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NANO I_g-NORMAL AND NANO I_g-REGULAR SPACES

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ABSTRACT. In this paper a new classes of nI_g-normal space and nI_g-regular space are introduced and various characterizations and properties are given. Further, we define a new notion is called nI_{rg}-closed set and establish their various characteristic properties are given.

1. INTRODUCTION

In 2013, Lellis Thivagar *et al.*, [3] introduced a nano topological space. Then the notions of an ideal nano topological space was introduced by Parimala *et al.*, [4, 6]. A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [6] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$. For background material, papers [1] to [10] may be perused. A subset H of a space $(U, \mathcal{N})$ is called ($n\alpha$-open, nr-open and np-open [3]), nI-open [4], ng-closed [2], $n\alpha g$-closed [10], nrg-closed [9] and nI_g-closed [5]. The family of all $n\alpha$-closed (resp. $\mathcal{N}^\alpha$) and the family of all $n\star$-closed(resp. $\mathcal{N}^\star$).

In this paper a new classes of nI_g-normal space and nI_g-regular space are introduced and various characterizations and properties are given. Further, we define a new notion is called nI_{rg}-closed set and establish their various characteristic properties are given.

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2. Preliminaries

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $nint(A)$ and $ncl(A)$ respectively. A nano topological space $(U, \mathcal{N})$ with an ideal I on U is called [6] an ideal nano topological space and is denoted by $(U, \mathcal{N}, I)$ in future is referred as a space.

Definition 2.1. *A subset H of a space $(U, \mathcal{N}, I)$ is called;*

(i) *nano $\star$-closed (briefly $n\star$-closed) [5] if $H_n^* \subseteq H$,*
(ii) *nano dense (briefly n-dense) [7] if $ncl(H) = U$.*

Definition 2.2. *[1] A space $(U, \mathcal{N})$ is said to be almost nano regular (briefly, almost n-regular) if for each n-regular closed set H and a point $a \in U - H$, there exist disjoint n-open sets M and N such that $a \in M$ and $H \subseteq N$.*

Definition 2.3. *[8] A space U is called nano-T_1 space (briefly nT_1-space)for $x, y \in U$ and $x \notin y$, there exists a nano-open sets G and H such that $x \in G$, $y \notin G$, and $y \in H$, $x \notin H$.*

3. nI_g-normal and nI_g-regular spaces

Definition 3.1. *An ideal nanotopological space $(U, \mathcal{N}, I)$ is said to be a nano I_g-normal space(briefly nI_g-normal space) if for every pair of disjoint n-closed set A and B, there exist disjoint nI_g-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.*

Definition 3.2. *An ideal nano I is said to be nano completely codense if $PO(U) \cap I = \{\phi\}$, where $PO(U)$ is the family of all np-open set in $(U, \mathcal{N})$.*

Theorem 3.1. *Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space. Then the following are equivalent.*

(i) *U is nI_g-normal.*
(ii) *For every pair of disjoint n-closed sets E and F, there exist disjoint nI_g-open sets G and H such that $E \subseteq F$ and $F \subseteq H$.*
(iii) *For every n-closed set E and a n-open set H containing E, there exists a nI_g-open set G such that $E \subseteq G \subseteq ncl^*(G) \subseteq H$*

Proof. (i) $\Rightarrow$ (ii) The proof follows from the definition of nI_g-normal spaces.

(ii) $\Rightarrow$ (iii) Let E be a n-closed set and H be a n-open set containing E. Since E and $U-H$ are disjoint n-closed sets, there exist disjoint nI_g-open sets G and S such that $E \subseteq G$ and $U-H \subseteq S$. Again, $G \cap S = \phi$ implies that $G \cap nint^*(S) = \phi$ and so $ncl^*(G) \subseteq U - nint^*(S)$. Since $U-H$ is n-closed and $S \subseteq H$. Thus, we have $E \subseteq G \subseteq ncl^*(G) \subseteq U - nint^*(S) \subseteq H$ which proves (iii).

(iii) $\Rightarrow$ (i) Let E and F be two disjoint n-closed subsets of U. By hypothesis, there exists a nI_g-open set G such that $E \subseteq G \subseteq ncl^*(G) \subseteq U - F$. If $S = U - ncl^*(G)$, then G and S are the required disjoint nI_g-open sets containing E and F respectively. So, $(U, \mathcal{N}, I)$ is nI_g-normal. □

Theorem 3.2. *Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space, where I is nano completely codense. If $(U, \mathcal{N}, I)$ is nI_g-normal then normal space.*

Proof. Suppose that I is nano completely codense. By Theorem 3.1, $(U, \mathcal{N}, I)$ is nI_g-normal if and only if for each pair of disjoint n-closed sets E and F, there exist disjoint nI_g-open sets G and H such that $E \subseteq G$ and $F \subseteq H$ if and only if G is normal. □

Theorem 3.3. *Let $(U, \mathcal{N}, I)$ be a nI_g-normal space. If P is n-closed and E is a ng-closed set such that $E \cap P = \phi$, then there exist disjoint nI_g-open sets G and H such that $E \subseteq G$ and $P \subseteq H$.*

Proof. Since $E \cap P = \phi$, $E \subseteq U - P$ where $U - P$ is n-open. Therefore, by hypothesis, $ncl(E) \subseteq U - P$. Since $ncl(E) \cap P = \phi$ and U is nI_g-normal, there exist disjoint nI_g-open sets G and H such that $ncl(E) \subseteq G$ and $P \subseteq H$. □

Theorem 3.4. *Let $(U, \mathcal{N}, I)$ be a normal ideal nanotopological space which is nI_g-normal. Then the following hold.*

(i) *For every n-closed set E and every ng-open set F containing E, there exists a nI_g-open set G such that $E \subseteq nint^*(G) \subseteq G \subseteq F$.*
(ii) *For every ng-closed set E and every n-open set F containing E, there a nI_g-closed set G such that $E \subseteq G \subseteq ncl^*(G) \subseteq F$.*

Proof. (i) Let E be a n-closed set and F be a ng-open set containing E. Then $E \cap (U - F) = \phi$, where E is n-closed and $U - F$ is ng-closed. By Theorem 3.3, there exist disjoint nI_g-open sets G and H such that $E \subseteq G$ and $U - F \subseteq H$.

Since $G \cap H = \phi$, we have $G \subseteq U - H$, then $E \subseteq nint^*(G)$. Therefore, $E \subseteq nint^*(G) \subseteq G \subseteq U - H \subseteq F$. This proves (i).

(ii) Let E be a ng-closed set and F be a n-open set containing E. Then $U - F$ is a n-closed set contained in the ng-open set $U - E$. By (i), there exists a nI_g-open set H such that $U - F \subseteq nint^*(H) \subseteq H \subseteq U - E$. Therefore, $E \subseteq U - H \subseteq ncl^*(U - H) \subseteq F$. If $G = U - H$, then $E \subseteq G \subseteq ncl^*(G) \subseteq F$ and so G is the required nI_g-closed set. □

Definition 3.3. *An ideal nanotopological space $(U, \mathcal{N}, I)$ is said to be nano gI-normal (briefly ngI-normal) if for each pair of disjoint nI_g-closed sets E and F, there exist disjoint n-open sets G and H in U such that $E \subseteq G$ and $F \subseteq H$.*

Theorem 3.5. *In an ideal nanotopological space $(U, \mathcal{N}, I)$, every n-closed set is nI_g-closed.*

Proof. Obvious. □

Proposition 3.1. *In an ideal nanotopological space $(U, \mathcal{N}, I)$, every ngI-normal space is normal. But the converse is not true as seen from the following Example.*

Example 1. *Let $U = \{n_1, n_2, n_3, n_4\}$ with $U/R = \{\phi, \{n_1\}, U\}$, $X = \{n_1\}$ and $I = \{\phi, \{n_1\}\}$. Since $\{n_1\}^*_n = \phi$, every n-open set is $n\star$-closed and so every subset of U is nI_g-closed. Now $A = \{n_1, n_2\}$ and $B = \{n_3, n_4\}$ are disjoint nI_g-closed sets, but they are not separated by disjoint n-open sets. So $(U, \mathcal{N}, I)$ is not ngI-normal. Since there is no pair of disjoint n-closed sets, $(U, \mathcal{N}, I)$ is normal.*

Theorem 3.6. *In an ideal nanotopological space $(U, \mathcal{N}, I)$, the following are equivalent.*

(i) *U is ngI-normal.*
(ii) *For every nI_g-closed set E and every nI_g-open set F containing E, there exists a n-open set G of U such that $E \subseteq G \subseteq ncl(G) \subseteq F$.*

Proof.
(i) $\Rightarrow$ (ii). Let E be a nI_g-closed set and F be a nI_g-open set containing E. Since E and $U - F$ are disjoint nI_g-closed sets, there exist disjoint n-open sets G and H such that $E \subseteq G$ and $U - F \subseteq H$. Now $G \cap H = \phi \Rightarrow ncl(G) \subseteq U - H$. Therefore, $E \subseteq G \subseteq ncl(G) \subseteq U - H \subseteq F$. This proves (ii).
(ii) $\Rightarrow$ (i). Suppose E and F are disjoint nI_g-closed sets, then the nI_g-closed set E is contained in the nI_g-open set $U - F$. By hypothesis, there exists a n-open

set G of U such that $E \subseteq G \subseteq ncl(G) \subseteq U - F$. If $H = U - ncl(G)$, then G and H are disjoint n-open sets containing E and F respectively. Therefore, $(U, \mathcal{N}, I)$ is ngI-normal. □

Theorem 3.7. *In an ideal space $(U, \mathcal{N}, I)$, the following are equivalent.*

(i) *U is ngI-normal.*
(ii) *For each pair of disjoint nI_g-closed subsets E and F of U, there exists a n-open set G of U containing E such that $ncl(G) \cap F = \phi$.*
(iii) *For each pair of disjoint nI_g-closed subsets E and F of U, there exists a n-open set G containing E and a n-open set H containing F such that $ncl(G) \cap ncl(H) = \phi$.*

Proof.
(i) $\Rightarrow$ (ii). Suppose that E and F are disjoint nI_g-closed subsets of U. Then the nI_g-closed set E is contained in the nI_g-open set $U - F$. By Theorem 3.6, there exists a n-open set G such that $E \subseteq G \subseteq ncl(G) \subseteq U - F$. Therefore, G is the required n-open set containing E such that $ncl(G) \cap F = \phi$.

(ii) $\Rightarrow$ (iii). Let E and F be two disjoint nI_g-closed subsets of U. By hypothesis, there exists a n-open set G containing E such that $ncl(G) \cap F = \phi$. Also, $ncl(G)$ and F are disjoint nI_g-closed sets of U. By hypothesis, there exists a n-open set H containing F such that $ncl(G) \cap ncl(H) = \phi$.

(iii) $\Rightarrow$ (i) The proof is clear. □

Theorem 3.8. *Let $(U, \mathcal{N}, I)$ be a ngI-normal space. If E and F are disjoint nI_g-closed subsets of U, then there exists disjoint n-open sets G and H such that $ncl^*(E) \subseteq G$ and $ncl^*(F) \subseteq H$.*

Proof. Suppose that E and F are disjoint nI_g-closed sets. By Theorem 3.7 (iii), there exists a n-open set G containing E and a n-open set H containing F such that $ncl(G) \cap ncl(H) = \phi$. Since E is nI_g-closed, $E \subseteq G \Rightarrow ncl^*(E) \subseteq G$. Similarly $ncl^*(F) \subseteq H$. □

Theorem 3.9. *Let $(U, \mathcal{N}, I)$ be a ngI-normal space. If E is a nI_g-closed set and F is a nI_g-open set containing E, then there exists a n-open set G such that $E \subseteq ncl^*(E) \subseteq G \subseteq nint^*(F) \subseteq F$.*

Proof. Suppose E is a nI_g-closed set and F is a nI_g-open set containing E. Since E and $U - F$ are disjoint nI_g-closed sets, by Theorem 3.8, there exist disjoint

n-open sets G and H such that $ncl^*(E) \subseteq G$ and $ncl^*(U-F) \subseteq H$. Now, $U - nint^*(F) = ncl^*(U-F) \subseteq H \Rightarrow U - H \subseteq nint^*(F)$. Again, $G \cap H = \phi \Rightarrow G \subseteq U - H$ and so $E \subseteq ncl^*(E) \subseteq G \subseteq U - H \subseteq nint^*(E) \subseteq F$. □

Definition 3.4. *A subset E of an ideal nanotopological space $(U, \mathcal{N}, I)$ is said to be a nano regular generalized closed set with respect to an ideal I (briefly nI_{rg}-closed) if $E_n^* \subseteq G$ whenever $E \subseteq G$ and G is n-regular open.*

E is called nI_{rg}-open if $U - E$ is nI_{rg}-closed.

Theorem 3.10. *In an ideal nanotopological space $(U, \mathcal{N}, I)$, every nI_g-closed set is nI_{rg}-closed.*

Proof. Follows from the Definitions 3.1 and 3.4

Lemma 3.1. *Let $(U, \mathcal{N}, I)$ be an ideal nanotopological spaces. A subset $E \subseteq U$ is nI_{rg}-open if and only if $P \subseteq nint^*(E)$ whenever P is nr-closed and $P \subseteq E$.*

Proof. Suppose that E is nI_{rg}-open. Let P be a nr-closed set contained in E. Then $U - E \subseteq U - P$ and $U - P$ is nr-open. Since $U - E$ is nI_{rg}-closed, $ncl^*(U-E) \subseteq U - P$ and so $P \subseteq U - ncl^*(U-E) = nint^*(E)$.

Conversely, suppose $U - E \subseteq G$ and G is nr-open. Then $U - G \subseteq E$ and $U - G$ is nr-closed. By our assumption, $U - G \subseteq nint^*(E)$ and so $U - nint^*(E) \subseteq G$ which implies that $ncl^*(U-E) \subseteq G$. Therefore, $U - E$ is nI_{rg}-closed and so E is nI_{rg}-open. □

Definition 3.5. *A space $(U, \mathcal{N})$ is said to be a mildly nano normal (briefly mildly n-normal), if disjoint nr-closed sets are separated by disjoint n-open sets.*

Theorem 3.11. *Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space, where I is nano completely codense. Then the following are equivalent.*

(i) *U is mildly n-normal.*
(ii) *For disjoint nr-closed sets E and F, there exist disjoint nI_g-open sets G and H such that $E \subseteq G$ and $F \subseteq H$.*
(iii) *For disjoint nr-closed sets E and F, there exist disjoint nI_{rg}-open sets G and H such that $E \subseteq G$ and $F \subseteq H$.*
(iv) *For a nr-closed set E and a nr-open set H containing E, there exists a nI_{rg}-open set G of U such that $E \subseteq G \subseteq ncl^*(G) \subseteq H$.*
(v) *For a nr-closed set E and a nr-open set H containing E, there exists a $n\star$-open set G of U such that $E \subseteq G \subseteq ncl^*(G) \subseteq H$.*

(vi) *For disjoint nr-closed sets E and F, there exist disjoint $n\star$-open sets G and H, such that $E \subseteq G$ and $F \subseteq H$.*

Proof.

(i) $\Rightarrow$ (ii). Suppose that E and F are disjoint nr-closed sets. Since U is mildly normal, there exists n-open sets G and H such that $E \subseteq G$ and $F \subseteq H$. But every n-open set is a nI_g-open set. This proves (ii).

(ii) $\Rightarrow$ (iii). The proof follows from the fact that every nI_g-open set is a nI_{rg}-open set.

(iii) $\Rightarrow$ (iv). Suppose E is nr-closed and F is a nr-open set containing E. Then E and $U - F$ are disjoint nr-closed sets. By hypothesis, there exists disjoint nI_{rg}-open sets G and H such that $E \subseteq G$ and $U - F \subseteq H$. Since $U - F$ is nr-closed and H is nI_{rg}-open, then $U - F \subseteq nint^*(H)$ and so $U - nint^*(H) \subseteq F$. Again, $G \cap H = \phi$ implies that $G \cap nint^*(H) = \phi$ and so $ncl^*(G) \subseteq U - nint^*(H) \subseteq F$. Hence G is required nI_{rg}-open set such that $E \subseteq G \subseteq ncl^*(G) \subseteq F$.

(iv) $\Rightarrow$ (v). Let E be a nr-closed set and H be a nr-open set containing E. Then there exists a nI_{rg}-open set J of U such that $E \subseteq J \subseteq ncl^*(J) \subseteq H$. By Lemma 3.1, $E \subseteq nint^*(J)$. If $G = nint^*(J)$, then G is a $n\star$-open set and $E \subseteq G \subseteq ncl^*(G) \subseteq ncl^*(J) \subseteq H$. Therefore, $E \subseteq G \subseteq ncl^*(G) \subseteq H$.

(v) $\Rightarrow$ (vi). Let E and F be disjoint nr-closed subsets of U. Then $U - F$ is a nr-open set containing E. By hypothesis, there exists a $n\star$-open set G of U such that $E \subseteq G \subseteq ncl^*(G) \subseteq U - F$. If $H = U - ncl^*(G)$, then G and H are disjoint $n\star$-open sets of U such that $E \subseteq G$ and $F \subseteq H$.

(vi) $\Rightarrow$ (i). Let E and F be disjoint nr-closed sets of U. Then there exist disjoint $n\star$-open sets G and H such that $E \subseteq G$ and $F \subseteq H$. Since I is nano completely codence, then $\mathcal{N}^\star \subseteq \mathcal{N}^\alpha$ and so $G, H \in \mathcal{N}^\alpha$. Hence $E \subseteq G \subseteq nint(ncl(nint(G))) = J$ and $F \subseteq H \subseteq nint(ncl(nint(H))) = K$. J and K are the required disjoint n-open sets containing E and F respectively. This proves (i). □

4. nI_g-REGULAR SPACES

Definition 4.1. *An ideal nanotopological space $(U, \mathcal{N}, I)$ is said to be a nano I_g-regular space(briefly nI_g-regular space) if for each pair consisting of a point y and a closed set F not containing y, there exist disjoint nI_g-open sets G and H such that $y \in G$ and $F \subseteq H$.*

Theorem 4.1. *In an ideal nanotopological space $(U, \mathcal{N}, I)$, the following are equivalent.*

(i) *U is nI_g-regular.*
(ii) *For every n-closed set F not containing $a \in U$, there exist disjoint nI_g-open sets G and H such that $a \in G$ and $F \subseteq H$.*
(iii) *For every n-open set H containing $a \in U$, there exists a nI_g-open set G of U such that $a \subseteq G \subseteq ncl^*(G) \subseteq H$.*

Proof.
(i) and (ii) are equivalent by the Definition 4.1.

(ii) $\Rightarrow$ (iii). Let H be a n-open subset such that $a \in H$. Then $U - H$ is a n-closed set not containing a. Therefore, there exist disjoint nI_g-open sets G and S such that $a \in G$ and $U - H \subseteq S$. Now, $U - H \subseteq S$ implies that $U - H \subseteq nint^*(S)$ and so $U - nint^*(S) \subseteq H$. Again, $G \cap S = \phi$ implies that $G \cap nint^*(S) = \phi$ and so $ncl^*(G) \subseteq U - nint^*(S)$. Therefore $a \in G \subseteq ncl^*(G) \subseteq H$. This proves (iii).

(iii) $\Rightarrow$ (i). Let F be a n-closed set not containing a. By hypothesis, there exists a nI_g-open set G such that $a \in G \subseteq ncl^*(G) \subseteq U - F$. If $S = U - ncl^*(G)$, then G and S are disjoint nI_g-open sets such that $a \in G$ and $F \subseteq S$. This proves (i). □

Theorem 4.2. *If $(U, \mathcal{N}, I)$ is a nI_g-regular, nT_1-space where I is nano completely codense, then U is nr-closed.*

Proof. Let F be a n-closed set not containing $a \in U$. By Theorem 4.1, there exists a nI_g-open set G of U such that $a \in G \subseteq ncl^*(G) \subseteq U - F$. Since U is a nT_1-space, $\{a\}$ is n-closed and so $\{a\} \subseteq nint^*(G)$. Since I is nano completely codense, $\mathcal{N}^\star \subseteq \mathcal{N}^\alpha$ and so $nint^*(G)$ and $U - ncl^*(G)$ are $\mathcal{N}^\alpha$-open sets. Now, $a \in nint^*(G) \subseteq nint(ncl(nint(nint^*(G)))) = J$ and $F \subseteq U - ncl^*(G) \subseteq nint(ncl(nint(U - ncl^*(G)))) = K$. Then J and K are disjoint n-open sets containing a and F respectively. Therefore, U is nr-closed. □

Theorem 4.3. *If every n-open subset of an ideal nanotopological space $(U, \mathcal{N}, I)$ is $n\star$-closed, then $(U, \mathcal{N}, I)$ is nI_g-regular.*

Proof. Suppose every n-open subset of U is $n\star$-closed. Then every subset of U is nI_g-closed and hence every subset of U is nI_g-open. If F is a n-closed set not containing a, then $\{a\}$ and F are the required disjoint nI_g-open sets containing a and F respectively. Therefore, $(U, \mathcal{N}, I)$ is nI_g-regular. □

Theorem 4.4. *Let $(U, \mathcal{N}, I)$ be an ideal topological space where I is nano completely codense. Then the following are equivalent.*

(i) *U is nr-closed.*

(ii) *For every n-closed set E and each $a \in U - E$, there exist disjoint $n\star$-open sets G and H such that $a \in G$ and $E \subseteq H$.*

(iii) *For every n-open set H of U and $a \in H$, there exists a $n\star$-open set G such that $a \in G \subseteq ncl^*(G) \subseteq H$.*

Proof.

(i) $\Rightarrow$ (ii). Let E be a n-closed subset of U and let $a \in U - E$. Then there exist disjoint n-open sets G and H such that $a \in G$ and $E \subseteq H$. But every n-open set is $n\star$-open. This proves (ii).

(ii) $\Rightarrow$ (iii). Let H be a n-open set containing $a \in U$. Then $U - H$ is n-closed and $a \in H$. By hypothesis, there exist disjoint $n\star$-open sets G and S such that $a \in G$ and $U - H \subseteq S$. Since $G \cap S = \phi$, we have $G \subseteq U - S$ and $U - S$ is $n\star$-closed. So $ncl^*(G) \subseteq U - S \subseteq H$. Therefore, G is the required $n\star$-open set such that $a \in G \subseteq ncl^*(G) \subseteq H$.

(iii) $\Rightarrow$ (i). Let E be a n-closed set and $a \notin E$. By (iii), there exists a $n\star$-open set G such that $a \in G \subseteq ncl^*(G) \subseteq U - E$. Let $H = U - ncl^*(G)$. Then $E \subseteq H$, and G and H are disjoint $n\star$-open sets. Since I is nano completely codense, $\mathcal{N}^\star \subseteq \mathcal{N}^\alpha$ and so G and H are $\mathcal{N}^\alpha$-open sets. Therefore, $G \subseteq nint(ncl(nint(G))) = J$ and $E \subseteq H \subseteq nint(ncl(nint(H))) = K$. Then J and K are disjoint n-open sets such that $a \in J$ and $E \subseteq K$. Hence U is n-regular.

□

Theorem 4.5. *Let $(U, \mathcal{N}, I)$ be an ideal nanotopological space, where I is nano completely codense. Then the following are equivalent.*

(i) *U is almost n-regular.*

(ii) *For each nr-closed set E and each $a \in U - E$, there exist disjoint $n\star$-open sets G and H such that $a \in G$ and $E \subseteq H$.*

(iii) *For each nr-open set H and $a \in H$, there exists a $n\star$-open set G such that $a \in G \subseteq ncl^*(G) \subseteq H$.*

Proof.

(i) $\Rightarrow$ (ii). Let $E \subseteq U$ be nr-closed and $a \in U - E$. Then there exist disjoint n-open sets G and H such that $a \in G$ and $E \subseteq H$. Since every n-open set is a $n\star$-open set, the proof follows.

(ii) $\Rightarrow$ (iii). Let H be nr-open and $a \in H$. By (ii), there exist disjoint $n\star$-open sets G and S such that $a \in G$ and $U - H \subseteq S$. Since $G \cap S = \phi$, we have $ncl^*(G) \subseteq U - S \subseteq H$. Therefore, G is the required $n\star$-open set such that $a \in G \subseteq ncl^*(G) \subseteq H$.

(iii) $\Rightarrow$ (i). Let E be nr-closed and $a \in U - E$. By hypothesis, there exists a $n\star$-open set G such that $a \in G \subseteq ncl^*(G) \subseteq U - E$. Let $H = U - ncl^*(G)$. Then $E \subseteq H$, and G, H are disjoint $n\star$-open sets. Since I is nano completely codense, $n\tau^\star \subseteq n\tau^\alpha$ and so G and H are $n\alpha$-open sets. Therefore, we have $a \in G \subseteq nint(ncl(nint(G))) = J$ and $E \subseteq H \subseteq nint(ncl(nint(H))) = K$. Then J and K are the required disjoint n-open sets such that $a \in J$ and $E \subseteq K$. Hence U is almost n-regular. $\square$

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