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SOME RESULTS FOR EQUITABLE EDGE COLORING OF SOME CYCLE AND SILICATE RELATED GRAPHS

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ABSTRACT. A Proper edge coloring of a Graph G is an assignment of colors to the edges of a graph G such that adjacent edges should receive different colors. An Equitable Edge Coloring of G is a proper edge coloring of G such that the number of edges of one color class differs from the number of edges of any other color class by at most one. In this Paper we are going to state the Equitable edge coloring of some cycle and silicate related Graphs.

1. INTRODUCTION

Suppose let us consider G to be a finite graph without self-loops and multiple edges. It is defined as a non-empty set of points V together with the set of all unordered pair of points E called edge set. A molecular graph is a graph which represents the arrangement of atoms in a molecule by the following way: Representing the atoms of the molecule by vertices and the covalent bond between them by edges.

1.1. **Prerequisites and Basic Definitions.**

Definition 1.1. (Proper Edge Coloring [1]) *An Edge Coloring of a graph G is an assignment of colors to the edges of G. An Edge Coloring of G is called Proper K edge Coloring if no two adjacent edges have assigned same color.*

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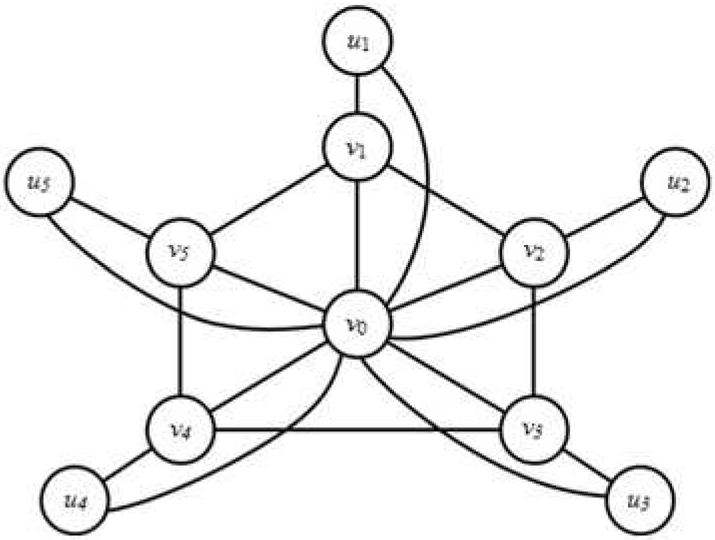
Definition 1.2. (Equitable Edge Coloring [7]) *A Proper Edge Coloring of G is called Equitable if $|l\,(s) - l(t)| \leq 1$ where $l(s)$ and $l(t)$ denote the number of edges in the color class s and t respectively.*

Definition 1.3. (Nearly Equitable Edge Coloring [7]) *A Proper Edge Coloring of G is called Nearly Equitable if $|l(s) - l(t)| \leq 2$ where $l(s)$ and $l(t)$ denote the number of edges in the color class s and t respectively.*

Definition 1.4. (Molecular Graph [11]) *A Molecular graph is a graph $G = (V, E)$ represents the arrangement of atoms in a molecule, where V is the set of all non-hydrogen atoms in particular carbon items and E is the set of all covalent bonds between them.*

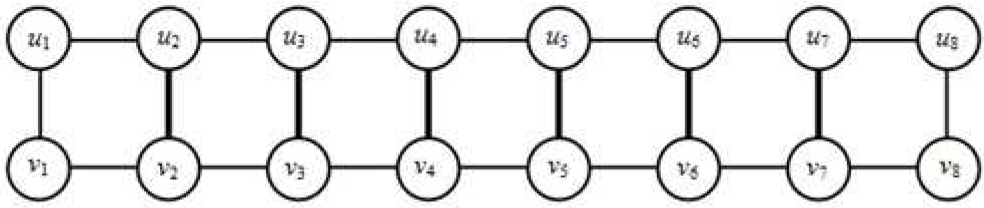
Definition 1.5. (Flower Graph Fl_n [3]) *A flower graph denoted by Fl_n is defined as a graph obtained from the Helm graph by joining each pendant vertex to the central vertex of the helm.*

Example 1. Fl_5



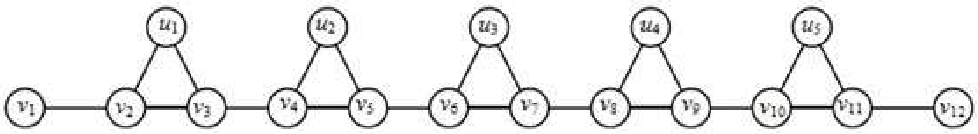
Definition 1.6. (Ladder Graph L_n [9]) *The ladder graph L_n is a planar undirected graph with $2n$ vertices and $n + 2(n - 1)$ edges defined as the Cartesian product of the two paths graphs P_n and P_2.*

Example 2. *LADDER GRAPH $\mathcal{L}_8$*



Definition 1.7. (Alternate Triangle snake graph $A(T_n)$) *An Alternate triangle snake graph is defined as the undirected planar graph with* $3n + 3$ *vertices and* $4n + 3$ *edges obtained from the path graph* P_n $(n \; is \; an \; even \; number)$ *by joining the vertices* v_{2i-1} *and* v_{2i} *with the new vertex* u_i *where* $i = 1, 2, 3, \ldots, n$.

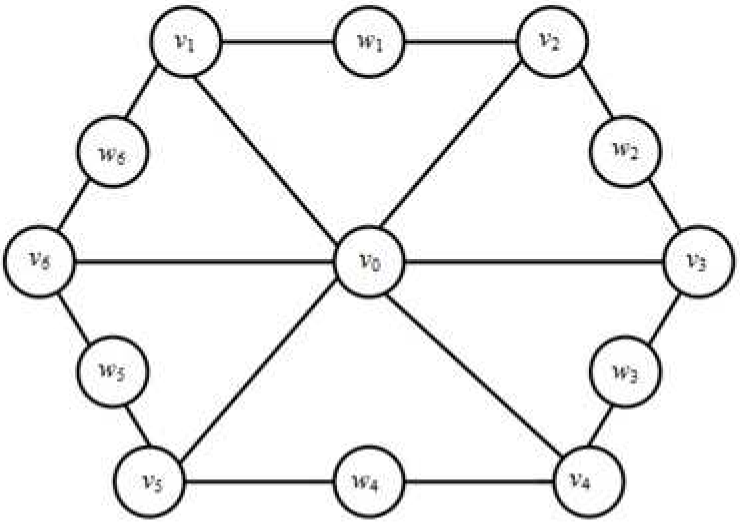
Example 3. *ALTERNATE TRIANGULAR SNAKE GRAPH* $A(\mathcal{T}_{\mathbf{12}})$



Definition 1.8. (Gear Graph) *The Gear graph represented by* G_n *is defined as a planar graph obtained from the wheel graph by inserting a new vertex between the adjacent vertices of the cycle* C_n *of the wheel graph.*

Theorem 1.1. (VIZING'S THEOREM [7] and [10]) *Every simple undirected graph* G *may be Edge colored using a number of colors that is at most one larger than the highest degree* Δ *of the graph* G.

Example 4. *GEAR GRAPH* $\mathcal{G}_{\mathbf{6}}$



2. Main Results

Algorithm 2.1. Construction algorithm for Helm-flower graph.

Step 1: Consider a Cycle $C_n, n \geq 3$ of length "n" whose vertices be designated as $v_1, v_2, \dots v_n$.
Step 2: Insert a new vertex say v_0 inside the cycle C_n and n vertices outside the cycle C_n say $u_1, u_2, \dots u_n$.
Step 3: Join the vertex v_0 with the vertices $u_1, u_2, \dots u_n$.
Step 4 : Join the vertex u_i with the vertex $v_i, i = 1, 2, 3, \dots n$. Thus we get a graph called Helm-flower Graph designated by Hfl_n

Observations 2.1. *In the above graph we have number of vertices $= |V| = 2n+1$, number of edges $= |E| = 3n$ and Sum of the degree of all the vertices of the graph $G = 6n$. Degree of the vertices $v_1, v_2, v_3, \dots vn$ are equals to 3. Degree of the central vertex V_0 is n. Degree of the vertices $u_1, u_2 \dots un$ are equal to 2. So at least Δ Colors needed for proper edge coloring of this graph where $\Delta = n$.*

Theorem 2.1. *The Helm- flower graph admits Equitable edge coloring and its equitable edge chromatic number is $\chi_e'(G) = n$.*

Proof. Let G be a Helm-Flower graph. By Theorem 1.1 [10] we need at least Δ colors for proper edge coloring of this graph. Now color the edges of G by the mapping $f : E(G) \to C$ where $C = \{1, 2, 3, \ldots, 2n\}$ such that

$$f(uv) = \begin{cases} i \ for \ u = v_i \ and \ v = v_{i+1}, \ i = 1, 2, 3, \ldots, n-1 \\ n \ for \ u = v_n \ and \ v = v_1 \\ i \ for \ u = v_0 \ and \ u_i, \ i = 1, 2, 3, \ldots, n \\ i+1 \ for \ u = v_i \ and \ v = u_i, \ i = 1, 2, 3 \ldots, n-1 \\ 1 \ for \ u = v_n \ and \ v = u_n \end{cases} .$$

Through this mapping we get $|l(s) - l(t)| \leq 1$, where $l(s)$ and $l(t)$ denotes the number of edges in the color classes s and t respectively.Thus the graph G admits equitable edge coloring. Also its equitable edge chromatic number is $\chi_e^{'}(G) = n$. □

Algorithm 2.2. Construction algorithm for Triangle-related ladder Graph
Step 1: Draw two paths of length $n-1$ say P_n and $P_n^{'}$ where the vertices of P_n and $P_n^{'}$ be $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ respectively.
Step 2: Join the vertices u_i and $v_i i = 1, 2, 3, n-1$.
Step 3: Add a new vertex called central vertex v_0.
Step 4: Join the vertices u_1 and v_1 with v_0. Thus we get a graph called Triangle-related ladder graph.

Observations 2.2. *In the Triangle-related ladder graph we have number of vertices $= |V| = 2n + 1$, number of edges $= |E| = 2n + 1$ and Sum of the degree of all the vertices of the graph $G = 4n - 4$. Degree of the vertices $v_i, u_i, i = 1, 2, 3 \ldots, n-1$ are equal to 3. Degree of the vertices u_n and v_n are 2. So at least Δ Color needed for proper edge coloring of this graph where $\Delta = 3$.*

Theorem 2.2. *The Triangle-related Ladder graph G admits equitable edge coloring with equitable edge chromatic number is $\chi_e^{'}(G) = 3$.*

Proof. Let G be a Triangle-related Ladder graph. By Theorem 1.1 we need at least Δ colors to coloring of the edges of G properly where $\Delta = 3$. . So we need 3 colours.

Now color the edges of G by the function $f : E(G) \to C$ where $C = \{1, 2, 3, \ldots, 2n\}$ such that

$$f(uv) = \begin{cases} 1 \ for \ u = v_0 \ and \ v = v_1 \\ 1 \ for \ u = v_{2i} \ and \ v = v_{2i+1}, i = 1, 2, 3, \ldots, \lfloor n/2 \rfloor \\ 1 \ for \ u = u2i - 1 \ and \ v = u_{2i}, i = 1, 2, 3 \ldots, \lfloor n/2 \rfloor \\ 2 \ for \ u = v_0 \ and \ v = u_1 \\ 2 \ for \ u = v_{2i-1} \ and \ v = v_{2i}, i = 1, 2, 3 \ldots, \lfloor n/2 \rfloor \\ 2 \ for \ u = u_{2i} \ and \ v = u_{2i+1}, i = 1, 2, 3 \ldots, \lfloor n/2 \rfloor \\ 3 \ if \ u = v_i \ and \ v = u_i, i = 1, 2, 3 \ldots, n \end{cases}.$$

Thus from this mapping we get $|l\,(s) - l(t)| \leq 1$ where $l(s)$ and $l(t)$ denote the number of edges in the color class s and t respectively and this proves that the Triangle-related Ladder graph admits the equitable edge coloring and its equitable edge chromatic number is $\chi_e'(G) = 3$. □

Algorithm 2.3. Construction Algorithm for Gear-Silicate Graph $\mathcal{GS}_n$

Step 1: Consider a Cycle $C_n, n \geq 3$ of length "n" whose vertices be designated as $v_1, v_2, \ldots, v_n$.

Step 2: Insert a new vertex say v_0 inside the cyclic graph C_n and join all the vertices of the cyclic graph C_n to the vertex v_0 graph to form the wheel graph W_n.

Step 3: Insert a new vertex $u_i, i = 1, 2, 3, \ldots, n$ for each triangular face of the wheel graph. Then join the newly added vertex u_i to all the vertices in that particular face of the wheel graph. This is repeated for all the faces of the wheel graph w_n. So we get a cyclic Silicate graph.

Step 4: Insert new vertices $w_i i = 1, 2, 3, \ldots, n$ between the two adjacent vertices on the cycle of the cyclic-silicate graph such that the vertex w_i is adjacent to v_i and $v_{i+1}, i = 1, i = 1, 2, 3, \ldots, n-1$ and the vertex w_n is adjacent to the vertices v_n and v_1.

Hence we get a graph called Gear-silicate Graph denoted by GS_n.

Observations 2.3. *In the above graph we have number of vertices $= |V| = 3n + 1$, number of edges $= |E| = 5n$ and Sum of the degree of all the vertices of the graph $G = 10n$. Degree of the vertices $v_1, v_2, v_3, \ldots, v_n$ are equals to 5. Degree of the central vertex V_0 is $2n$. Degree of the vertices $w_1, w_2 \ldots, w_n$ are equal to 2 and* *degree of the vertices $u_i, i = 1, 2, 3, \ldots, n$ are equal to 3. So at least $\Delta = 2n$(By theorem 1.1) Colors needed for proper edge coloring of this graph.*

Theorem 2.3. *The Gear-silicate Graph admits Equitable edge coloring. Also its equitable edge chromatic number is $\chi'_e = 2n$.*

Proof. Let us consider G to be a Gear- Silicate graph. Now color the edges of by the function
$f : E(G) \to C$ where $C = \{1, 2, 3, \ldots, 2n\}$ such that

$$f(uv) = \begin{cases} 2i\ for\ u = v_i\ and\ v = w_i, i = 1, 2, 3, \ldots, n \\ 2i - 1\ for\ u = v_0\ and\ v = v_i, i = 1, 2, 3 \ldots, n \\ 2j - 1\ for\ u = u_j\ and\ v = v_{i+1}, i, j = 1, 2, 3, \ldots, n \\ 2j + 1\ for\ u = u_j\ and\ v = v_i i, j = 1, 2, 3, \ldots, n \\ 1\ for\ u = u_n\ and\ v = v_n \\ 2j\ for\ u = v_0\ and\ v = u_j, j = 1, 2, 3 \ldots, n - 1 \\ 2i - 2\ for\ u = w_i\ and\ v = v_{i+1} i = 2, 3.4, \ldots, n - 1 \\ 2n - 2\ for\ u = w_n\ and v = v_1 \\ 2n\ for\ u = w_1\ and\ v = v_2 \end{cases}.$$

Therefore through this mapping we have $|l\,(s) - l(t)| \leq 1$ where $l(s)$ and $l(t)$ denote the number of edges in the color class s and t respectively and this proves that the Gear-silicate graph admits the equitable edge coloring and its equitable edge chromatic number is $\chi'_e(G) = 2n$. □

Algorithm 2.4. Construction Algorithm for Odd-Alternate triangle snake graph

Step 1: Draw a path of odd length with n vertices $v_1, v_2, \ldots, v_n$.
Step 2: Add $m = n/2$ new vertices $u_1, u_2, \ldots, u_m$.
Step 3: Join the vertex u_i with the vertices v_{2i-1} and v_{2i}, $i = 1, 2, 3, \ldots, m$
Step 4: Hence we get a graph called Odd-path Alternate triangular snake graph and it is denoted by OAT_n.

Observations 2.4. *In this graph we have total number of vertices $= |V| = 3n$, total number of edges $= |E| = 4n - 1$. Degree of the vertices $u_1, u_2, \ldots, u_m$ are equal to 2, degree of the vertices v_1, v_n are equal to 2 and degree of the vertices*

$v_2, v_3 \ldots, v_{n-1}$ are equal to 3.Total degree of all the vertices $= 8n - 2$. So at least $\Delta = 4$ colors needed for proper edge coloring of this graph.

Theorem 2.4. *The Odd Alternate triangular snake graph admits the equitable edge coloring with $\chi_e^{'}(OAT_n.) = 4$.*

Proof. Let G be a Odd-Alternate triangular snake graph .By Theorem 1.1 here we need at least $\Delta = 3$ colors for proper edge coloring of this graph.
Let us coloring this graph by define a mapping $f : E(G) \to \{1, 2, 3\}$ as

$$f(uv) = \begin{cases} 1\ for\ u = v_i\ and\ v = v_{i+1}\ i\ is\ odd\ and\ i \leq n/2 \\ 2\ for\ u = v_i\ and\ v = v_{i+1},\ i\ is\ even\ and\ i \leq n/2 \\ 3\ for\ u = v_i\ and\ v = u_i,\ i\ is\ odd\ and\ i \leq n/2 \\ 4\ for\ u = v_i\ and\ v = u_i,\ i\ is\ even\ and\ i \leq n/2 \end{cases}.$$

Hence from this mapping we see that $|l\,(s) - l(t)| \leq 1$ where $l(s)$ and $l(t)$ denote the number of edges in the color class s and t respectively and this proves that the Odd-Alternate Triangular Snake graph admits the equitable edge coloring and its equitable edge chromatic number is $\chi_e^{'}(G) = 3$. □

3. Conclusion

Hence in this work, we have stated the algorithms for cycle and silicate related graphs and also stated their equitable edge coloring.

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