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LIAR DOMINATION ON BIPOLAR FUZZY GRAPHS

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ABSTRACT. A set $B \subseteq V$ in a bipolar fuzzy graph is called a liar dominating set if B satisfies the following axioms:

(1) Each $v_i \in V(G)$ is dominated by minimum two vertices in bipolar fuzzy graph G.

(2) Each pair $v_i, v_j \in V(G)$, jointly dominated by minimum three vertices in bipolar fuzzy graph G.

x is said to be dominated by y if $x \in N[\ y]$ where $N[\ y] = y \cup \{x \in V(G), \mu^{\infty}(x,y) = \mu(x,y)\}$. In this manuscript, we discussed liar domination on bipolar fuzzy graphs and some of their properties.

1. INTRODUCTION

In 1735, Euler presented the graph theory [14]. In graph theory there are many applications that solve combinatorial problems in various fields including computer science, economics, social science, physics, chemistry, civil engineering, electrical engineering, biology, etc. In 1965, Lotfy Zadeh [12] introduced fuzzy set of a crisp set. Fuzzy theory is applied in various areas such as medical and life Sciences, management science, artificial intelligence, networking and decision making, etc. In 2008, Peter J. Slater [11] presented liar domination in theory of graphs. Liar domination is advancement of domination which is

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used to detect the intruder location in a network. B.S. Panda [9] analyzed that liar domination set is applied in deploying protection devices with minimum number of nodes so that the fault can be identified and reported correctly. In 1973, Kauffman [2] initiated fuzzy graphs. In 1975, Rosenfeld [10] developed fuzzy graph theory. In 1987, Battacharya [1] discussed some remarks on fuzzy graphs. In 1994, J.N. Moderson et al. [4] studied operations on fuzzy graphs. In 2002, M.S. Sunitha et. al. [13] studied complement of fuzzy graphs.

In 1994, Zhang [15, 16] introduced bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy set whose membership value is $[\,-1,1]$. In an bipolar fuzzy sets membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $(0,1]$ of an element represents that the element somewhat satisfies the property and membership degree $[\,-1,0)$ of an element represents that the element somewhat satisfies the implicit counter property. M. Akram [5–7] discussed bipolar fuzzy graphs, some of their properties and applications. M.G. Karunambigai et al. [3] developed the concepts of domination, independent and irredundant on bipolar fuzzy graphs. Further, M. Akram et al. [8] presented novel applications of bipolar fuzzy graphs to decision making problem. Motivated by the above works, in this manuscript, we discussed the liar domination and some of its properties on bipolar fuzzy graphs.

2. Preliminaries

In this section, we recall some definitions which are very useful to prove our main results.

Definition 2.1. *An edge (u,v) is said to be strong edge in bipolar fuzzy graph (in short BFG), if*

$$\mu_2^+(u,v) = (\mu_2^+)^\infty(u,v)$$

and

$$\mu_2^-(u,v) = (\mu_2^-)^\infty(u,v),$$

where $(\mu_2^+)^\infty(u,v) = \max\{(\mu_2^+)^k(u,v), k = 1,2,\ldots,n\}$. That is, $(\mu_2^+)^\infty(u,v)$ = maximum of μ^+ strengths of all paths between u and v; and $(\mu_2^-)^\infty(u,v) = \min\{(\mu_2^-)^k(u,v), k = 1,2,\ldots,n\}$. That is, $(\mu_2^-)^\infty(u,v)$ = minimum of μ^- strengths of all paths between u and v.

Definition 2.2. *Let u be a vertex in bipolar fuzzy graph. Then, $N[\,u] = \{u\} \cup \{v :$ $v \in V$ and (u, v) is strong edge in BFG $\}$ is called closed neighbourhood of the vertex u in G.*

Definition 2.3. *A vertex u is said to be a isolated vertex if $N(u) = \phi$.*

Remark 2.1. *An isolated vertex u is dominated by u itself.*

Definition 2.4. *We say that u dominates v if $v \in N[\,u]$. This implies that u dominates itself. Obviously, domination satisfies symmetric relation on V. That is, u dominates v iff v dominates $u, \forall\, u, v \in V$.*

Remark 2.2. *If no edge is strong edge in BFG, then we can not form liar domination set for bipolar fuzzy graphs, since every node is single dominant node in this case.*

3. Results and Discussion

In this section, we present and prove the main results. Before we prove the main theorem, first we list some of the properties of liar dominating set.

(1) A set $B \subseteq V$ in a BFG is called liar dominating set if B satisfies the following two conditions:
   (a) every $u \in v$ is dominated by minimum two nodes in BFG.
   (b) every $u, v \in V$, together are dominated by minimum three nodes in BFG.
(2) The minimum fuzzy cardinality of liar domination set B in BFG is called liar domination number of BFG and is denoted by $\lambda(BFG)$.
   Example:

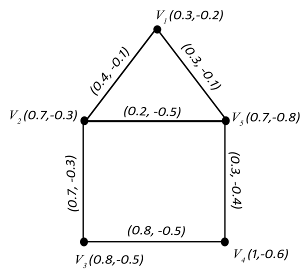

FIGURE 1. $\lambda(BFG) = \{V_1, V_2, V_4, V_5\}$ forms liars domination set.

(3) A liar's domination set B' in BFG, is called a minimal liar domination set if no proper subset of B' is a liar domination set.
(4) A liar domination set B in BFG is called a minimum liar domination set, if there is no liar domination set B' such that $|B'| < |B|$.
(5) A liar dominating set B in BFG is called a maximum liar domination set if there is no liar domination set B' such that $|B'| > |B|$.

Example:

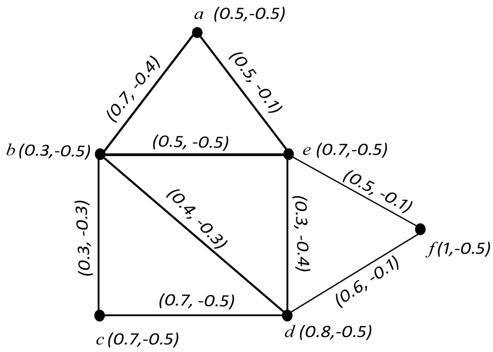

FIGURE 2. $\{a, b, d, c, d, e, f\}$ is maximum liar dominating set.

Theorem 3.1. *A liar dominating set B in BFG $G =< V, E >$ is a minimal liar dominating set if for each $t_m \in B, m = 1, 2, \ldots, i, \ldots, j, \ldots, n$, then one of the following conditions hold:*

(i) *t_i (for some i) is a strong neighbour to only one vertex in B;*
(ii) *There are nodes $v \in V_B$ such that $N(v) \cap B = \{t_i, t_j\}, t_i, t_j \in B$.*

Proof. Let B are a minimal liar dominating set in BFG. Let $t_i \in B$. Then $B - \{t_i\}$ is not a liar domination set in G. This implies that some $u \in V$ is not dominated by $B - \{t_i\}$.

Suppose that $u = t_i$, then t_i is strong neighbour to only one vertex in B. Otherwise, if $v \neq t_i$, (i.e) $v \in V - B$, then there are two strong neighbours to v in B. That is, $N(v) \cap B = \{t_i, t_j\}$ where $t_i, t_j \in B$.

Conversely, let us assume that the given two conditions (i) & (ii) hold in BFG.

Suppose B is not a minimal liar dominating set. If $v \neq t_i \in B$, then there are two strong neighbours to v in B, then this violates condition (i).

Let $v \in V - B$, then each v is dominated by at least three vertices in B. Condition (ii) fails in this case.

Therefore B must be minimal liar dominating set in BFG. □

Theorem 3.2. *In a complete bipolar fuzzy graphs all arcs are strong.*

Proof. Let k_n be the complete bipolar fuzzy graph with $n \geq 3$ vertices. Then

$$\mu_2^+(u,v) = \min(\mu_1^+(u), \mu_1^+(v))$$
$$\mu_2^-(u,v) = \min(\mu_1^-(u), \mu_1^-(v)) \text{ for all } u, v \in V(k_n).$$

Let

$$\mu_2^+(u,v) = \min(\mu_1^+(u), \mu_1^+(v)) = \mu_1^+(v);\ \mu_2^-(u,v) = \min(\mu_1^-(u), \mu_1^-(v)) = \mu_1^-(v).$$

There are one or more than one fuzzy paths joining the nodes $u\&v$.

Suppose $u - w - v$ is one fuzzy path, and $u - x - v$ is another fuzzy path. If $\mu_1^+(w) > \mu_1^+(v)$, then $\mu_2^+(wv) = \mu_1^+(v)$ and if $\mu_1^+(x) > \mu_1^+(v)$, then $\mu_2^+(xv) = \mu_1^+(v)$. Hence, $(\mu_2^+)^\infty(u,v) = \mu_2^+(u,v)$. Otherwise, $(\mu_2^+)^\infty(u,v) = \mu_2^+(w)$. Since $\mu_1^+(u) > \mu_1^+(u)$ and $\mu_1^+(w) < \mu_1^+(v)$, we have $\mu_2^+(uw) > \mu_1^+(w)$. Then strength of $u - w - v$ fuzzy path is $\mu_1^+(w)$. Then, $(\mu_2^+)^\infty(uw) = \mu_1^+(w)$.

Similarly, we can prove that, $(\mu_2^-)^\infty(u,v) > \mu_2^+(u,v)$. Hence uv is strong arc. Similarly, we can show each arc is strong arc. Thus, in a complete bipolar fuzzy graph all arcs are strong.

Example: □

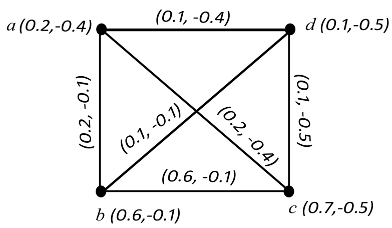

FIGURE 3

Theorem 3.3. *Liar dominating number of complete bipolar fuzzy graph is*

$$\lambda(k_n) = \sum_{i=1}^{3} \left[\min_{v_i \in V} \left\{ \frac{1 + \mu^+(v_i) + \mu^-(v_i)}{2} \right\} \right].$$

In other words,

$$\lambda(k_n) = \min\{|v_i|, v_i \in V\} + \min\{|v_j|, v_j \in V - v_j\} + \min\{|v_k|, v_k \in V - v_i - v_j\}.$$

Proof. From Theorem 3.2, all edges in complete bipolar fuzzy graphs are strong. Therefore

$$(\mu_2^+)^\infty(u,v) = \mu_2^+(u,v)$$

and

$$(\mu_2^-)^\infty(u,v) = \mu_2^-(u,v), \text{ for all } u,v \in V(k_n).$$

Liar domination set contains at least three vertices, in any bipolar fuzzy graph. Therefore, three vertices which have minimum membership value form minimum liar domination set and hence, we have

$$\lambda(k_n) = \sum_{i=1}^{3} \left[\min_{v_i \in V} \left\{ \frac{1+\mu^+(v_i)+\mu^-(v_i)}{2} \right\} \right].$$

□

Theorem 3.4. *Liar domination number of complete bipolar fuzzy graph is*

$$\lambda(k_{v_1,v_2}) = \sum_{i=1}^{3} \left[\min_{u_i \in V_1} \left\{ \frac{1+\mu^+(u_i)+\mu^-(u_i)}{2} \right\} \right] + \sum_{j=1}^{3} \left[\min_{u_j \in V_2} \left\{ \frac{1+\mu^+(u_j)+\mu^-(u_j)}{2} \right\} \right].$$

Proof. Let k_{v_1,v_2} be a complete bipartite bipolar fuzzy graph. Then

$$\mu_2^+(u_i,u_j) = \min(\mu_1^+(u_i), \mu_1^+(u_j))$$

and

$$\mu_2^-(u_i,u_j) = \max(\mu_1^-(u_i), \mu_1^-(u_j)) \text{ for all } u_i \in v_1 \text{ and } u_j \in v_2.$$

Using Theorem 3.2, all the edges in K_{V_1,V_2} are strong.

To satisfy the conditions of liar domination set, liar dominating set B must contain at least three vertices and hence

$$\lambda(k_{v_1,v_2}) = \sum_{i=1}^{3} \left[\min_{u_i \in V_1} \left\{ \frac{1+\mu^+(u_i)+\mu^-(u_i)}{2} \right\} \right] + \sum_{j=1}^{3} \left[\min_{u_j \in V_2} \left\{ \frac{1+\mu^+(u_j)+\mu^-(u_j)}{2} \right\} \right].$$

□

Theorem 3.5. *Liar dominating set in a bipolar fuzzy graph contains atleast three vertices.*

Proof. Suppose liar dominating set B is a singleton set. That is, it contains only one vertex x.If the device which is identify the intruder location fixed in x and fails to report the intruder location correctly, then B will not be a liar dominating set.

Suppose B contains two vertices y, z. Let the devices be fixed in y and z. z reports the intruder location which is in y correctly. But y reports the intruder location which is in z wrongly, then also B can not be a liar dominating set.

And, now B contains three vertices x, y, z. Let the devices be fixed in $x, y\&z$. x reports y as intruder location, y reports itself as intruder location, but z reports x as intruder location, then one can easily identify the intruder's location which is at y. And, hence liar dominating set B in BFG must contain at least three vertices. □

Theorem 3.6. *Liar dominating set B exists in a bipartite fuzzy graph if and only if every vertex in V_1 & V_2 are dominated by at least two vertices in V_1 & V_2 respectively, and every pair of vertices in V_1 & V_2 are dominated by atleast three vertices in V_1 & V_2 respectively.*

Proof. Proof comes from the definition of liar dominating set. □

Theorem 3.7. *Liar dominating set B exists in a bipolar fuzzy graphs iff $|N(u) \cap B| \geq 2$ and $|N(u) \cup N(v) \cap B| \geq 3$, where $u, v \in V(G)$.*

Proof. Let Liar dominating set B exists in a bipolar fuzzy graph. Then,
$\Longleftrightarrow$ every vertex $u \in G$ is dominated by minimum two vertices in B
$\Longleftrightarrow$ This implies that, $|N(u) \cap B| \geq 2$.
$\Longleftrightarrow$ every pair $u, v \in G$ together are dominated by minimum three vertices in B.

This shows that $|N(u) \cup N(v) \cap B| \geq 3$. □

4. Conclusion

Bipolar fuzzy graphs are generalization of fuzzy graphs. When we handle spatial information, bipolar information occurs in image processing or in application for spatial reasoning. When we identify the position of some object in a positive information to represent set of possible places and negative information to represent set of impossible places. In this manuscript, we presented liar domination on bipolar fuzzy graphs. Also we discussed some of their characteristics.

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