

DETERMINE THE VALUE OF STRESS AND STRAIN AT VARIOUS VALUES OF EXPONENTS

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ABSTRACT. In this present study value of stress as well as strain along tangential and radial direction of a rotating disk is calculated for different value of exponent, i.e., $n = 3, 5, 8$ by taking volume, temperature and particle size constant. It has been observed that by increasing the value of exponent the stress and strain decreases.

1. INTRODUCTION

Rotating disks have many engineering applications so rotating disks provide an area of research and studies such as steam, turbo generators, fly wheels, computer hard disk drives, pumps, compressors and aero plans [4, 6, 7, 8]. Material made from two or more material having different physical and chemical properties is called composite material [3]. The composite material have many applications in different field like automobiles are engine cylinder liners, combustion chamber, CNG storage cylinders, brake rotors, drive shafts, diesel energy pistons, flywheels, motorcycle, drive sprocket, pulley. In sub-marine are propulsion shaft, cylindrical pressure hull, composite piping systems, boats hull. In industrial and commercial are computer hard disk drive, needle for carpet-weaving machine, electronic packaging, thermal management, pressure vessels, wind turbine blades, laptop cases, electric motors, MRI scanner cryogenic tubes,

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Wheel chairs, Eyeglass frames, artificial ligaments, Hip joint implant, camera tripods, Drilling tubes, Musical instruments, Drilling motor shaft, X- ray table, Heart valves, Beams, Helmets. In aerospace tools, space structure as well as structures is rocket nozzle, Engines parts, Spacecraft truss structure, Wind tunnel blades, Solar panels, Reflectors, Turbine rotor, Turbine wheels and Space shuttle. In aircraft, Missile structures are wings, Engine casing, Rotary launchers, Drive shaft, Landing gear doors, Propeller blades, Helicopter components, Main rotor blades, and Mast mount. In sports are golf shafts, Tennis rackets, racing bicycle and fishing rod.

ESTIMATION OF CREEP PARAMETERS

The secondary stage deformation Al-SiC composite of different composition be describe in conditions of Sherby threshold stress based model known as

$$(1.1) \quad \dot{\epsilon} = [M (\sigma - \sigma_0)]^n ,$$

where,

$$M = \frac{1}{E} \left[\frac{AD}{|\vec{b}_r|'} \right]^{1/n} ,$$

σ is effective stress, "M" is material creep constant, D. lattice diffusivity, "A" constant, b magnitude of Burger's vector, λ sub grain size , * E* young's modulus, σ_0 threshold stress.

The standards creep parameter "M", σ_0 take from creep outcomes of Al-SiC composite disk given by (Pandey et. al, 1992)

$$(1.2) \quad \begin{aligned} \ln M &= -34.91 + 0.2112 \ln P + 4.89 \ln T - 0.591 \ln V \\ \sigma_0 &= -0.02050P + 0.01378T + 1.033V - 4.9695, \end{aligned}$$

where P is particle size, T is temperature and V is volume content.

MATHEMATICAL FORMULATION

Let aluminum silicon carbide particulate composite disk having inner radii and outer radii are "a" and "b" respectively.

Constitutive general ga^2 used for deformation in isotropic composite obtain in following type as suggestion frame is use as along principal directions r, θ and z

$$\dot{\epsilon}_r = \frac{\dot{\epsilon}}{2\sigma} [2\sigma_r - \sigma_\theta] \quad \dot{\epsilon}_\theta = \frac{\dot{\epsilon}}{2\sigma} [2\sigma_\theta - \sigma_r]$$

$$(1.3) \quad \dot{\varepsilon}_z = \frac{\dot{\bar{\varepsilon}}}{2\bar{\sigma}} [-(\sigma_r + \sigma_\theta)]$$

where, $\varepsilon_r, \varepsilon_\theta, \varepsilon_z$ are strain rates and $\sigma_r, \sigma_\theta, \sigma_z$ are stresses rates resp. equally inside the direction r, θ, z as indicate by subscripts.

The effective stress use for biaxial condition of stress as well as effective stress, $\bar{\sigma}$ is base on mises criterion (1913) be known as,

$$(1.4) \quad \sigma = \frac{1}{\sqrt{2}} [\sigma_r^2 + \sigma_\theta^2 + (\sigma_r - \sigma_\theta)^2]^{1/2}.$$

Using above equations we get

$$(1.5) \quad \begin{aligned} \varepsilon_r &= \frac{d\mu_r}{dr} = \frac{[M(\bar{\sigma} - \sigma_0)]^n (2u - 1)}{2[u^2 - u + 1]^{1/2}} \\ \varepsilon_\theta &= \frac{\mu_r}{r} = \frac{[M(\bar{\sigma} - \sigma_0)]^n (2 - u)}{2[u^2 - u + 1]^{1/2}} \\ \varepsilon_z &= \frac{-[M(\bar{\sigma} - \sigma_0)]'' (u + 1)}{2[u^2 - u + 1]^{1/2}}. \end{aligned}$$

Wherever $u = \sigma_r / \sigma_\theta$ is ratio of radial stress with tangential stress on equations (1.6) and (1.7) can be used to find σ_θ which is written below,

$$(1.6) \quad \sigma_\theta = \frac{(\dot{u}_a)^{1/n}}{M} \psi_1 + \psi_2$$

where

$$(1.7) \quad \begin{aligned} u_a^{1/n} &= \frac{\int_a^b M \sigma_\theta dr - \int_a^b M \psi_2 dr}{\int_a^b \psi_1 dr} \\ \psi_1 &= \frac{\psi}{[u^2 - u + 1]^{1/2}} \\ \psi_2 &= \frac{\sigma_0}{[u^2 - u + 1]^{1/2}} \\ \psi &= \left[\frac{2[u^2 - u + 1]^2}{r(2 - u)} \exp \int_a^b \frac{g}{r} dr \right]^{\frac{1}{n}} \end{aligned}$$

also

$$(1.8) \quad g = \frac{2u - 1}{2 - u}.$$

Inside radial direction forces of equilibrium are

$$(1.9) \quad \frac{d}{dr} [r\sigma_r] - \sigma_\theta + \rho\omega^2 r^2 = 0.$$

Integration equation (1.9) taking limits $r = a$ to $r = b$ and by taking boundary circumstance $\sigma_r = 0$ at $r = a$ also $\sigma_r = 0$ at $r = b$ we gets

$$(1.10) \quad \int \sigma_\theta dr = \rho\omega^2 (b^3 - a^3) / 3.$$

In the first iteration, $\sigma_\theta = \sigma_{\theta \text{ avg}}$, where $\sigma_{\theta \text{ avg}}$ is the average tangential stress on cross section of disk, therefore equation (1.10) write in the given form,

$$(1.11) \quad u_a^{1/n} = \frac{\sigma_{\theta \text{ avg}} \int_a^b M dr - \int_a^b M \psi_2 dr}{\int_a^b \psi_1 dr}.$$

The value of σ_r can be calculating by taking integration of equations (1.11) by using limits $r = a$ to b w.r.t. "r" are given below,

$$(1.12) \quad \sigma_r = (1/r) \int_a^r (\sigma_\theta) dr - \frac{\rho\omega^2 (r^3 - a^3)}{3r}.$$

Finding the values of σ_θ from equation (1.9), σ_r is radial stress find by equation (1.8) at various point in disk moreover strain rate $\dot{\epsilon}_r$ and $\dot{\epsilon}_\theta$ be calculated from equations (1.6) and (1.7) respectively.

DISCUSSION AND GRAPHICAL REPRESENTATION:

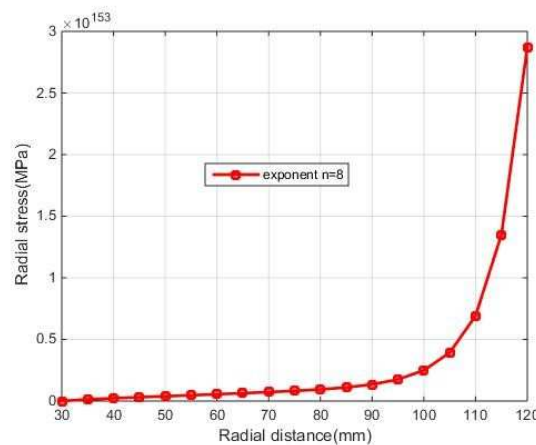


Fig.1. The change of radial stress at various radius with disk at exponent $n = 8$

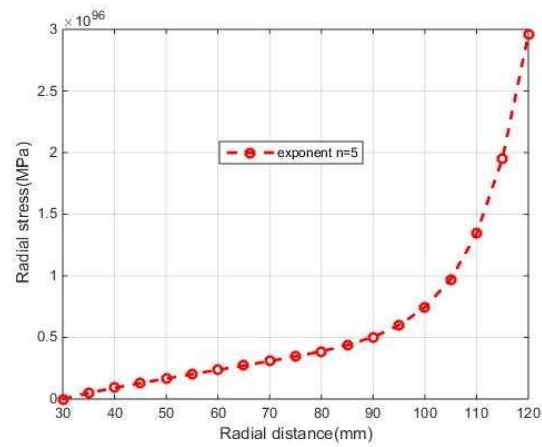


Fig.2. The change of radial stress at various radius with disk at exponent $n = 5$

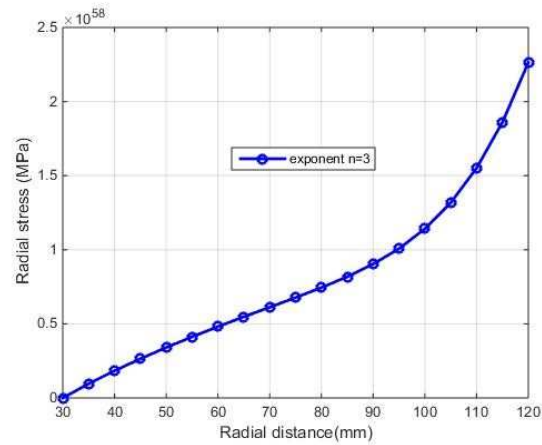


Fig.3. The change of radial stress at various radius with disk at exponent $n = 3$

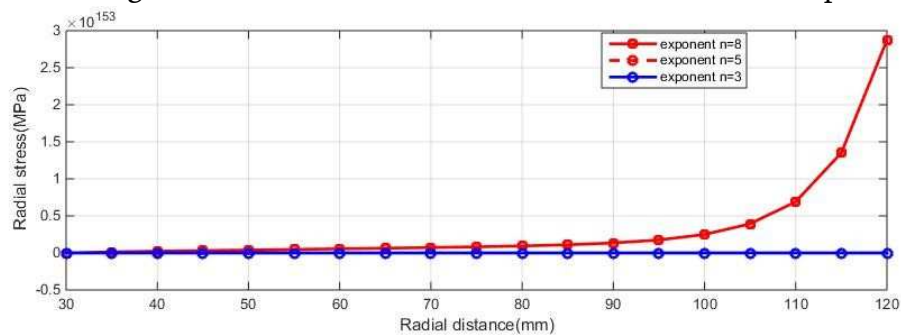


Fig.4 The change of radial stress at various radius with disk at exponent $n = 8, 5, 3$

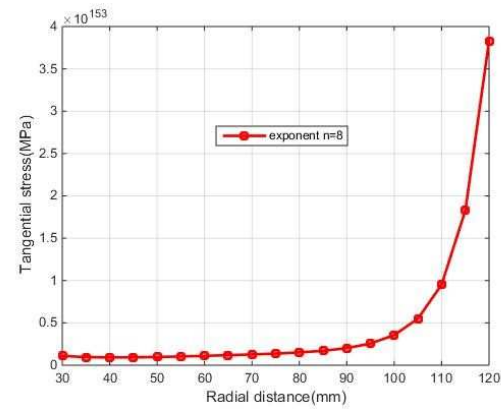


Fig.5. The change of tangential stress at various radius with disk at exponent $n = 8$

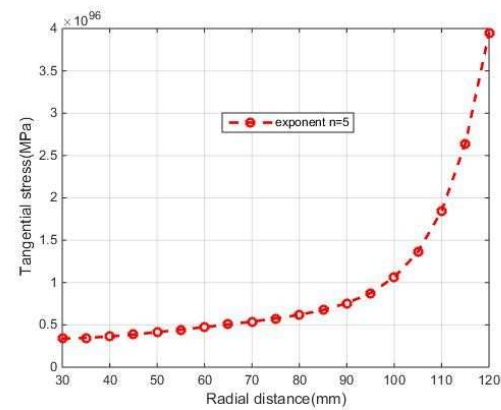


Fig.6. The change of tangential stress at various radius with disk at exponent $n = 5$

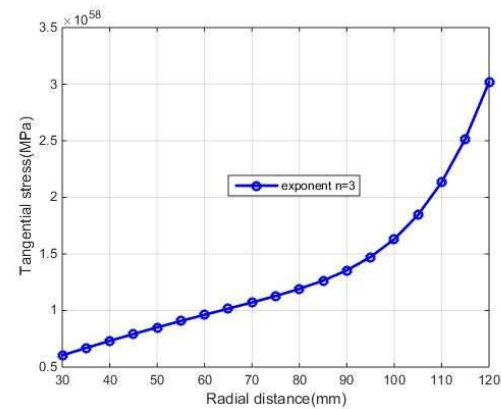


Fig.7. The change of tangential stress at various radius with disk at exponent $n = 3$

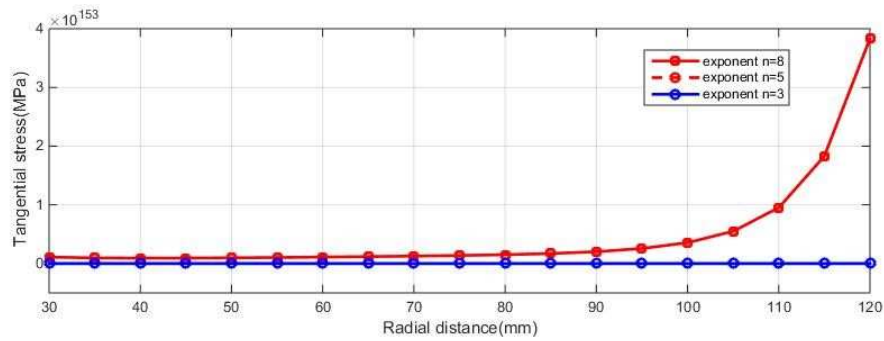


Fig.8. The change of tangential stress at various radius with disk at exponent $n = 3, 5, 8$

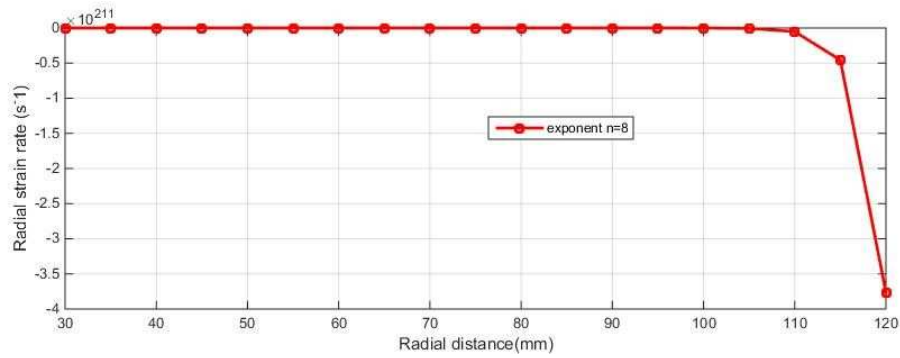


Fig.9. The change of radial strain at various radius with disk at exponent $n = 8$

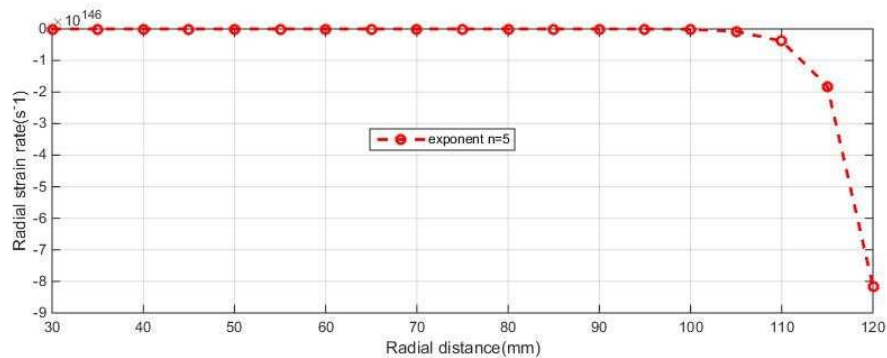


Fig.10. The change of radial strain at various radius with disk at exponent $n = 5$

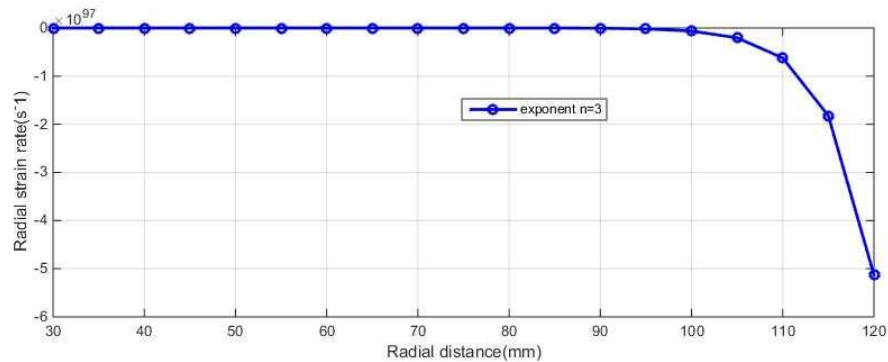


Fig.11 The change of radial strain at various radius with disk at exponent $n = 3$

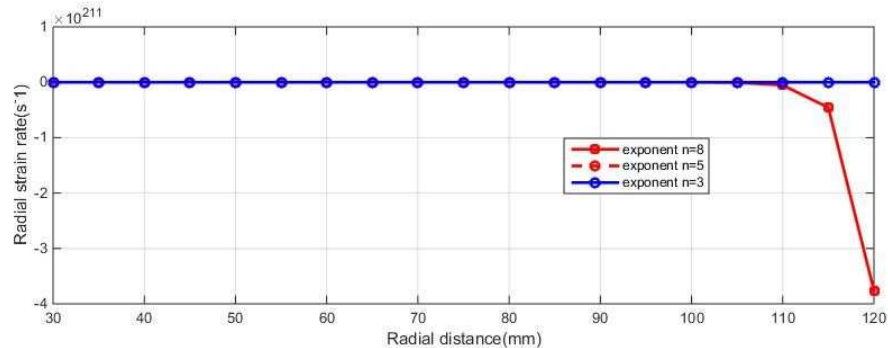


Fig.12 The change of radial strain at various radius with disk at exponent $n = 3, 5, 8$

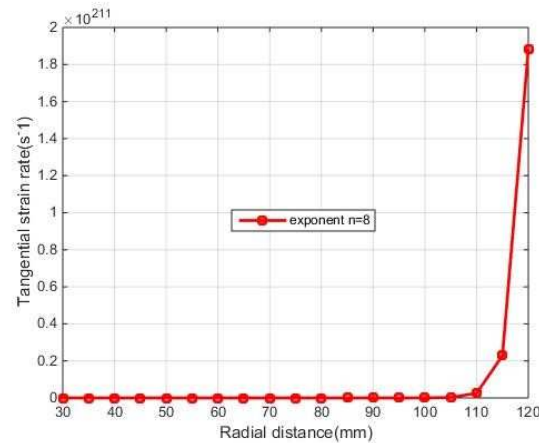


Fig.13 The change of tangential strain at various radius with disk at exponent $n = 8$

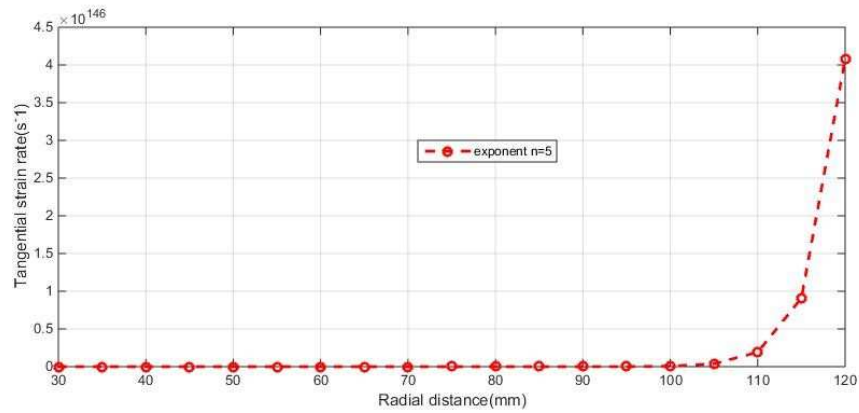


Fig.14 The change of tangential strain at various radius with disk at exponent
 $n = 5$

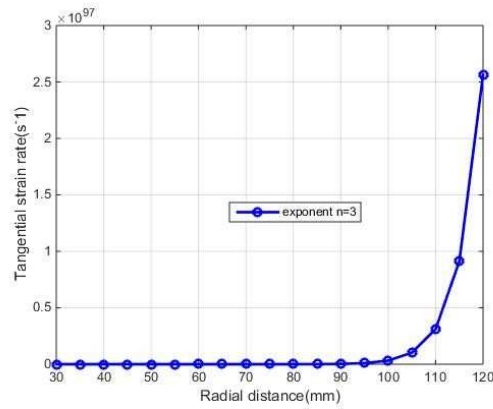


Fig.15 The change of tangential strain at various radius with disk at exponent
 $n = 3$

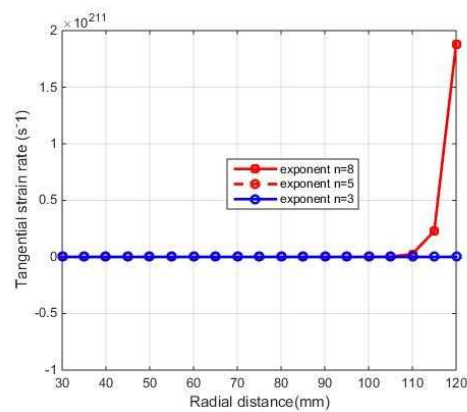


Fig.16 The change in tangential strain at various radius with disk at exponent
 $n = 3, 5, 8$

In this given figure 1,2,3 the radial stress along radius at $n = 8, 5, 3$ respectively there observed that radial stress rate increased as we move from inner radii to outer radii. In figure 4, radial stress along radius at exponent $n = 8, 5, 3$ we observed that increases the value of exponent radial stress rate decreases.

In this given figure 5,6,7 the tangential stress along radius at $n = 8, 5, 3$ respectively there observed that tangential stress increased as we move from inner radii to outer radii. In figure 8, tangential stress along radius at exponent $n=8,5,3$ in fig 8 observed that tangential stress rate decreases with increased the value of exponent.

In this given figure 9,10,11 the radial strain along radius at $n = 8, 5, 3$ respectively there observed that radial strain rate decreases as we moves inner radii to outer radii. In figure 12, radial strain rate along radius at exponent $n=8,5,3$ in fig 12 observed that radial strain rate decreased with increases the value of exponent.

In this given figure 13, 14, 15 the tangential strain along radius at $n = 8, 5, 3$ respectively there observed that tangential strain rate increases from inner radii to outer radii. In figure 16, tangential strain rate along radius at exponent $n = 8, 5, 3$ it is observed increasing the value of exponent the tangential strain rate decreases.

2. CONCLUSIONS

We concluded that with increases the value of exponent in moving composite disk having volume 20%, angular speed 16000rpm at inner radii 30mm and outer radii 120mm the stress also strain rates decreases with increases the value of exponent $n = 3, 5, 8$. So in order to reduce distortion take the value of exponent $n = 8$.

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