot (K_1^* \nabla T) - C_E \ddot{T} + T_0 M \cdot \nabla V \ddot{} - \dot{r} \text{ on } U \times [0, \infty). \quad (19)$$

Satisfying the initial conditions as:

$$V(., 0) = V_0 \text{ and } \dot{V}(., 0) = \dot{V}_0 \text{ and } T(., 0) = T_0 \text{ and } \dot{T}(., 0) = \dot{T}_0 \text{ on } U. \quad (20)$$

The boundary conditions are

$$V = V' \text{ and } T = T' \text{ on } \partial U \times [0, \infty). \quad (21)$$

Here V represents displacement vector and U as elastic heat flow vector.

Equation (19) represent the undamped thermoelastic waves as it does not contain change of temperature component. Therefore it is known as GN theory without energy dissipation. **Green-Nagadhi-III(GN-III):** (GN-III) [10] theory is commonly known as GN model of thermoelasticity with energy dissipation.

4. Dual Phase Lag of Thermo elasticity: In 1995, D.Y. Tzou [11] formulated Dual phase lag concept in heat transfer using delay in heat flux and temperature gradient that involves small scale response in space. In this theory, microstructural influence into time delayed response in microscopic formulation lag arises due to phonon-electron interactions at microscopic level. Due to such interactions two phase lag like heat flux vector (τ_q) and temperature gradient (τ_T) is considered as delayed response.

Consider the equation that shows lagging behaviour in heat transfer. This equation represent

$$q(P, t + \tau_q) = -K \nabla T(P, t + \tau_T). \quad (22)$$

The temperature gradient (τ_T) is generated across volume located at a point P at the particular time $t + \tau_T$ result in the heat flow at particular instant of time $t + \tau_q$ where both the lags are positive by expanding the equation (22) as Taylor series with respect to t . The equation can be transformed to

$$q(P, t) + \tau_q \frac{\partial q}{\partial t}(P, t) = -K[\nabla T(P, t) + \tau_T \frac{\partial}{\partial t}(\nabla T(P, t))]. \quad (23)$$

The equation (23) can be combined with energy equation

$$-\nabla \cdot q(P, t) = C_p \frac{\partial T}{\partial t}(P, t). \quad (24)$$

By eliminating the heat flux vector from equation (23) combining with (24)

$$\nabla^2 T + \tau_T \frac{\partial}{\partial t}(\nabla^2 T) = \frac{1}{\alpha} \frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha} \frac{\partial^2 T}{\partial t^2}. \quad (25)$$

By eliminating the temperature, results in the following equation:

$$\nabla(\nabla \cdot q) + \tau_T \frac{\partial}{\partial t}(\nabla(\nabla \cdot q)) = \frac{1}{\alpha} \frac{\partial q}{\partial t} + \frac{\tau_q}{\alpha} \frac{\partial^2 q}{\partial t^2}. \quad (26)$$

At the condition $\tau_T = 0$ both the equations (25) and (26) represents the thermal wave equation. In case of $\tau_q = \tau_T = 0$ equation (22) reduce to fourier law in heat conduction and diffusion equation shown by (25). Microscopic influence becomes vital when the response time reduce to zero. The dual phase lag provide simple transition from macroscopic analysis to microscopic analysis.

5. H-I thermoelastic model at low temperature: In 1996, Hetnarski and Ignaczak [12-14] proposed a thermoelastic model for soliton like thermoelastic waves at low temperature. The main feature of this model is that heat flux vector and free energy depends upon absolute temperature, strain tensor and heat flow in elastic medium satisfying evolution equation.

Consider thermoelastic wave flows in homogenous isotropic body satisfied by the (T, V, U) on $\bar{U} \times [0, \infty)$ and satisfy the field equations in non dimensional form on $U \times [0, \infty)$:

$$\nabla^2 T - \dot{T} + U \cdot \frac{\nabla T}{T} - \nabla \cdot U - T \nabla \cdot \dot{V} = -R \quad (27)$$

$$\nabla(\nabla \cdot V) - k_1(\nabla \times (\nabla \times V)) - r^2 \ddot{V} - \epsilon^* \nabla T = -F \quad (28)$$

$$\omega \dot{U} + \frac{nabla T}{T} = 0 \quad (29)$$

with the initial condition

$$T(., 0) = T_0, U(., 0) = U_0, V(., 0) = V_0 \text{ and } \dot{V}(., 0) = (\dot{V}_0) \text{ on } U;$$

and the boundry condition

$$V = \dot{V} \text{ on } \partial U_1 \times (0, \infty)$$

$$[2k_1(\hat{\nabla} V) + (1 - 2k_1)(\nabla \cdot V)1 - \epsilon^*(T - 1)1]N = \hat{S} \text{ on } \partial U_2 \times (0, \infty)$$

$$T = \hat{T} \text{ on } \partial U_3 \times (0, \infty)$$

$$(-\nabla T + U) \cdot \hat{N} = \hat{Q} \text{ on } \partial U_4 \times (0, \infty).$$

Here T , V , U represent absolute temperature, displacement vector and elastic heat flow vector respectively. ω , ϵ^* and r represent parameter at low temperature ($\omega \ll 1$). Here coupling constant in terms poisson ratio (v_1) is defined as $k_1 = \frac{1-2v_1}{1+2v_1}$; $\hat{\nabla}$ represent the symmetric gradient and $\hat{N}$ is unit vector normal to the surface ∂U at 1 as the unit tensor, $R, F, T_0, U_0, V_0, \dot{V}_0, \hat{V}, \hat{S}, \hat{T}$ and $\hat{Q}$ are the function.

In case of absence of body force and heat supply fields (27) can be converted into dimensionless equation

$$\left[\frac{\partial^2 \emptyset}{\partial t^2} - \frac{\partial^3 \emptyset}{\partial x^2 \partial t} + \omega \frac{\partial^2 \emptyset}{\partial x \partial t} \right] e^{(-\omega \frac{\partial \emptyset}{\partial t})} - \frac{\partial \emptyset}{\partial x} \frac{\partial^2 \emptyset}{\partial x \partial t} - \omega^{-1} \frac{\partial^2 \emptyset}{\partial x^2} - \omega^{-1} \frac{\partial^2 u}{\partial x \partial t} e^{(\omega \frac{\partial \emptyset}{\partial t})} = 0 \quad (30)$$

$$\omega^{-1} \left[\frac{\partial^2 u}{\partial x^2} - r^2 \frac{\partial^2 u}{\partial t^2} \right] + \epsilon^* \frac{\partial^2 \emptyset}{\partial x \partial t} e^{(-\omega \frac{\partial \emptyset}{\partial t})} = 0. \quad (31)$$

$\emptyset$ and u denote heat flow potential and displacement respectively in x -direction at time t .

$$T = e^{(-\omega \frac{\partial \emptyset}{\partial t})} \quad (32)$$

$$q = \frac{\partial \emptyset}{\partial x} + \omega \frac{\partial^2 \emptyset}{\partial x \partial t} \quad (33)$$

$$\sigma = \frac{\partial u}{\partial x} - \epsilon^* (T - 1) \quad (34)$$

Here T , q , σ represent the absolute temperature, total heat flux and stress respectively.

The equations (30) and (31) represent energy flow in balance form and equation of motion respectively. The value of T is adjusted in such a way that it satisfy (29). Equation (34) represent modified fourier law. Eqs(30)-(34) represent non rigid conductor at low temperature at the condition $u=0$ at $\epsilon^* = 0$.

6. C-T Model of thermoelasticity: In 1998, Chandrasekharaiah and Tzou [11,15-16] formulated the theory by modification of Classical thermoelastic model in which fourier law is used in its modify form as: $q(x, t + \tau_q) = -K \nabla T(x, t + \tau_T)$ for every point (x, t) belongs to $\bar{U} \times [0, \infty)$. Here $\bar{U}$ is the heat flow vector where τ_q represent phase-lag of heat flux, τ_T represent phase-lag of temperature gradient. This model propagates the wave in thermoelastic medium if above equation is approximated as Taylor series:

$$q + \tau_q \frac{\partial q}{\partial t} = -K \nabla (T + \tau_T \frac{\partial T}{\partial t}) \text{ here } 0 \leq \tau_T \leq \tau_q \quad (35)$$

or by

$$q + \tau_q \frac{\partial q}{\partial t} + \frac{1}{2} \tau_q^2 \frac{\partial^2 q}{\partial t^2} = -K \nabla (T + \tau_T \frac{\partial T}{\partial t}) \text{ here } \tau_T > 0, \tau_q > 0. \quad (36)$$

Equation (35) represent energy balance equation of a rigid heat conductor that leads to heat conduction in Hyperbolic form.

7. Three Phase Lag model of thermoelasticity: In 2007, Roy [17-19] proposed a thermoelastic model that involves the phase lag of thermal displacement gradient, heat flux and temperature gradient. As the energy equation for homogenous isotropic thermoelastic medium is given as:

$$-\nabla \cdot \vec{q} + \rho Q_1 = \rho C_v \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial \Delta}{\partial t}, \quad (37)$$

where Q_1 is source of heat per mass per unit time, C_v denotes specific heat, ρ denotes the density of the medium. The parameter $\gamma = (3\lambda + 2\mu)\alpha(t)$, λ, μ are lame constants and $\alpha(t)$ is the coefficient of linear form thermal expansion, and Δ is the dilation term.

Taking the divergence on the both sides to the equation

$$\vec{q} + \tau_q \frac{\partial \vec{q}}{\partial t} = -[\tau_v^* \nabla T + K \tau_T \frac{\partial \nabla T}{\partial t} + K^* \nabla V], \quad (38)$$

here $\tau_v^* = K + K^* \tau_v$, K represent thermal conductivity of the material and K^* represent material constant characteristic, we obtain

$$\nabla \cdot \vec{q} + \tau_q \frac{\partial}{\partial t} (\nabla \cdot \vec{q}) = -[\tau_v^* \nabla^2 T + K \tau_T \frac{\partial \nabla^2 T}{\partial t} + K^* \nabla^2 V]. \quad (39)$$

Take the time differential to this equation and use $\frac{\partial V}{\partial t} = T$. We have

$$\nabla \cdot \frac{\partial \vec{q}}{\partial t} + \tau_q \frac{\partial^2 (\nabla \cdot \vec{q})}{\partial t^2} = -[\tau_v^* \nabla^2 \frac{\partial T}{\partial t} + K \tau_T \frac{\partial^2 \nabla^2 T}{\partial t^2} + K^* \nabla^2 T]. \quad (41)$$

Differentiating the energy equation with respect to t and eliminate $\nabla \cdot \frac{\partial \vec{q}}{\partial t}$, therefore the above equation becomes

$$(1 + \tau_q \frac{\partial}{\partial t} (\rho C_v \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^2 \Delta}{\partial t^2} - \rho \frac{\partial Q_1}{\partial t}) = \tau_v^* \nabla^2 \frac{\partial T}{\partial t} + K \tau_T \nabla^2 \frac{\partial^2 T}{\partial t^2} + K^* \nabla^2 T. \quad (42)$$

As the displacement equation of motion in thermoelastic medium

$$\mu \nabla^2 \vec{u} + (\lambda + \mu) \vec{\nabla} (\vec{\nabla} \cdot \vec{u}) - \gamma \vec{\nabla} T + \rho \vec{F} = \rho \frac{\partial^2 \vec{u}}{\partial t^2}. \quad (43)$$

Both the equations in coupling form formulated the thermoelastic theory for three phase lag model in isotropic material.

If K is quite large as compared to K^* or set $K^* = 0$, then the above equation is reduced to dual phase lag model of thermoelasticity.

Furthermore if $K \ll K^*$, then modified fourier law reduces to original as $\vec{q} = -K^* \nabla V$

$$\text{and, } (1 + \tau_q \frac{\partial}{\partial t} (\rho C_v \frac{\partial^2 T}{\partial t^2} + \gamma T_0 \frac{\partial^2 \Delta}{\partial t^2} - \rho \frac{\partial Q_1}{\partial t}) = K^* (1 + \tau_v \frac{\partial}{\partial t} \nabla^2 T. \quad (44)$$

In case of $\tau_q = \tau_v = \tau_T = 0$, the above equation reduces to Green Nagadhi model of thermoelasticity without energy dissipation.

2. CONCLUDING REMAKS

The main emphasis of this review article to look into various models in thermoelasticity that contribute immensely in the development of theory of thermoelasticity. It has been observed that at the initial stage of this theory, uncoupled

and coupled thermoelasticity plays an important role. The results of various models in generalized thermoelasticity has been discussed. The main contributor in this thermoelasticity are Lord-Shulman (L-S) model, Green-Lindsay (G-L) model, Green-Nagadhi(G-N)model, Dual Phase Lag(DPL model), H-I model,C-T model and Three Phase Lag (TPL) model.

1. L-S model and G-L models is applicable to the propagation of undamped thermoelastic waves,this results gives the motivation to Green-Nagadhi (G-N) model to formulate the thermoelastic model without energy dissipation.

2. C-T model and DPL model is the extension of L-S model that taken into account of phase lag of heat flux and phase lag temperature gradient that depends upon Taylor series approximation of modified fourier law of heat conduction.

3. H-I model is strongly non-linear which is applicable for soliton like thermoelastic waves only at low temperature and this model gives the motivation to other reasearchers in developing a new model that propagates classical thermoelastic waves.

4. TPL model is extension of DPL model as proposed by Tzou. TPL model consider the phase lag of heat flux, temperature gradient and thermal displacement gradient. In the absence of all these phase lag, this model reduces to G-N model without energy dissipation.

The results in this review article should be quite helpful for the reseachers working in the area in the development of theory of thermoelasticity and different fields like material science, design of materials.

REFERENCES

- [1] M.A. BIOT: *Thermoelasticity and Irreversible Thermodynamics*, J. Appl. Phys., **27** (1956), 40.
- [2] M.A. BIOT: *Theory of Propagation of Elastic Waves in Fluid-Saturated Porous Solid*, The J.of soc.of Acoustical society of America, **28** (1956), 168-177.
- [3] B.A. BOLEY, I.S. TOLINS: *Transient coupled thermoelastic boundry value problems in the half space*, Journal of applied mechanics, **18** (1962), 637-646.
- [4] 'NOWACKI: *problems of thermoelasticity*, Progress in aerospace sciences, **10** (1964), 1-63.
- [5] H. W. LORD AND Y. H. SHULMAN: *A Generalized Dynamical Theory of Thermoelasticity*, Mech.Phy.Solids, **15** (1967), 299-309.
- [6] A. E.GREEN AND K. A. LINDSAY: *Thermoelasticity*, J. Elasticity, **2** (1972), 1-7.
- [7] A. E.GREEN AND N.LAWS: *On the Entropy Production Inequality*, Arch.Ration. Mech.Anal., **45** (1972), 47-53.

- [8] A. E. GREEN AND P. M. NAGHDI: *A re-examination of basic postulates of thermomechanics*, Proc.R.Soc.London, **432** (1991), 171-194.
- [9] A. E. GREEN AND P. M. NAGHDI: *On Undamped Heat Waves in an Elastic Solid*, Thermal Stresses, **15** (1992), 253-264.
- [10] A. E. GREEN AND P. M. NAGHDI: *Thermoelasticity Without Energy Dissipation*, J. Elast., **31** (1993), 189-208.
- [11] D. Y. TZOU: *A Unified Field Approach for Heat Conduction from Macro to Micro-Scales*, ASME J.Heat Transfer, **117** (1995), 8-16.
- [12] R. B. HETNARSKI AND J. IGNACZAK: *Soliton-like Waves in a Low-temperature Nonlinear thermoelstic Solid*, Int. J. Engrg. Sci., **34** (1996), 1767-1787.
- [13] R. B. HETNARSKI AND J. IGNACZAK: *On Soliton-like Thermoelastic Waves*, Appl. Anal., **65** (1997), 183-204.
- [14] R. B. HETNARSKI AND J. IGNACZAK: *Soliton-like Waves in a Low-temperature non linear rigid heat Conductor*, Int. J. Engrg. Sci., **33** (1995), 1725-1741.
- [15] D. S. CHANDRASEKHARAIAH: *A Uniqueness Theorem in the Theory of Thermoelasticity without Energy Dissipation*, J. Thermal Stresses, **19** (1996), 267-272.
- [16] D. S. CHANDRASEKHARAIAH: *Hyperbolic Thermoelasticity; A Review of Recent Literature*, Appl. Mech. Rev., **51** (1998), 705-729.
- [17] S. K. ROY CHOUDHURI AND N. BANDYOPADHYAY: *Thermoelastic Wave Propagation in rotating Elastic Medium Without Energy Dissipation*, Int. Math. Math. Sci., **1** (2005), 99-107.
- [18] K. ROY CHOUDHURI AND P. S.DUTTA: *Thermoelastic Interaction Without Energy dissipation in an infinite Solid with Distributed Periodically Varying Heat Sources*, Int. Solids structures, (2005), 4192-4203.
- [19] S. K. ROY CHOUDHURI AND M. CHATTOPADHYAY: *Magneto-elastic Plane Waves in Rotating media in Thermoelasticity of Type II*, Int. J. Math. Math. Sci., **71** (2004), 917-929.

DEPARTMENT OF MATHEMATICS,
CHANDIGARH UNIVERSITY,
GHARUAN, MOHALI (PUNJAB)

DEPARTMENT OF MATHEMATICS,
CHANDIGARH UNIVERSITY,
GHARUAN, MOHALI (PUNJAB)