Advances in Mathematics: Scientific Journal **9** (2020), no.8, 5573–5580
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.8.26

AN ASSESSMENT ON BOND VALUE APPROXIMATION USING TAYLOR SERIES

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ABSTRACT. The impact of change in yield to maturity on future value of cash flows for bonds is determined by duration and convexity of bonds. A non linear relationship has been established by Taylor's expansion between price of bond and rate of return for a bond evaluated around its initial value. This paper seeks to describe the relationship of bond price function and its sensitivity with change in yield to maturity.

1. INTRODUCTION

The variation in rates of interest always has an impact on performance of business. All participants of market are exposed, to a high level or low level degree of volatility of interest rates. Higher volatility in rates of interest for the past few years, outline interest rate risk as one of the most noteworthy dangers. Consequently, it becomes crucial to handle risks of this kind appropriately. It is not possible to balance these risks entirely. However it is necessary to minimize the risk. Assessment of the impact of change in bond price because of change in yield to maturity enables better assessment of effects of risk that helps required management. The mathematical method, the Taylor series expansion can be applied for this purpose. This series expansion establishes a non-linear relationship between price of a bond and the yield to maturity about its initial value.

2010 *Mathematics Subject Classification.* 91B82, 91G10.
Key words and phrases. Duration, yield to maturity, present value, cash flow.

Frederick Macaulay used this concept while developing bond valuation using duration and convexity of bonds.

Taylor series

A function can expressed as a series of infinite terms using Taylor series. The terms expressed the derivatives determined from of the function's value at a given single point. This arrangement develops the estimation of value of the function in any neighborhood point anytime. A quantitative gauge of the mistake is given by Taylor's approximation develop some errors that can be estimated with arrangement estimate of Taylor's theorem. The Taylor polynomial is shaped by taking limit of some initial terms of the Taylor arrangement development. The Taylor arrangement of a capacity is the restriction of that capacity's Taylor polynomials as the degree increments, given that the breaking point exists. Taylor arrangement around any point is

$$f(x) = f0(a) + f1(a)(x - a) + f2(a)(x - a)2/2! + f3(a)(x - a)3/3! + ...$$

where $f^0(a)$ is value of function at point, f^1, f^2, f^3 represents first second and third derivatives respectively at given point.

Taylor's theorem gives an important series technique for the developed the duration concept for bonds which was also applied by Black and Scholes when they formulate their option pricing model.

Bond

A bond is security instrument of loan issued by business house or an administration unit for the raising of funds. The bonds might be given at standard, premium or below par. The standard is the sum expressed in face of bond. It expresses the sum the firm borrows and promises to repay at time of maturity.

Characteristics of Bonds

Face value is the amount of money the bond is sold at and will have hold at the end of maturity period. Periodic payments are made to buyer as reward of investment known as coupon. The coupon rate is the pace of intrigue the security backer will pay on the assumed worth of the security, communicated as a rate. The dates on which the interest payments are made to buyer are known as coupon dates. Payments period can be annual, semiannual, quarterly, monthly etc, but the most usual period is semiannual.

Maturity

Maturity of a bond is the length of time untills the coupon payments are to be made and bondholder receives the par value. The bond is more sensitive to change in price with respect to change in rate of return which have longer time period to maturity than the shorter time period. The date on which the bond will mature and the bond issuer will pay the total amount to bondholder is maturity date. The price at which the bonds are originally sold is called the issue price.

Yield to Maturity (YTM)

YTM is the effective rate of return for a bond if the bond is held until the finish of its lifetime. It is the interior rate of return of an interest in a bond if the financial specialist holds the bond until maturity and if all installments are made as planned. Interest rate is a rate charged for the utilization of the cash An expansion (decline) in the necessary yield diminishes (builds) the current estimation of its normal incomes and in this way diminishes (expands) the security's cost. This relationship isn't direct. The state of the value yield relationship for any alternative free security is alluded to as a curved relationship.

Duration

Duration is a proportion of the affectability of the cost of a security or other debt instrument to an adjustment in financing costs. Duration gauges to what extent it takes, in years, for a financial specialist to be reimbursed the bond's cost by the bond's all out incomes. Simultaneously, duration is a proportion of affectability of a security's or fixed salary portfolio's cost to changes in financing costs. Time to maturity and coupon rate are elements can influence a security's length. The duration is directly proportional to time hence more time implies greater risk. The security with the higher coupon rate will take care of its unique costs quicker than the security with a lower yield. The higher the coupon rate, the lower the term, and the lower the loan cost chance. The Macaulay Duration is the weighted normal time until all the bond's incomes are paid. By representing the current estimation of future bond installments

$$D = \left[\frac{1C}{1+r} + \frac{2C}{(1+r)^2} + \frac{3C}{(1+r)^3} + \ldots + \frac{nC}{(1+r)^n} + \frac{nM}{(1+r)^n}\right] \frac{1}{P}.$$

The Macaulay Duration enables a speculator to assess and look at bonds autonomous of their term or time to development. The adjusted length of a bond assists financial specialists with seeing how much a bond's cost will rise or fall if the YTM rises or falls by 1%. This is a significant number if a financial specialist is concerned that loan fees will be changing temporarily. Macaulay Duration is a normal or powerful development. Modified Duration truly quantifies how little changes in the respect development affect the cost of the bond. Modified duration= $\frac{D}{(1+r)}$.

Convexity

Convexity is an extent of the twist, or the degree of the twist, in the association between security expenses and security yields. Convexity shows how the length of a security changes as the credit cost changes.. In the event that a security's term increments as yields increment, the security is said to have negative convexity. In the event that a security's term rises and yields fall, the security is said to have positive convexity. Changed Duration relationship doesn't completely catch the genuine connection between security costs and respect development. So as to all the more completely catch this, specialists use Convexity.

The price function and Taylors series

The cost of a security is the current estimation of all it s incomes, discou nted at the fitting interior pace of return (which turns into the respect development). It is given by:

$$P = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n},$$

where P is price of bond, C is coupon, r is rate of return, n is number of payments

$$\begin{aligned}
\frac{dP}{dr} &= \frac{(-1)C}{(1+r)^2} + \frac{(-2)C}{(1+r)^3} + \ldots + \frac{(-n)C}{(1+r)^{n+1}} + \frac{(-n)M}{(1+r)^{n+1}} \\
\frac{dP}{dr} &= -\frac{1}{(1+r)}\left(\frac{1C}{1+r} + \frac{2C}{(1+r)^2} + \ldots + \frac{nC}{(1+r)^n} + \frac{nM}{(1+r)^n}\right) \\
\frac{dP}{dr}\frac{1}{P} &= -\frac{1}{(1+r)}\left(\frac{1C}{1+r} + \frac{2C}{(1+r)^2} + \ldots + \frac{nC}{(1+r)^n} + \frac{nM}{(1+r)^n}\right)\frac{1}{P} \\
\frac{dP}{dr}\frac{1}{P} &= -\frac{1}{(1+r)}D \\
\frac{dP}{dr}\frac{1}{P} &= -MD.
\end{aligned}$$

If price function is

$$P = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t},$$

then by Taylor's series

$$\Delta P = \frac{\delta P}{\delta t}\Delta t + \frac{\delta P}{\delta r}\Delta r + \frac{1}{2}\frac{\delta^2 P}{\delta r^2}\Delta r + \dots.$$

Keeping t constant as We want to hold time constant and measure the change in price due to an instantaneous change in yield. For our purposes we can ignore the derivative with respect to t,

$$\frac{\Delta P}{P} = \frac{1}{P}\frac{\delta P}{\delta r}\Delta r + \frac{1}{2P}\frac{\delta^2 P}{\delta r^2}\Delta r.$$

This implies $\%$ change in Price =-Modified Duration$*\Delta r + 12$*convexity*$(\Delta r)^2$ For instance, the six-year 6.1$\%$ coupon security had a respect development of 10$\%$ and a semi-yearly Macaulay Duration of 10.014 (5.007 yearly Macaulay Duration). The Modified Duration of this bond is 10.014/(1+.05) or 9.537 on a semi-yearly premise or 9.537/2 = 4.77 years on a yearly premise. Accepting that the respect development of 10$\%$ increments by 25 premise focuses to 10.25$\%$, in view of the Modified Duration of 4.77 years the cost of the bond should change by ΔP/P=-D .Δr= - 4.77 (.25$\%$)= - 1.19$\%$ The bond cost should drop by 1.19$\%$ from 827.17 to 817.31 ($827.17 * (1 - .0119)$ =817.31). The genuine determined cost at a respect development of 10.25$\%$ is $\$817.38$.

The accompanying table shows the Modified Duration value change and the genuine determined value change for various changes in respect development.

Bond Data
Coupon = 6.1$\%$
Maturity= 6 years
Assumed worth = $\$1,000$
Respect Maturity= 10$\%$
Cost = $\$827.17$
Modifed Duration= 4.770

Modified Duration accept that the value changes are direct as for changes in the respect development. From table over, the genuine connection between the security's cost and the respect development isn't direct. The Column with the distinctions is constantly positive and increments. Including the convexity modification adjusts for the way that Modified Duration minimizes the real bond cost. For instance above, at a yield of 12% the rate value change utilizing just Modified Duration was - 9.54%, while the genuine was - 9.01%. On the off chance that we utilize the Convexity esteem we simply determined, the anticipated rate value change would be

$$\%\Delta Price = -4.77(.02) + (1/2)27.72(.02)2 = -.0954 + .00554 = -.0899.$$

This is - 8.99%, which is a lot nearer to the genuine rate value change of - 9.01%.

Table 1

New Yield to Maturity	**Change in Yield**	**- D*Change in Yield**	**Predicted Price**	**Actual % change**	**Actual price**	**Difference**
12.00%	**2.00%**	**-9.54%**	**748.26**	**-9.01%**	**752.68**	**4.42**
11.75%	1.75%	-8.35%	758.12	-7.94%	761.52	3.40
11.50%	1.50%	-7.16%	767.99	-6.85%	770.50	2.52
11.25%	1.25%	-5.96%	777.85	-5.75%	779.61	1.76
11.00%	1.00%	-4.77%	787.71	-4.63%	788.85	1.13
10.75%	0.75%	-3.58%	797.58	-3.50%	798.22	0.64
10.50%	0.50%	-2.39%	807.44	-2.35%	807.73	0.29
10.25%	0.25%	-1.19%	817.31	-1.18%	817.38	0.07
10.00%	0.00%	0.00%	827.17	-0.00%	827.17 -	
9.75%	-0.25%	1.19%	837.03	1.20%	837.10	0.07
9.50%	-0.50%	2.39%	846.90	2.42%	847.18	0.28
9.25%	-0.75%	3.58%	856.76	3.66%	857.40	0.64
9.00%	-1.00%	4.77%	866.63	4.91%	867.78	1.15
8.75%	-1.25%	5.96%	876.49	6.18%	878.31	1.82
8.50%	-1.50%	7.16%	886.35	7.47%	889.00	2.64
8.25%	-1.75%	8.35%	896.22	8.79%	899.84	3.62
8.00%	-2.00%	9.54%	906.08	10.12%	910.84	4.76

2. Conclusion

Taylor series, an important concept in mathematics plays a crucial role in estimating change in price of bond due to change in yield to maturity. Measure of price sensitivity helps to minimize investment risks for which Taylors series enable to do the calculation of effect of change.

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