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A COMMON FIXED POINT THEOREM IN NON-ARCHIMEDEAN MENGER PROBABILITISTIC METRIC SPACE

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ABSTRACT. We demonstrated the presence of normal fixed point hypotheses in Non-Archimedean Menger Probabilistic Metric Space utilizing the R-weakly commuting maps. The presented theorem extends some already known results of literature [1, 2].

1. INTRODUCTION

In 1942 Menger [3] presented the idea ofprobabilistic metric spaces (quickly, PM-spaces) as a generalization of a metric space which prompts the examination of physical quantities and probabilistic functions. Istratescu and Crivat [4] had characterized the Non-Archimedean (quickly, N.A) PM-space and clarified essential topological basics of N.A Menger PM-space in [4]. Istratescu et al. demonstrated the presence of fixed point of contractive maps in N.A Menger PM-space in [4, 5] which was the generalization of the existing. In 1994, Pant [6] presented the idea of R-weakly commuting maps in metric spaces. Vasuki [7] explained some common fixed point theorems for R-weakly commuting maps in fuzzy metric spaces. The motive of the presented paper is to prove the existence

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of common fixed point theorem in N.A Menger PM-spaces Space utilizing the R-weakly commuting maps. We generalized the result of K.P.R. Sastry, G.A. Naidu, I. Laxmi Gayatri and S.S.A. Sastri [2] to prove our result.

2. PRELIMINARIES

The succeed classifications and consequences will be used subsequently.

Definition 2.1. *([8]) Let X be any non-empty set and the arrangement of all left continuous distribution functions be indicated as D. A pair (X, θ) is characterized to be the N.A.PM-space, if $\emptyset$ is a mapping from $X \times X \to D$ fulfil the accompanying conditions:*

(i) $\theta_{x,y}(\tau_i) = 1 \forall \tau > 0$ *if and only if* $x = y$;
(ii) $\theta_{x,y}(\tau) = \theta_{y,x}(\tau_i)$;
(iii) $\theta_{x,y}(0) = 0$;
(iv) If $\theta_{x,y}(\tau_1) = \theta_{y,z}(\tau_2) = 1,$ *then* $\theta_{x,z}(\max\{\tau_1, \tau_2\})) = 1.$

Definition 2.2. *([8,9]) If* $\theta_{x,z}(\max\{\tau_1, \tau_2\}) \geq \theta_{xy}(\tau_1)\, \delta\theta_{y,z}(\tau_2) \forall x, y, z \in X, \tau_1, \tau_2 \geq 0$. *Then, PM-space* $(\mathrm{X}, \theta, \delta)$ *is known as* N.A.

Definition 2.3. A *PM-space* $(\mathrm{X}, \theta, \delta)$ *is Archimedean iff* $\exists x, y, z \in X, \tau_{13} \geq 0$ *such that* $\theta_{x,z}(\tau_i) < \theta_{x,y}(\tau_{ij})\, \delta\theta_{y,z}(\tau_2)$.

Definition 2.4. *An arrangement* $\{x_n\}$ *in a N.A Menger PM-space* $(\mathrm{X}, \theta, \delta)$ *coincides to* x *iff each* $\varepsilon > 0, \lambda > 0 \exists M(\varepsilon, \lambda)$ *where* $g(\theta(x_n, x, \varepsilon)) < g(1 - \lambda) \forall n, m > M$.

Definition 2.5. *An arrangement* $\{x_n\}$ *in a N.A Menger PM-space* $(\mathrm{X}, \theta, \delta)$ *coincides to* x *iff each* $\varepsilon > 0$ $\lambda > 0 \exists$ *an integer* $M(z, \lambda)$ *where* $g(\theta(x_n, x_{n+p}, \varepsilon)) < g(1 - \lambda) \forall n$ *and* $n \geq M$ *and* $p \geq 1$.

Definition 2.6. *Two maps G and H of a N.A Menger PM-space* $(\mathrm{X}, \theta, \delta)$ *into itself is said to be R weakly commuting of type* A_s *if for* $x \in X$ *and* $R > 0$ $g(\theta(GHx, HHx, \tau)) \leq g\left(\theta\left(Gx, Hx, \frac{\tau}{R}\right)\right)$.

Definition 2.7. *Two maps G and H of a N.A Menger PM-space(X,* θ, δ *) into itself is said to be R weakly commuting of type* A_T *if for* $x \in X$ *and* $R > 0$ $g(\theta(GHx, HHx, \tau)) \leq g\left(\theta\left(Gx, Hx, \frac{\tau}{R}\right)\right)$.

Example 1. *Let* (X,d) *be a metric space with* d *defined as* $d(x,y)=\left\{\begin{array}{l} 0 \text{ if } x=y \\ 1 \text{ if } x\neq y\} \end{array}\right\}$ *and* δ *be any* τ *-norm. then* $(\mathrm{X},\theta,\delta)$ *is a N.A Menger PM-space iff* $\theta_{xy}\left(\tau_{\vdots}\right)=\frac{\tau}{\tau+d(x+y)}\forall\tau>0.$

Theorem 2.1. *([1]) Let G and H be two continuous self-maps of a complete N.A Menger space (X,* θ,δ *), where* δ *is continuous and firmly expanding* τ *-norm. Let* A *be self map of* X *fulfilling:*

- (i) $\{\mathrm{A_2G}\}$ *and* $\{\mathrm{A},\mathrm{H}\}$ *are point wise* R*-weakly commuting and* $\mathrm{A(X)}\subseteq\mathrm{G(X)}\cap\mathrm{H(X)}$*;*
- (ii) $g\left(\theta_{Ax,Ay}(\tau)\right)\leq$ $\varphi[\max\{g\left(\theta_{Gx,Hy}(\tau)\right),g\left(\theta_{Gx,Ax}(\tau)\right),g\left(\theta_{Gx,Ay}(\tau)\right),g\left(\theta_{Hy,Ay}(\tau)\right)\}]$ *for all* $x,y\in X$, *and* $\varphi:[0,\infty)\to[0,\infty)$ *is upper semi-continuous from the right.*

Then there exists a unique common fixed point A,G *and* H *in* X*.*

Theorem 2.2. *([1]) Let G and H be two continuous self-maps of a complete N.A Menger space (X,* θ,δ *), where* δ *is least* τ*-norm. Let* A *be self map of* X *fulfilling:*
(i) $\{\mathrm{A},\mathrm{G}\}$ *and* $\{\mathrm{A},\mathrm{H}\}$ *are point wise* R *-weakly commuting and* $\mathrm{A(X)}\subseteq\mathrm{G(X)}\cap\mathrm{H(X)}$.
(ii) $g\left(\theta_{Ax,Ay}(\tau)\right)\leq\varphi\left[\max\left\{\begin{array}{l} g\left(\theta_{Ax,Gx}(\tau)\right),g\left(\theta_{Ax,Gy}(\tau)\right),g\left(\theta_{Ax,Hy}(\tau)\right), \\ g\left(\theta_{Ay,Gx}(\tau)\right),g\left(\theta_{Ay,Gy}(\tau)\right),g\left(\theta_{Ay,Hy}(\tau)\right) \\ g\left(\theta_{Gx,Gy}(\tau)\right),g\left(\theta_{Cx,Hy}(\tau)\right),g\left(\theta_{Gy,Hy}(\tau)\right) \end{array}\right\}\right.$
for all $x,y\in X,\tau>0,$ *for some* $g\in\Omega$ *and* $\varphi\in\phi$*.*
Then A,G *and* H *have a unique common fixed point in* X*.*

3. Main Results

The following theorem is an extension of Theorem 2.1 with $\varphi\in\phi$.

Theorem 3.1. *Let* S,T *and* P *be three continuous self-maps of a complete* N*. A Menger space* (X,F,δ) *where* δ *is a continuous t-norm. Let* A *be self map of* X *satisfying:*
(i) $\{\mathrm{A},\mathrm{S}\},\{\mathrm{A},\mathrm{T}\}$ *and* $\{\mathrm{A},\mathrm{P}\}$ *are point wise* R*-weakty commuting and* $\mathrm{A(X)}\subseteq\mathrm{S(X)}\cap\mathrm{T(X)}\cap\mathrm{P(X)}$.
(ii) $g\left(F_{Ax,Ay}(t)\right)\leq$

$$\varphi\left[\max\left\{\begin{array}{l} g\left(F_{Ax,Sx}(t)\right), g\left(F_{Ax,Sy}(t)\right), g\left(F_{Ax,Ty}(t)\right), g\left(F_{Ax,Py}(t)\right), \\ g\left(F_{Ay,Sx}(t)\right), g\left(F_{Ay,Sy}(t)\right), g\left(F_{Ay,Ty}(t)\right), g\left(F_{Ay,Py}(t)\right) \\ g\left(F_{Sx,Ty}(t)\right), g\left(F_{Sx,Py}(t)\right), g\left(F_{Sy,Ty}(t)\right), g\left(F_{Sy,Py}(t)\right) \end{array}\right\}\right.$$

for all $x, y \in X, t > 0$, *for some* $g \in \Omega$ *and* $\varphi \in \phi$.

Let $x_0 \in X$. *Define a sequence* $\{x_n\}$ *and* $\{y_n\}$ *by* $y_n = Ax_n = Sx_{n+1} = Tx_{n+2} = Px_{n+2}$ *for all* $= 0, 1, 2 \ldots$ *Suppose* $\lim_{n\to\infty} F_{y_n,y_{n+1}}(t) = 1 \forall t > 0$.

Then $\{y_n\}$ *is a Cauchy Sequence in* X.

Proof. Suppose $\{y_n\}$ is not a Cauchy sequence in X. Then there exist $\varepsilon_0 \in (0,1), t_0 > 0$ and two sequences $\{m_i\}, \{n_i\}$ of positive integers such that $m_i > n_{i+1}$ and $n_i \to \infty$ as $i \to \infty$; and $F_{y_{m_i} y_{m_i}}(t_0) < 1 - \varepsilon_0$ and $F_{y_{m_{l-1}} y_{n_l}}(t_0) \geq 1 - \varepsilon_0$ for $i = 1, 2, 3 \ldots$. By taking $x = x_{m_l}$ and $y = y_{n_{i+2}}$ in condition (ii), we get,

$$g\left(F_{Ax_{m_i},Ax_{n_{i+2}}}(t)\right) \leq$$

$$\varphi\left[\max\left\{\begin{array}{c} g\left(F_{Ax_{m_i}}sx_{m_i}(t)\right), g\left(F_{Ax_{m_i}Sx_{n_{i+2}}}(t)\right), g\left(F_{Ax_{m_i},Tx_{n_{i+2}}}(t)\right), \\ g\left(F_{Ax_{m_i}Px_{n_{i+2}}}(t)\right), g\left(F_{Ax_{n_{i+2}},sx_{m_i}}(t)\right), g\left(F_{Ax_{n_{i+2}},Sx_{n_{i+2}}}(t)\right) \\ g\left(F_{Ax_{n_{i+2}},Tx_{n_{i+2}}}(t)\right), g\left(F_{Ax_{i_{i+2}},Px_{n_{i+2}}}(t)\right), g\left(F_{Sx_{m_i},Tx_{n_{i+2}}}(t)\right), \\ g\left(F_{Sx_{m_i},Px_{n_{i+2}}}(t)\right), g\left(F_{Sx_{n_{i+2}},Tx_{n_{i+2}}}(t)\right), g\left(F_{Sx_{n_{i+2}},Px_{n_{i+2}}}(t)\right) \end{array}\right\}\right.$$

$$g(F_{y_{m_i}y_{n_{i+2}}}(t)) \leq$$

$$\varphi\left[g(\min\left\{\begin{array}{c} \left(F_{y_{m_i},y_{m_{i-1}}}(t)\right), \left(F_{y_{m_i},y_{n_{i+1}}}(t)\right), \left(F_{y_{m_i},y_{n_i}}(t)\right), \left(F_{y_{m_i},y_{n_{i-1}}}(t)\right), \\ \left(F_{y_{n_{i+2}},y_{m_{i-1}}}(t)\right), \left(F_{y_{n_{i+2}},y_{n_{i+1}}}(t)\right), \left(F_{y_{n_{i+2}},y_{n_i}}(t)\right), \left(F_{y_{n_{i+2}},y_{n_{i-1}}}(t)\right), \\ \left(F_{y_{m_{i-1}},y_{n_i}}(t)\right), \left(F_{y_{m_{i-1}},y_{n_{i-1}}}(t)\right), \left(F_{y_{n_{i+1}},y_{n_i}}(t)\right), \left(F_{y_{n_{i+1}},y_{n_{i-1}}}(t)\right) \end{array}\right\})\right].$$

since, X is N. We have, $1 - \varepsilon_0 > F_{y_{m_i} y_{n_i}}(t) \geq F_{y_{m_l} y_{m_{l-1}}}(t) \delta F_{y_{m_{i-1}} y_{m_i}}(t)$. It follows that

I. $\lim_{n\to\infty}\left(F_{y_{m_l}} y_{a_{n_{i+1}}}(t)\right) = (1 - \varepsilon_0)$

II. $\lim_{n\to\infty}\left(F_{y_m,y_{n_i}}(t)\right) = (1 - \varepsilon_0)$

III. $\lim_{n\to\infty}\left(F_{y_m,y_{m_{(-1)}}}(t)\right) = (1 - \varepsilon_0)$

IV. $\lim_{n\to\infty}\left(F_{y_{i+2}} y_{im_{i-1}}(t)\right) = (1 - \varepsilon_0)$

V. $\lim_{n\to\infty}\left(F_{y_m} l = 1, y_{n_i}(t)\right) = (1 - \varepsilon_0)$

On letting $i \to \infty$, by using the results $\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}, \mathrm{V}, \mathrm{VI}$ and continuity of g, the inequality becomes,

$$g(1-\varepsilon_0) \leq \varphi[g(\min[1,(1-\varepsilon_0),(1-\varepsilon_0),(1-\varepsilon_0),(1-\varepsilon_0),1,1,1,(1-\varepsilon_0),(1-\varepsilon_0),1,1]]$$
$$g(1-\varepsilon_0) \leq \varphi[g(1-\varepsilon_0)] < g(1-\varepsilon_0)$$

which is a contradiction. Therefore, $\{y_n\}$ is a Cauchy sequence in X. □

Lemma 3.1. *If δ is a min t-norm and $\alpha,\beta,\gamma \in [0,1]$ be such that $\alpha\delta\beta \leq \gamma, \beta\delta\gamma \leq \alpha, \gamma\delta a \leq \beta$ then $a > \beta$ implies $\beta = \gamma$.*

Theorem 3.2. *Let $\underline{S}, \underline{I}$ and$P$ be three continuous self-maps of a complete$N$. A Menger space (X,F,δ) whereδ is a minimum t-norm. Let A be self map of X satisfying:*

(i) $\{\mathrm{A},\mathrm{S}\},\{\mathrm{A},\mathrm{T}\}$ *and* $\{\mathrm{A},\mathrm{P}\}$ *are point wise* R*-weakly commuting and* $\mathrm{A(X)} \subseteq \mathrm{S(X)} \cap \mathrm{T(X)} \cap \mathrm{P(X)}$*;*

(ii) $g(F_{Ax,Ay}(t)) \leq \varphi[\max\{g(F_{Ay,Sy}(t)), g(F_{Ay,Ty}(t)), g(F_{Ay,Py}(t)), g(F_{Sx,Ty}(t)), g(F_{Sx,Py}(t)), g(F_{Sy,Ty}(t)),\}]$

for all $x,y \in X, t>0,$ for some $g \in \Omega$ and $\varphi \in \phi$.

Then $\underline{A}, S, T$ and P have a unique common fixed point in X.

In fact for any $x_0 \in X$, the sequence $\{y_n\}$ defined as $y_n = Ax_n = Sx_{n+1} = Tx_{n+2} = Px_{n+3}$ for $\mathrm{n} = 0,1,2\ldots$, then $\{y_n\}$ converges to the unique common fixed point of $\underline{A}, \underline{S}, T$ and P in X.

Proof. Firstly, we show that $\lim_{n\to\infty}\left(F_{y_{n+1},y_{n+2}}(t)\right) = 1 \forall t > 0$ By taking $x = x_n$ and $y = x_{n+1}$ in condition (ii). We get,

$$g\left(F_{Ax_{m_i},Ax_{n_{i+2}}}(t)\right) \leq$$
$$\varphi\left[\max\left\{\begin{array}{c} g\left(F_{Ax_{m_i}}sx_{m_i}(t)\right), g\left(F_{Ax_{m_i}Sx_{n_{i+2}}}(t)\right), g\left(F_{Ax_{m_i},Tx_{n_{i+2}}}(t)\right), \\ g\left(F_{Ax_{m_i}Px_{n_{i+2}}}(t)\right), g\left(F_{Ax_{n_{i+2}},sx_{m_i}}(t)\right), g\left(F_{Ax_{n_{i+2}},Sx_{n_{i+2}}}(t)\right) \\ g\left(F_{Ax_{n_{i+2}},Tx_{n_{i+2}}}(t)\right), g\left(F_{Ax_{i_{i+2}},Px_{n_{i+2}}}(t)\right), g\left(F_{Sx_{m_i},Tx_{n_{i+2}}}(t)\right), \\ g\left(F_{Sx_{m_i},Px_{n_{i+2}}}(t)\right), g\left(F_{Sx_{n_{i+2}},Tx_{n_{i+2}}}(t)\right), g\left(F_{Sx_{n_{i+2}},Px_{n_{i+2}}}(t)\right) \end{array}\right\}\right.$$

$$g\left(F_{y_{n+1}\cdot y_{n+2}}(t)\right) \leq \varphi\left[g\left(\min\left\{\left(F_{y_ny_{n+1}}(t)\right),\left(F_{y_{n+1}y_{n+2}}(t)\right),\left(F_{y_n,y_{n+2}}(t)\right)\right\}\right)\right] \tag{3.1}$$

Case-I: Suppose for some 'n', $F_{y_{n+2},y_{n+2}}(t) = 1$. Then from (3.1) we can write,

$$g\left(F_{y_{n+2},y_{n+8}}(t)\right) \leq \varphi\left[g\left(\min\left\{\left(F_{y_{n+1},y_{n+2}}(t)\right),\left(F_{y_{n+2},y_{n+3}}(t)\right),\left(F_{y_{n+1},y_{n+3}}(t)\right)\right\}\right)\right].$$

Next,

(3.2) $$\alpha = F_{y_{n+2}\mathscr{Y}_{n+2}}(t), \beta = F_{y_{n+2},y_{n+3}}(t), \gamma = F_{y_{n+2},y_{n+8}}(t).$$

Then, since X is $N.A$ and $\alpha = 1$, equation (3.2) satisfies the hypothesis Lemma 3.2, we get, $g(\beta) \leq \varphi(g(\beta)) < g(\beta) > 0,$ a contradiction. Hence $(\beta) = 0,$ so that $g\left(F_{y_{n+2},y_{n+8}}(t)\right) = 0$. Consequently, $F_{y_{n+2},y_{n+3}}(t) = 1 \forall t > 0$.

Hence by induction; $F_{y_{n+1}y_{n+2}}(t) = 1$ for $\mathrm{m} \geq n$. Therefore,

$$\lim_{m\to\infty}\left(F_{y_{m+2},y_{m+2}}(t)\right) = 1 \forall t > 0.$$

Case-II: Suppose $F_{y_{n+2},y_{n+2}}(t) < 1 \forall n$. Write
$\alpha = F_{y_n,y_{n+1}}(t), \beta = F_{y_{n+1},y_{n+3}}(t), \gamma = F_{y_n,y_{n+2}}(t)$. If $\alpha > \min\{\alpha,\beta,\gamma\}$, then from Lemma 3.2 we get, $\beta = \gamma = \min\{\alpha,\beta,\gamma\}$. Then from inequality (3.1), we have,

$$g(\beta) \leq \varphi(g(\beta)) < g(\beta) > 0,$$

which is a contradiction. Therefore $\alpha = \min\{\alpha,\beta,\gamma\}$. This implies $g(\beta) \leq \varphi(g(\alpha))$. Hence,

$$g\left(F_{y_{n+1}}y_{n+2}(t)\right) \leq \varphi\left[g\left(F_{y_n,y_{n+1}}(t)\right)\right] \leq \cdots \leq \varphi^n\left[g\left(F_{y_0,y_1}(t)\right)\right] \to \infty \text{ as } i \to \infty.$$

Therefore, from both the cases I and II, we obtain,

$$\lim_{m\to\infty}\left(F_{y_{m+1},y_{m+2}}(t)\right) = 1 \forall t > 0.$$

Hence, $\{y_n\}$ is a Cauchy sequence in X. Since $(\mathrm{X_1F},\delta)$ is complete, the sequence $\{y_n\}$converges to a point z,say in X. Now by definition of the sequence of $\{y_n\}$, we have

$$y_n \to z, Ax_n \to z, Tx_n \to z, Px_n \to z \text{ asn } \to \infty$$
$$\Rightarrow Sy_n \to Sz, Ty_n \to Tz, \quad Py_n \to Pz$$

and

$$SAx_n \to Sz, TAx_n \to Tz, PAx_n \to Pz,$$

since $\{\mathrm{A_2S}\}, \{\mathrm{A},\mathrm{T}\}$ and $\{\mathrm{A},\mathrm{P}\}$ are point wise R-weakly commuting.

Hence,

$$F_{A\backslash x_n,\mathrm{sax}_n}(t) \geq F_{Ax_n,sx_n}\left(\frac{t}{R}\right), R > 0, F_{ATx_n,TAx_n}(t) \geq F_{Ax_n,Tx_n}\left(\frac{t}{R}\right), R > 0$$

and

(3.3) $$F_{APx_n,TAx_n}(t) \geq F_{Ax_n,Px_n}\left(\frac{t}{R}\right), R > 0.$$

From (3.3), we can write

$$\lim_{n\to\infty} F_{ASx_n,\mathrm{SA}\,x_n}(t) = \lim_{n\to\infty} F_{Ay_n,sy_n}(t) = \lim_{n\to\infty} F_{Ay_n,s_2}(t)$$
$$\geq \lim_{n\to\infty} F_{Ax_nsx_n}(t) = \lim_{n\to\infty} F_{z,z}(t) = 1$$

and $Ay_n \to Sz$ as $n \to \infty$.

Similarly, $\lim_{\eta\to\to\infty} F_{ATx_n,TAx_n}(t) \geq 1$. Now

$$1 \geq \lim_{n\to\infty} F_{Ay_n}, \tau_z(t) \geq 1 \Rightarrow \lim_{n\to\infty} F_{Ay_n,Tz}(t) = 1 \Rightarrow Ay_n \to Tz$$

as $n \to \infty$ and $\lim_{n\to\infty} F_{APx_n,PAx_n}(t) \geq 1$.

Now

$$1 \geq \lim_{n\to\infty} F_{Ay_n} p_z(t) \geq 1$$

implies

$$\lim_{n\to\infty} F_{Ay_n,Pz}(t) = 1 \Rightarrow Ay_n \to Pz \text{ asn } \to \infty$$

.

From the above analysis, we get, $Sz = Tz = Pz$.

Now by taking, $x = Sx_n$ and $y = z$ in condition (ii), we get, $Sz = Az$. Hence, $Az = Sz = Tz = Pz$.

By taking $x = x_{n+1}$ and $y = z$ in condition (ii), we get, $z = Az$. Hence, $z = Az = Sz = Tz = Pz$.

Therefore z is a common fixed point of $\underline{A}, S, T$ and P.

Let x be a common fixed point of A, S, T and P. From condition (ii), we have,

$$g(F_{Ax,Az}(t)) \leq \varphi\left[\max\left\{\begin{array}{c} g\left(F_{Ax,Sx}(t)\right), g\left(F_{Ax,Sz}(t)\right), g\left(F_{Ax,Tz}(t)\right), g\left(F_{Ax,Pz}(t)\right), g\left(F_{Az,Sx}(t)\right), \\ g\left(F_{Az,Sz}(t)\right), g\left(F_{Az,Tz}(t)\right), g\left(F_{Az,Pz}(t)\right), g\left(F_{Sx,Tz}(t)\right), g\left(F_{Sx,Pz}(t)\right), g\left(F_{Sz,Tz}(t)\right), \\ g\left(F_{Sz,Pz}(t)\right) \end{array}\right\}\right]$$

$$g(F_{x,z}(t)) \leq \varphi\left[\max\left\{\begin{array}{c} g\left(F_{x,x}(t)\right), g\left(F_{x,z}(t)\right), g\left(F_{x,z}(t)\right), g\left(F_{x,z}(t)\right), g\left(F_{z,x}(t)\right), \\ g\left(F_{z,z}(t)\right), g\left(F_{z,z}(t)\right), g\left(F_{z,z}(t)\right), g\left(F_{x,z}(t)\right), g\left(F_{x,z}(t)\right), g\left(F_{z,z}(t)\right), \\ g\left(F_{z,z}(t)\right) \end{array}\right\}\right]$$

$$g(F_{x,z}\,(t)) \leq \varphi[\max\{0, g\,(F_{x,z}\,(t))\}$$
$$g\,(F_{x,z}\,(t)) = 0\;\forall\; t > 0$$
$$F_{x,z}\,(t) = 1$$

$$\Rightarrow x = z$$

Therefore, z has a unique common fixed point of A, S, T and P. □

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