ALMOST NANO REGULAR SPACES

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ABSTRACT. In main aim of this paper, we introduce a new separation axiom is called almost nano regular space. Also various characterizations of almost nano regular space are given.

1. INTRODUCTION

In 2013, M. Lellis Thivagar *et al,*. [2] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space. In main aim of this paper, we introduce a new separation axiom is called almost nano regular space. Also various characterizations of almost nano regular spaceare given.

2. PRELIMINARIES

Definition 2.1. *[3] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

(i) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.*

(ii) *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.*

(iii) *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. *[2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms;*

(i) *U and $\phi \in \tau_R(X)$,*

(ii) *The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,*

(iii) *The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.*

Thus $\tau_R(X)$ is a topology on U called the nano topology with respect to X and $(U, \tau_R(X))$ is called the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed.

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $nint(A)$ and $ncl(A)$ respectively.

Definition 2.3. *[2] A subset H of a space $(U, \mathcal{N})$ is called nano α-open (briefly $n\alpha$-open) if $H \subseteq nint(ncl(nint(H)))$,*

The complements of the above mentioned set is called their respective closed set.

Definition 2.4. *[2] A subset H of a space $(U, \mathcal{N})$ is called nano regular closed (briefly nr-closed) if $H = ncl(nint(H))$.*

The complements of the above mentioned set are called their respective an open set.

Definition 2.5. *[1] A subset H of a nano topological space $(U, \mathcal{N})$ is called a nano t-set if $nint(H) = nint(ncl(H))$.*

Definition 2.6. *[4] A subset H of a nano topological space $(U, \mathcal{N})$ is said to be nano semi regular if H is nano semi open and a nano t-set.*

3. Almost Nano Regular Spaces

Definition 3.1. *A space $(U, \mathcal{N})$ is said to be almost nano regular (briefly almost n-regular), if for each n-regular closed set H and a point $a \in U - H$, there exist disjoint n-open sets S_1 and S_2 such that $a \in S_1$ and $H \subseteq S_2$.*

Theorem 3.1. *For a nano topological space $(U, \mathcal{N})$, the following are equivalent:*

(i) *$(U, \mathcal{N})$ is almost n-regular.*

(ii) *For each point $x \in U$ and each n-regular open set F containing x, there exists a n-regular open set E such that $x \in E \subseteq ncl(E) \subseteq F$.*

(iii) *For each point $x \in U$ and each neighbourhood J of x, there exists a n-regular open neighbourhood F of x such that $ncl(F) \subseteq nint(ncl(J))$.*

(iv) *For each point $x \in U$ and each neighbourhood J of x there exists a n-open neighbourhood F of x such that $ncl(F) \subseteq nint(ncl(J))$.*

(v) *For every n-regular closed set A and each point x not belonging to A, there exist n-open sets E and F such that $x \in E$, $A \subseteq F$ and $ncl(E) \cap ncl(F) = \phi$.*

(vi) *Every n-regular closed set L is expressible as an intersection of some n-regular closed neighbourhood of L.*

(vii) *Every n-regular closed set L is identical with the intersection of all n-closed neighbourhoods of L.*

(viii) *For every set G and every n-regular open set H such that $G \cap H \neq \phi$, there exists a n-open set M such that $G \cap M \neq \phi$ and $ncl(M) \subseteq H$.*

(ix) *For every non-empty set G and every n-regular closed set H satisfying $G \cap H \neq \phi$, there exist disjoint n-open sets M and N such that $G \cap M \neq \phi$ and $H \subseteq N$.*

Proof. $(i) \Rightarrow (ii)$. If F is a n-regular open set containing x, then $\{x\}$ is disjoint from the n-regular closed set $U - V$. Therefore, there exist n-open sets E_1 and E_2 such that $\{x\} \subseteq E_1$, $U - F \subseteq E_2$ and $E_1 \cap E_2 = \phi$. Then $ncl(E_1) \cap E_2$ is also empty and hence $ncl(E_1) \subseteq U - E_2 \subseteq F$. Thus, $x \in E_1 \subseteq ncl(E_1) \subseteq F$.

Again, $E_1 \subseteq nint(ncl(E_1)) \subseteq ncl(E_1) \subseteq F$. Therefore, if $nint(ncl(E_1)) = E$, then $E_1 \subseteq E \subseteq ncl(E) = ncl(E_1) \subseteq F$. Hence $x \in E \subseteq ncl(E) \subseteq F$ where E is n-regular open.

$(ii) \Rightarrow (iii)$. Obvious.

$(iii) \Rightarrow (iv)$. Obvious.

$(iv) \Rightarrow (v)$. If G is n-regular closed and $x \notin G$, then $U - G$ is a neighbourhood of x. Therefore, there exists a n-open set F such that $x \in F \subseteq ncl(F) \subseteq U - G$. Again, F is a neighbourhood of x. Therefore there exists a n-open set E such that $x \in E \subseteq ncl(E) \subseteq F$. Then E and $U - ncl(F)$ are n-open sets with disjoint nano closures containing x and G respectively.

$(v) \Rightarrow (vi)$. If L is a n-regular open set, then for each $x \notin L$, there exist n-open sets M_x and N_x such that $L \subseteq M_x$, $x \in N_x$ and $ncl(N_x) \cap ncl(M_x) = \phi$. Thus, $L \subseteq M_x$ and $x \notin ncl(M_x)$. It can be seen easily that $L = \bigcap ncl(M_x)$. Also, each $ncl(M_x)$ is a n-regular closed neighbourhood of L.

$(vi) \Rightarrow (vii)$. Obvious.

$(vii) \Rightarrow (viii)$. Let G be any set and let H be n-regular open such that $G \cap H \neq \phi$. Then, there exists a point $x \in G \cap H$. Therefore, $U - H$ is a n-regular closed set and hence $U - H = \bigcap_{i \in I} J_i$ where $\{J_i : i \in I\}$ is the family of n-closed neighbourhoods of $U - H$. Since $x \in H$, therefore $x \notin \bigcap J_i$ and thus $x \notin J_i$ for all i. Since J_i is a neighbourhood of $U - H$, therefore there exists a n-open set N such that $U - H \subseteq N \subseteq J_i$. Let $V = U - J_i$. Then V is a n-open set containing x and also $x \in G$. Thus $x \in V \cap G$, that is, $V \cap G \neq \phi$. Also $U - N$ is n-closed, therefore $V = ncl(U - J_i) \subseteq U - N \subseteq H$.

$(viii) \Rightarrow (ix)$. Obvious.

$(ix) \Rightarrow (1)$. If G is n-regular closed and $x \notin G$, then $\{x\}$ is non-empty and $G \cap \{x\} = \phi$. Therefore there exist disjoint n-open sets M and N such that $\{x\} \cap M \neq \phi$ and $G \subseteq N$. Thus $x \in M$, $G \subseteq N$ and $M \cap N = \phi$. Hence U is almost n-regular. $\square$

Theorem 3.2. *Every almost n-regular, n-semi regular space is n-regular.*

Proof. Suppose U is an almost n-regular, n-semi regular space. Let J be any n-open set containing a point $x \in U$. Since U is n-semi regular, there exists a n-open set E such that $x \in E \subseteq nint(ncl(E)) \subseteq J$. Again, since E is a n-open neighbourhood of x and U is almost n-regular, there is a n-open set F such that

$x \in F \subseteq ncl(F) \subseteq nint(ncl(E))$. Hence $x \in F \subseteq ncl(F) \subseteq J$. Thus U is n-regular. $\qquad\square$

Theorem 3.3. *Let $(U, \mathcal{N})$ is almost n-regular if and only if for each $x \in U$ and each n-regular open neighbourhood E of x, there exists a n-regular open set F such that $x \in F \subseteq ncl(F) \subseteq E$.*

Proof. Suppose that U is an almost n-regular space. Let $x \in U$ and let E be a n-regular open set containing x. But then $U - E$ is a n-regular closed set with $x \notin U - E$. But U is almost n-regular. Hence there exists disjoint n-regular open sets F and T such that $x \in F$ and $U - E \subseteq T$. But then $F \subseteq U - T$ and $U - T \subseteq E$. Therefore, $x \in F \subseteq ncl(F) \subseteq E$.

Conversely, suppose that for each $x \in U$ and for each n-regular open neighbourhood E of x, there is a n-regular open set F such that $x \in F \subseteq ncl(F) \subseteq E$. Let L be a n-regular closed set in U and let $x \notin L$. Then there exists a n-regular open set F such that $x \in F \subseteq ncl(F) \subseteq U - L$ implies $x \in F$ and $L \subseteq U - ncl(F)$. Then the sets F and $U - ncl(F)$ are n-regular open sets with $F \subseteq ncl(F)$. Therefore F and $U - F$ are disjoint. Hence U is almost n-regular. $\qquad\square$

Theorem 3.4. *Let P and Q be non-empty nano topological spaces. The product $P \times Q$ is almost n-regular if and only if both P and Q are almost n-regular.*

Proof. Suppose that $P \times Q$ is almost n-regular. Let $p_0 \in P$. Let E be any n-open neighbourhood of p_0. Let $q_0 \in Q$. Then $E \times Q$ is a n-regular open neighbourhood of (p_0, q_0). But $P \times Q$ is almost n-regular. Hence there exists a n-regular open set T in $P \times Q$ such that $(p_0, q_0) \in T \subseteq ncl(T) \subseteq E \times Q$. Let $H_1 \times H_2$ be a basic n-regular open subset of $P \times Q$ such that $(p_0, q_0) \in H_1 \times H_2 \subseteq T$. Then $(p_0, q_0) \in H_1 \times H_2 \subseteq ncl(H_1 \times H_2) \subseteq ncl(T) \subseteq E \times Q$. Implies that $(p_0, q_0) \in H_1 \times H_2 \subseteq ncl(H_1) \times ncl(H_2) \subseteq E \times Q$. Therefore $p_0 \in H_1 \subseteq ncl(H_1) \subseteq E$. Hence P is almost n-regular. Similarly we can show that Q is almost n-regular.

Conversely, suppose that P and Q are almost n-regular. Let $(p_0, q_0) \in P \times Q$. Let T be a n-regular open neighbourhood of (p_0, q_0). Then there exists a basic n-regular open set E_1 and H_1 such that $p_0 \in E_1 \subseteq ncl(E_1) \subseteq E$ and $q_0 \in H_1 \subseteq ncl(H_1) \subseteq H$. Hence $(p_0, q_0) \in E_1 \times H_1 \subseteq ncl(E_1) \times ncl(H_1) \subseteq E \times H \subseteq T$. Implies that $(p_0, q_0) \in E_1 \times H_2 \subseteq ncl(E_1 \times H_2) \subseteq E \times H \subseteq T$. Hence $P \times Q$ is almost n-regular. $\qquad\square$

Theorem 3.5. *If J and K are subsets of $(U, \mathcal{N})$, where J and K are n-regular open and $J \subseteq K$ then $ncl(J) \subseteq ncl(nint(ncl(K)))$.*

Proof. Let $J \subseteq K$. Since $J \subseteq ncl(K)$ then $J \subseteq nint(ncl(K)$ because J is n-regular open. Hence $ncl(J) \subseteq ncl(nint(ncl(K)))$. □

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