

A NEW APPROACH OF STRONG DOMINATION IN HESITANCY FUZZY GRAPHS USING VERTEX CARDINALITY

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ABSTRACT. In this paper, the vertex degree and strong neighbor of the vertex are introduced by using membership degree, non-membership degree and hesitancy degree of vertex in the Hesitancy fuzzy graph $G(V, E)$ is defined and strong domination number of Hesitancy fuzzy graph $G(V, E)$ also defined. Further some properties and bounds of strong domination number of the hesitancy fuzzy graph are discussed.

1. INTRODUCTION

Zadeh LA introduces the concept Fuzzy set theory. Furthermost of the actual world complications are enormously multipart and hold vague data. In mandate to quantity the lack of certainty, Torra V introduced extraprogress to Fuzzy sets and named the progression it as Hesitant Fuzzy Sets(HFSs). HFSs are encouraged to handle the mutual trouble that performs in setting the membership degree of ancomponent from some potential standards. This circumstances is slightlymutual in decision making problems moreover while an professional is asked to assign different degrees of membership to a set of elements $\{x, y, z, \dots\}$ in a set A. Frequently problems arise due to uncertain issues and situations hence one is faced with hesitant moments. The investigator had to find ways and means to take the problems and arrive at a solution. Therefore investigators have taken up the learning and application of

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HFS. HFSs have been extended Xu Z. and Zhu B, from different perspectives such as, both quantitative and qualitative.

In this paper, the vertex degree and strong neighbor of the vertex are introduced by using membership, non-membership and hesitancy degree of vertex in the Hesitancy fuzzy graph $G(V, E)$ is defined and strong domination number of a Hesitancy fuzzy graph $G(V, E)$ also defined. Further some properties and bounds of strong domination number of the hesitancy fuzzy graph are discussed.

2. PRELIMINARIES

A fuzzy graph with V as the underlying set is a pair $G(\sigma, \mu)$ where $\sigma : V \rightarrow [0, 1]$ is a fuzzy subset, $\mu : V \times V \rightarrow [0, 1]$ is a fuzzy relation on the fuzzy subset σ , such that $\mu(uv) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. Two nodes x and y are said to be neighbor if $\mu(xy) > 0$.

A fuzzy graph $G(\sigma, \mu)$ with the underlying set V , the order of G is defined and denoted by $O(G) = \sum_{u \in V} \sigma(u)$ and size of G is define and denoted by $S(G) = \sum_{u, v \in V} \mu(uv)$.

Let $G(\sigma, \mu)$ be a fuzzy graph. The degree of a node is defined as $d(u) = \sum_{v \neq u, v \in V} \mu(uv)$. An edge in a fuzzy graph $G(\sigma, \mu)$ is said to be an strong edge if $\mu(uv) = \sigma(u) \wedge \sigma(v)$.

A fuzzy graph $G(\sigma, \mu)$ is said to be a complete fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. A fuzzy graph $G : (\sigma, \mu)$ is said to be a strong fuzzy graph if $\mu(uv) = \sigma(u) \wedge \sigma(v)$ for all $uv \in E$. Let $G(\sigma, \mu)$ be a fuzzy graph and u be a node in G then there exist a node v such that (u, v) is a strong arc then we say that u dominates v .

Let $G(\sigma, \mu)$ be a fuzzy graph. A set D of V is said to be fuzzy dominating set of G if every $v \in V - D$ there exit $u \in D$ such that u dominates v . A fuzzy dominating set D of a fuzzy graph G is called minimal fuzzy dominating set of G , if every node $v \in D$, $D - \{v\}$, is not a fuzzy dominating set. The fuzzy dominating number $\gamma(G)$ of the fuzzy graph G is the minimum cardinality taken over all minimal fuzzy dominating set of G .

A Hesitancy fuzzy graph $G(V, E)$, where the vertex sex V is a triplet fuzzy functions it is defined by $\mu_1 : V \rightarrow [0, 1]$, $\nu_1 : V \rightarrow [0, 1]$ and $\beta_1 : V \rightarrow [0, 1]$, these functions are called as membership, non-membership and hesitancy of

the vertex $v_i \in V$ respectively and $\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1$, $\beta_1(v_i) = 1 - [\mu_1(v_i) + \gamma_1(v_i)]$. The edge set of $G(V, E)$ is a triplet fuzzy functions it is defined by $\mu_2 : V \times V \rightarrow [0, 1]$, $\gamma_2 : V \times V \rightarrow [0, 1]$ and $\beta_2 : V \times V \rightarrow [0, 1]$, such that $\mu_2(uv) \leq \mu_1(u) \wedge \mu_1(v)$, $\beta_2(uv) \leq \beta_1(u) \wedge \beta_1(v)$ and $0 \leq \mu_2(uv) + \gamma_2(uv) + \beta_2(uv) \leq 1$ for every $uv \in E$.

3. STRONG DOMINATION IN HESITANCY FUZZY GRAPH

3.1. Degree and Order of a Hesitancy Fuzzy Graph using Vertex Cardinality.

In this section introduce the degree order of the vertex in hesitancy fuzzy graph using vertex cardinality and also define strong dominating set.

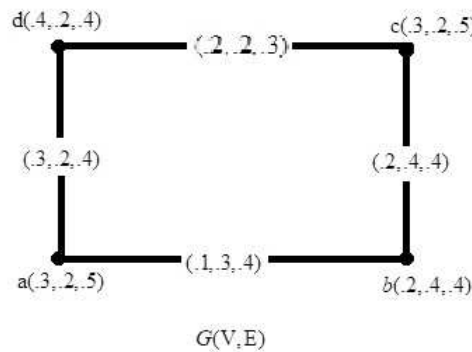
Definition 3.1. The cardinality of the vertex $v \in V$ in the hesitancy fuzzy graph $G(V, E)$ is defined by $|v| = \left\lfloor \frac{1 + \mu_1(v) + \beta_1(v) - \gamma_1(v)}{3} \right\rfloor$.

Definition 3.2. An edge $uv \in E$ in a hesitancy fuzzy graph $G(V, E)$, is said to be an strong edge such that $\mu_2(uv) = \mu_1(u) \wedge \mu_1(v)$, $\beta_2(uv) = \beta_1(u) \wedge \beta_1(v)$. The vertex u and v are said to be adjacent vertices and neighborhood vertices. The neighborhood set $N(u)$ is set all vertices that are adjacent to the vertex u .

Definition 3.3. The neighborhood degree and effective neighborhood degree of the vertex $u \in V$ in the hesitancy fuzzy graph $G(V, E)$ is defined by $d_N(u) = \sum_{v \in N(u)} |v|$ and $d_E(u) = \sum_{v \in N(u)} \left\lfloor \frac{1 + \mu_2(uv) + \beta_2(uv) - \gamma_2(uv)}{3} \right\rfloor$.

Definition 3.4. The order of the vertex $u \in V$ in the hesitancy fuzzy graph $G(V, E)$ is defined by $O(G) = \sum_{v \in V} |v|$.

Example 1.



In the above example cardinality of the vertices are $|a| = 0.53, |b| = 0.4, |c| = 0.53, |d| = 0.53$, the degree of the vertices are $d_N(a) = 0.53, d_N(b) = 0.53, d_N(c) = 0.4, d_N(d) = 0.53$ and the order of the graph is $O(G) = 1.99$. The minimal strong dominating set and minimal weak dominating set are $S = \{a, c\}$ and $W = \{b, d\}$.

3.2. Strong Domination in Hesitancy Fuzzy Graph.

In this section, introduce the strong dominating set in hesitancy fuzzy graphs. Further investigate some bounds and properties of strong domination number

Definition 3.5. In hesitancy fuzzy graph $G(V, E)$ the vertex u strongly dominates v if

- (i) uv is a strong edge in $G(V, E)$;
- (ii) $d_N(u) \geq d_N(v)$. The set $D \subseteq V$ is said to be a strong dominating set of $G(V, E)$ such that every vertex $v \in V - D$ is strongly dominated by a vertex $u \in D$.

Definition 3.6. Minimum cardinality among the strong dominating set is called a minimal dominating set and the cardinality of the set is called the minimum domination number $\zeta_s(G)$ of the hesitancy fuzzy graph $G(V, E)$.

Theorem 3.1. Let $G(V, E)$ be a hesitancy fuzzy graph and u is vertex having maximum degree of the graph $G(V, E)$, then $V - N(u)$ is a strong dominating set of $G(V, E)$.

Proof. Let $G(V, E)$ be a hesitancy fuzzy graph and u is vertex having maximum degree of the graph $G(V, E)$, i.e $d_N(u) = \Delta_N(G)$. Therefore we get $d_N(u) > d_N(v) \forall v \in N(u)$ this implies u strongly dominates every vertex in its Neighborhood. If $v \in V - N(u)$ and $v \in N(u)$ such that $d_N(u) > d_N(v)$ therefore u strongly dominates v . If $v \notin N(u)$ such that $d_N(v) = d_N(v)$ therefore v strongly dominates v . This implies we get $V - N(u)$ is a strong dominating set of $G(V, E)$. Hence proved. $\square$

Theorem 3.2. Let $G(V, E)$ be a hesitancy fuzzy graph and v is vertex having minimum degree of the graph $G(V, E)$, then $V - N(v)$ is a weak dominating set of $G(V, E)$.

Proof. Let $G(V, E)$ be a hesitancy fuzzy graph and u is vertex having minimum degree of the graph $G(V, E)$, i.e $d_N(u) = \delta_N(G)$. Therefore we get

$d_N(v) < d_N(u) \forall u \in N(v)$ this implies u weakly dominates every vertex in its neighborhood. If $u \in V - N(v)$ and $u \in N(u)$ such that $d_N(u) > d_N(v)$ therefore u weakly dominates v . If $u \notin N(v)$ such that $d_N(u) = d_N(v)$ therefore v weakly dominates v . This implies we get $V - N(v)$ is a weak dominating set of $G(V, E)$. Hence proved. $\square$

Theorem 3.3. *Let $G(V, E)$ be a hesitancy fuzzy graph and D is a minimal strong dominating set, then $V - D$ is a weak dominating set of $G(V, E)$.*

Proof. Let $G(V, E)$ be a hesitancy fuzzy graph and D is a minimal strong dominating set of $G(V, E)$ therefore every there is a vertex $u \in D$ such that $d_N(u) \geq d_N(v) \forall v \in V - D$. Now we prove $V - D$ is a weak dominating set of $G(V, E)$. Let the vertex $v \in V - D$ such that $d_N(v) \leq d_N(u) \forall u \in D$. Therefore we get $V - D$ is a weak dominating set of $G(V, E)$. Hence proved. $\square$

Theorem 3.4. *Let $G(V, E)$ be a hesitancy fuzzy graph and D is a minimal weak dominating set, then $V - D$ is a strong dominating set of $G(V, E)$.*

Proof. Let $G(V, E)$ be a hesitancy fuzzy graph and D is a minimal weak dominating set of $G(V, E)$ therefore every there is a vertex $u \in D$ such that $d_N(u) \leq d_N(v) \forall v \in V - D$. Now we prove $V - D$ is a strong dominating set of $G(V, E)$. Let the vertex $v \in V - D$ such that $d_N(v) \geq d_N(u) \forall u \in D$. Therefore we get $V - D$ is a strong dominating set of $G(V, E)$. Hence proved. $\square$

Theorem 3.5. *Let $G(V, E)$ be a hesitancy fuzzy graph of order p , then*

- (1) $\zeta(G) \leq \zeta_s(G) \leq p - \Delta_N(G) \leq p - \Delta_E(G)$;
- (2) $\zeta(G) \leq \zeta_w(G) \leq p - \delta_N(G) \leq p - \delta_E(G)$.

Proof. Let $G(V, E)$ be a hesitancy fuzzy graph of order p and u is vertex having maximum degree of the graph $G(V, E)$, i.e $d_N(u) = \Delta_N(G)$. Let D is a minimal strong dominating set of $G(V, E)$ note that every strong dominating set is a dominating set but it is not a minimal dominating set S of $G(V, E)$ therefore we get

$$|S| \leq |D|$$

$$(3.1) \quad \Rightarrow \zeta(G) \leq \zeta_s(G).$$

From above theorem 4.2 $V - N(u)$ is a strong dominating set of $G(V, E)$ but not a minimal strong dominating set of $G(V, E)$ therefore we get

$$|D| \leq |V - N(u)|$$

$$(3.2) \quad \Rightarrow \zeta_s(G) \leq p - \Delta_N(u).$$

Clearly the neighborhood degree is less than effective neighborhood degree of the vertex in $G(V, E)$, therefore we get

$$\Delta_N(G) \leq \Delta_E(G).$$

$$(3.3) \quad \Rightarrow p - \Delta_N(u) \leq p - \Delta_E(u)$$

From (1), (2) and (3) we get $\zeta(G) \leq \zeta_s(G) \leq p - \Delta_N(G) \leq p - \Delta_E(G)$.

Similarly we prove this result for weak dominating set Let D is a minimal weak dominating set of $G(V, E)$ note that every weak dominating set is a dominating set but it is not a minimal dominating set S of $G(V, E)$ therefore we get

$$|S| \leq |D|$$

$$(3.4) \quad \Rightarrow \zeta(G) \leq \zeta_w(G).$$

From above theorem 4.2, u is vertex having minimum degree of the graph $G(V, E)$, then $V - N(u)$ is a weak dominating set of $G(V, E)$ but not a minimal weak dominating set of $G(V, E)$ therefore we get

$$|D| \leq |V - N(u)|$$

$$(3.5) \quad \Rightarrow \zeta_w(G) \leq p - \Delta_N(u).$$

Clearly the neighborhood degree is less than effective neighborhood degree of the vertex in $G(V, E)$, therefore we get

$$\delta_N(G) \leq \delta_E(G)$$

$$(3.6) \quad \Rightarrow p - \delta_N(u) \leq p - \delta_E(u).$$

From (1), (2) and (3) we get $\zeta(G) \leq \zeta_w(G) \leq p - \delta_N(G) \leq p - \delta_E(G)$. $\square$

Theorem 3.6. Let $G(V, E)$ be a complete hesitancy fuzzy graph and u and v are the vertex having minimum cardinality and maximum cardinality respectively, then $\zeta_s(G) = |u|$, $\zeta_w(G) = |v|$.

Proof. Let $G(V, E)$ be a complete hesitancy fuzzy graph. Therefore we get $|u| = \sum_{v \neq u} |v|$ since $G(V, E)$ be a complete hesitancy fuzzy graph, there is a strong edge between every pair of vertices. Let u is a vertex having the minimum cardinality. This implies $d_N(u) \geq d_N(v)$ for all $v \in V$. The set $D \subseteq V$ is a minimum strong dominating set of $G(V, E)$. Therefore $D = \{u\}$

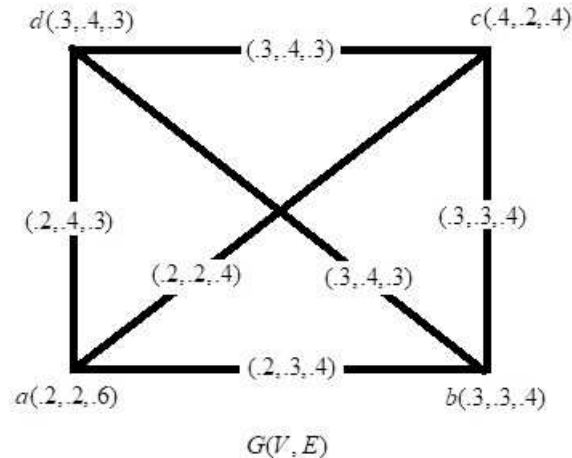
$$\Rightarrow \zeta_s(G) = |u|.$$

Similarly assume v is the vertex having the maximum cardinality therefore we get $|v| = \sum_{u \neq v} |u|$ since $G(V, E)$ be a complete hesitancy fuzzy graph, there is a strong edge between every pair of vertices. Let u is a vertex having the maximum cardinality. This implies $d_N(v) \geq d_N(u)$ for all $u \in V$. The set $D \subseteq V$ is a minimum weak dominating set of $G(V, E)$. Therefore $D = \{v\}$

$$\Rightarrow \zeta_w(G) = |v|.$$

Hence proved. □

Example 2.



In the above example $G(V, E)$ is a complete hesitancy fuzzy graph cardinality of the vertices are $|a| = 0.53$, $|b| = 0.47$, $|c| = 0.53$, $|d| = 0.4$, the degree of the vertices are $d_N(a) = 1.4$, $d_N(b) = 1.4$, $d_N(c) = 1.4$, $d_N(d) = 1.53$. The strong dominating number and weak dominating number are $\zeta_s(G) = 0.4$ and $\zeta_w(G) = 0.53$.

Theorem 3.7. Let $G(V, E)$ be a regular hesitancy fuzzy graph, then $\zeta_s(G) = \frac{O(G)}{2}$.

Proof. Let $G(V, E)$ be a regular hesitancy fuzzy graph, therefore every vertex $u \in V$ having the same degree. Let D is a minimal strong dominating set of $G(V, E)$ this implies every vertex $v \in V - D$ such that $d_N(u) \geq d_N(v)$ here $G(V, E)$ is a regular hesitancy fuzzy graph this implies $d_N(u) = d_N(v)$ the minimum strong dominating set

$$\zeta_s(G) = \min\{|V - N(u)|, |D|\} \Rightarrow \zeta_s(G) = \min\{|D|, |D|\} \text{ since } G \text{ is regular}$$

$$\Rightarrow \zeta_s(G) = \frac{O(G)}{2}$$

Hence proved. □

4. CONCLUSION

The concept of domination in Hesitancy fuzzy graphs is very rich both in theoretical developments and applications. In this paper, the vertex degree and strong neighbor of the vertex are introduced by using membership, non-membership and hesitancy degree of vertex in the Hesitancy fuzzy graph is defined and strong domination number of a Hesitancy fuzzy graph also defined. Further some properties and bounds of strong domination number of the hesitancy fuzzy graph are discussed.

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