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ψ-$\mathcal{I}$-CLOSED SET, WEAKLY ψ-$\mathcal{I}$-CLOSED SET AND CONTRA ψ-$\mathcal{I}$-CONTINUOUS MAPPING IN IDEAL TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce a new class of sets namely ψ-$\mathcal{I}$-closed sets, weakly ψ-$\mathcal{I}$-closed sets and contra-ψ-$\mathcal{I}$ continuous mappings in ideal topological spaces and investigate their properties and relations.

1. INTRODUCTION

N.Levine[14] and M.E.Abd El-Monsef et al. [1] introduced semi-open sets and β-sets respectiely. Levine [18] generalised the concept of closed sets to generalised closed sets. Bhattacharaya and Lahiri [6] generalized the concept of closed sets to semi-generalized closed sets via semi-open sets. The complement of a semi-open(resp. semi-generalized closed) set is called semi-closed [7] (resp. semi-generalized open [6])set. In 2000 M.K.R.S. Veera Kumar [25] introduced ψ-closed sets and in 1992 Jankovic and Hamlett [16] introduced the notion of I-open sets in an ideal topological spaces via ideals. Abd El-Monsef et al[1] further investigated $\mathcal{I}$-open sets and $\mathcal{I}$-continuous functions. The purpose of this paper is to give a new class of ψ-$\mathcal{I}$-closed and weakly ψ-$\mathcal{I}$-closed sets in an ideal topological space and derive some characterizations . Later S. Jafari and T. Noiri [15] introduced Contra -α-continuous between topological spaces. Throughout this paper, $int(A)$ and $cl(A)$ denote the interior and closure of A,

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respectively. An ideal $\mathcal{I}$ on a topological space(X, τ) is a nonempty collection of subsets of x which satisfies (i) A $\in \mathcal{I}$ and B $\subset$ A implies B $\in$ A implies B $\in \mathcal{I}$ and (ii) A $\in \mathcal{I}$ and B $\in \mathcal{I}$ implies A $\cup$ B $\in \mathcal{I}$. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\mathcal{P}$(X) is the set of all subsets of X, then the set operator $(.)^*$: $\mathcal{P}$(X)$\rightarrow$ $\mathcal{P}(X)$, called the local function of A with respect to τ and $\mathcal{I}$, is defined as follows: For A $\subset$ X, $A^*(\tau, \mathcal{I}) = \{x \in X : U \cap A \notin \mathcal{I}\}$, for every open set U of X containing x. A Kuratowski closure operator cl^*(.) for a topology $\tau^*(\tau, \mathcal{I})$ called the $*$-topology, finer than τ is defined by cl^*(A) = A $\cup A^*(\tau, \mathcal{I})$ when there is no chance of confusion. $A^*(\mathcal{I})$ is denoted by A^*. If $\mathcal{I}$ is an ideal on X, then (X, τ, $\mathcal{I}$) is called an ideal topological space.

2. PRELIMINARIES

Here we will recall some definitions used in sequel.

Definition 2.1. *A subset A of a topological space (X, τ) is said to be:*

(i) *sg-closed [6] if scl(A)$\subseteq$U, whenever $A \subseteq U$ and U is semi-open in (X, τ),*

(ii) *ψ-closed [24] if scl(A)$\subseteq$U, whenever $A \subseteq U$ and U is semi-generalised open in (X, τ).*

The complement of sg-closed and ψ-closed sets are called sg-open and ψ-open sets respectively.

Definition 2.2. *A subset A of a topological space (X, τ) is said to be*

(i) *pre-closed [18] if cl(int((A))$\subseteq$ A;*

(ii) *α-closed [22] if cl(int(cl(A)))$\subseteq$ A;*

(ii) *β-closed [2] if int(cl(int(A)))$\subseteq$ A.*

The complement of pre-closed,α-closed and β-closed sets are called pre-open, α-open and β-open sets respectively.

Definition 2.3. *A subset A of an ideal topological space (X, τ, I) is said to be*

(i) *$\mathcal{I}$-closed [1] if $cl(A^*)$ $\subseteq$A,*

(ii) *pre-$\mathcal{I}$-closed [9] if $cl^*(int$(A))$\subseteq$A ,*

(iii) *$semi$-$\mathcal{I}$-closed [14] if $int(cl^*$(A))$\subseteq$A,*

(iv) *α-$\mathcal{I}$-closed [13] if $cl^*(int(cl^*$(A)))$\subseteq$ A,*

(v) *β-$\mathcal{I}$-closed [12] if $int(cl^*(int$(A)))$\subseteq$ A.*

Definition 2.4. *A subset A of an ideal topological space (X, τ, I) is said to be*

(i) *$\mathcal{I}_g$-closed [10] if $A^* \subseteq U$, whenever $A \subseteq U$ and U is open in (X, τ, I),*
(ii) *$\alpha\mathcal{I}_g$-closed [18] if $A^* \subseteq U$, whenever $A \subseteq U$ and U is α-open in (X, τ, I),*
(iii) *$\mathcal{I}_{\widehat{g}}$-closed [4] if $A^* \subseteq U$, whenever $A \subseteq U$ and U is semi-open in (X, τ, I).*

Definition 2.5. *([13]) A function f :$(X, \tau) \to (Y, \sigma)$ is said to be $contra$-continuous if for every V$\in \sigma$, $f^{-1}(V)$ is open in (X, τ).*

Definition 2.6. *A function f :$(X, \tau, I) \to (Y, \sigma)$ is said to be*

(i) *$semi$-I-continuous [14] if for every V$\in \sigma$, $f^{-1}(V)$ is semi-I-open,*
(ii) *α-I-continuous [13] if for every V$\in \sigma$, $f^{-1}(V)$ is pre-α-open,*
(iii) *$semi$-continuous [17] if for every V$\in \sigma$, $f^{-1}(V)$ is semi-open,*
(iv) *α-continuous [22] if for every V$\in \sigma$, $f^{-1}(V)$ is α-open.*

Definition 2.7 (23). *A function f : (X, τ,$\mathcal{I}$) $\to$(Y, σ) is called contra strongly-α-$\mathcal{I}$-continuous if $f^{-1}(V)$ is strongly α-$\mathcal{I}$-open in (X, τ, I) as well as B_I set for every closed set V of Y.*

Lemma 2.1. *If (X, τ, I) is any ideal topological space and A $\subseteq$ X then the following are equivalent.[21,Theorem 3.4]*

(i) *A is $\alpha\mathcal{I}_g$-closed;*
(ii) *$cl^*(A) \subseteq$ U whenever $A \subseteq$ U and U is α-open in X.*

Lemma 2.2. *If (X, τ, I) is any ideal topological space and A $\subseteq$ X then the following are equivalent ([4,Theorem 2.5]):*

(i) *A is $\mathcal{I}_g$-closed;*
(ii) *$cl^*(A) \subseteq$ U whenever $A \subseteq$ U and U is $semi$-open in X.*

3. ψ-$\mathcal{I}$-CLOSED SET

Definition 3.1. *A subset A of an ideal topological space (X, τ, I) is said to be ψ-$\mathcal{I}$-Closed if $cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is ψ-open in (X, τ, I).*

Theorem 3.1. *Every closed set is ψ-$\mathcal{I}$-closed but not conversely.*

Proof. Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $cl^*(A) \subseteq cl(A)$ and A is closed, $cl^*(A) \subseteq cl(A)$ = A $\subseteq$ U. Therefore A is ψ-$\mathcal{I}$-closed set in (X, τ, I). □

Example 1. *Let $X = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$ and $\mathcal{I} = \{\phi, \{a\}\}$. Then ψ-$\mathcal{I}$-closed sets are $\{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ and closed sets are $\{X, \phi, \{a, b\}, \{a\}, \{b\}\}$. It is clear that $\{a\}$ is ψ-$\mathcal{I}$-closed set but it is not closed.*

Theorem 3.2. *Let (X, τ, $\mathcal{I}$) be a ideal topological space. Then the following statements holds:*

(i) *Every ψ-$\mathcal{I}$-closed set is $\mathcal{I}_g$-closed;*
(ii) *Every ψ-$\mathcal{I}$-closed set is $\alpha\mathcal{I}_g$-closed;*
(iii) *Every ψ-$\mathcal{I}$-closed set is semi-closed;*
(iv) *Every ψ-$\mathcal{I}$-closed set is semi-$\mathcal{I}$-closed;*
(v) *Every ψ-$\mathcal{I}$-closed set is $\mathcal{I}_{\widehat{g}}$-closed;*
(vi) *Every ψ-$\mathcal{I}$-closed set is ψ-closed;*
(vii) *Every ψ-$\mathcal{I}$-closed set is β-$\mathcal{I}$-closed.*

Proof.

(i) Let $A \subseteq U$ and U is ψ-open in (X, τ, I). By hypothesis $cl^*(A) \subseteq$ U whenever $A \subseteq$ U and U is open in X. Since every ψ-open is open, $cl^*(A) \subseteq$ U whenever $A \subseteq$ U and U is open in X. Hence A is $\mathcal{I}_g$ closed in X.

(ii) Let $A \subseteq U$ and U is ψ-open in (X, τ, I). By hypothesis $cl^*(A) \subseteq$ U whenever $A \subseteq$ U and U is α-open in X. By Lemma[2.5].Since every ψ-open is α-open, cl^*(A)$\subseteq$ U whenever $A \subseteq$ U and U is α-open in X. Hence A is $\alpha\mathcal{I}_g$-closed in X.

(iii) Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $int(cl(A)) \subseteq cl^*(A)$ and A is ψ-$\mathcal{I}$-closed, $int(cl(A)) \subseteq cl^*(A)$ = A $\subseteq$ U. Therefore A is semi-closed set in X.

(iv) Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $int(cl^*(A)) \subseteq cl^*(A)$ and A is ψ-$\mathcal{I}$-closed, $int(cl^*$(A))$\subseteq$ cl^*(A) = A $\subseteq$ U. Therefore A is semi-$\mathcal{I}$-closed set in (X, τ, I).

(v) Let $A \subseteq U$ and U is ψ-open in (X, τ, I). By hypothesis $cl^*(A) \subseteq$ U whenever $A \subseteq$ U and U is semi-open in X.By Lemma[2.6]. Since every ψ-open is semi-open, cl^*(A)$\subseteq$ U whenever $A \subseteq$ U and U is semi-open in X. Hence A is $\mathcal{I}_{\widehat{g}}$-closed in (X, τ, I).

(vi) Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $int(cl(A)) \subseteq cl^*(A)$ and A is ψ-$\mathcal{I}$-closed, $int(cl(A)) \subseteq cl^*(A)$ = A $\subseteq$ U. Therefore A is ψ-closed set in (X, τ, I).

(vii) Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $int(cl^*(int(A))) \subseteq cl^*(A)$ and A is ψ-$\mathcal{I}$-closed, $int(cl^*(int(A))) \subseteq cl^*(A)$ = A $\subseteq$ U. Therefore A is β-$\mathcal{I}$-closed set in X.

□

But the converse of the above theorem need not be true as shown in the following examples.

Example 2.

(i) *Let X =* $\{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. *Then* $\mathcal{I}_g$*-closed sets are* $\{X, \phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. *and* ψ-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}\}$. *It is clear that* $\{a, d\}$ *is* $\mathcal{I}g$ *closed set but it is not* ψ-$\mathcal{I}$*-closed.*

(ii) *Let X =* $\{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c, d\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}\}$. *Then* α-$\mathcal{I}_g$*-closed sets are* $\{X, \phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$ *and* ψ-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{b\}, \{b, c, d\}, \{b, d\}, \{a, b, d\}\}$. *It is clear that* $\{a, b\}$ *is* $\alpha\mathcal{I}_g$ *closed set but it is not* ψ-$\mathcal{I}$*-closed.*

(iii) *Let X =* $\{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c, d\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}\}$. *Then semi-closed sets are* $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}\}$ *and* ψ-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{b\}, \{b, c, d\}, \{b, d\}, \{a, b, d\}\}$. *It is clear that* $\{a, b\}$ *is semi-closed set but it is not* ψ-$\mathcal{I}$ *closed.*

(iv) *Let X =* $\{a, b, c, d\}$, $\tau = \{X, \phi, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, b, d\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. *Then semi-*$\mathcal{I}$*-closed sets are* $\{X, \phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{c, d\}, \{b, d\}\ \{b, c, d\}\}$ *and* ψ-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}\}$. *It is clear that* $\{b, d\}$ *is semi-*$\mathcal{I}$*-closed set but it is not* ψ-$\mathcal{I}$*-closed.*

(v) *Let X =* $\{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}\}$. *Then* $\mathcal{I}_{\widehat{g}}$*-closed sets are* $\{X, \phi, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ *and* ψ-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. *It is clear that* $\{b, d\}$ *is* $\mathcal{I}_{\widehat{g}}$*-closed but it is not* ψ-$\mathcal{I}$*-closed.*

(vi) *Let X =* $\{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. *Then* ψ*-closed sets are* $\{X, \phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$ *and* ψ-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}\}$. *It is clear that* $\{a, d\}$ *is* $\mathcal{I}_g$ *closed set but it is not* ψ-$\mathcal{I}$*-closed.*

(vii) *Let X =* $\{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$ *and* $\mathcal{I}= \{\phi, \{c\}, \{d\}, \{c, d\}\}$. *Then* β-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{b, c, d\}\}$, *and* ψ-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}\}$. *It is clear that* $\{a, d\}$ *is* β-$\mathcal{I}$ *closed set but it is not* ψ-$\mathcal{I}$*-closed.*

Theorem 3.3.

(i) *Every pre-closed set and* ψ-$\mathcal{I}$*-closed set in* (X, τ, I) *are independent to each other.*

(ii) *Every* $\mathcal{I}$*-closed set and* ψ-$\mathcal{I}$*-closed set in* (X, τ, I) *are independent to each other.*

Proof. Follows from the following examples. □

Example 3.

(i) *Let X =*$\{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{c\}, \{a, c, d\}\}$ *and* $\mathcal{I}= \{\phi, \{b\}\}$. *Here pre-*$\mathcal{I}$*-closed sets are* $\{X, \phi, \{b\}, \{b, d\}, \{b, c, d\}\}$ *and* ψ-$\mathcal{I}$*-closed sets are* $\{X, \phi, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. *It is clear that* $\{b\}$ *is pre-closed but it is not* ψ-$\mathcal{I}$*-closed and also* $\{a, c, d\}$ *is* ψ-$\mathcal{I}$*-closed but not pre-closed.*

(ii) *Let X =*$\{a, b, c, d\}$, τ=$\{X, \phi, \{a\}, \{b\}, \{c, d\}, \{a, b\}, \{a, c, d\}, \{a, c, d\}, \{b, c, d\}\}$ *and* $\mathcal{I}= \{\phi, \{c\}\}$. *Here* $\mathcal{I}$*-closed sets are* $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, d\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$ *and* ψ *-* $\mathcal{I}$*-closed sets are* $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. *It is clear that* $\{d\}$ *is* $\mathcal{I}$*-closed but it is not* ψ-$\mathcal{I}$*-closed and also* $\{a, c, d\}$ *is* ψ-$\mathcal{I}$*-closed but not* $\mathcal{I}$*-closed.*

Remark 3.1. *The union of two* ψ-$\mathcal{I}$*-closed sets need not be a* ψ-$\mathcal{I}$*-closed set.*

Example 4. *Consider the ideal toplogical space* (X, τ, I)*, where Let X =*$\{a, b, c, d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\} \{a, b, c\}\}$ *and* $\mathcal{I}= \{\phi, \{a\}\}$. *In this ideal space the sets* $\{a\}$ *and* $\{b, c\}$ *are* ψ-$\mathcal{I}$*-closed sets but their union* $\{a, b, c\}$ *is not a* ψ-$\mathcal{I}$*-closed set.*

Remark 3.2. *The intersection of two* ψ-$\mathcal{I}$*-closed sets need not be a* ψ-$\mathcal{I}$*-closed set.*

Example 5. *Consider the ideal toplogical space* (X, τ, I)*, where Let X =*$\{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ *and* $\mathcal{I}= \{\phi, \{a\}\}$. *In this ideal space the sets* $\{a, b\}$ *and* $\{b, c\}$ *are* ψ-$\mathcal{I}$*-closed sets but their intersection* $\{b\}$ *is not a* ψ-$\mathcal{I}$*-closed set.*

Remark 3.3. *In the following diagram we denote by arrows the implications between ψ-$\mathcal{I}$-closed sets and other sets.*

$$\begin{array}{ccccc}
\mathbf{CS} & \rightarrow & \mathbf{SCS} \rightarrow \mathcal{I}\mathbf{CS} & & \\
 & \searrow & \uparrow & \nearrow & \\
\alpha\mathcal{I}_{\mathbf{g}}\mathbf{CS} & \leftarrow & \psi - \mathcal{I}\mathbf{CS} \rightarrow \beta - \mathcal{I}\mathbf{CS} & & \\
\uparrow & \swarrow & \downarrow & \searrow & \\
\mathcal{I}_{\widehat{\mathbf{g}}}\mathbf{CS} & \rightarrow & \mathcal{I}_{\mathbf{g}}\mathbf{CS} & & \psi - \mathbf{CS}
\end{array}$$

4. Weakly ψ-$\mathcal{I}$-closed set

Definition 4.1. *A subset A of an ideal topological space (X,τ,I) is said to be weakly ψ-$\mathcal{I}$-closed if $cl^*(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is ψ open in (X,τ,I).*

Theorem 4.1. *Every closed set is weakly ψ-$\mathcal{I}$-Closed but not conversely.*

Proof. Let $A \subseteq U$ and U is ψ-open in (X,τ,I). Since $cl^*(int(A)) \subseteq cl(A)$ and A is closed, $cl^*(int(A)) \subseteq cl(A)$ = A $\subseteq$ U. Therefore A is weakly ψ-$\mathcal{I}$-closed set in (X,τ,I). □

Example 6. *Let $X = \{a,b,c,d\}$, $\tau = \{X,\phi,\{a\},\{b\},\{b,c\}\ \{a,b,c\}\}$ and $\mathcal{I} = \{\phi,\{a\}\}$. Here closed sets are $\{X,\phi,\{b,c,d\},\{a,c,d\},\{c,d\},\{a,d\},\{d\}\}$ and weakly ψ $\mathcal{I}$-closed sets are $\{X,\phi,\{b,c,d\},\{a,c,d\},\{c,d\},\{a,d\},\{a\},\{d\}\}$. It is clear that $\{a\}$ is weakly ψ-$\mathcal{I}$-closed but it is not closed.*

Theorem 4.2. *Every pre-$\mathcal{I}$-closed set is weakly ψ-$\mathcal{I}$-closed but not conversely*

Proof. Let $A \subseteq U$ and U is ψ-open in (X,τ,I). By hypothesis $cl^*(int(A)) \subseteq$ U and A is weakly ψ-$\mathcal{I}$-closed in (X,τ,I). □

Example 7. *Let $X = \{a,b,c,d\}$, $\tau = \{X,\phi,\{a\},\{b,c\}\}$ and $\mathcal{I} = \{\phi,\{c\}\}$. Here pre-$\mathcal{I}$-closed sets are $\{X,\phi,\{a\},\{c\},\{a,c\},\{b,c\}\}$ and weakly ψ-$\mathcal{I}$-closed sets are $\{X,\phi,\{a\},\{b\},\{c\},\{a,c\},\{b,c\}\}$. It is clear that $\{b\}$ is weakly ψ-$\mathcal{I}$-closed but it is not pre-$\mathcal{I}$-closed.*

Theorem 4.3. *Every α-$\mathcal{I}$-closed set is weakly ψ-$\mathcal{I}$-Closed but not conversely*

Proof. Let $A \subseteq U$ and U is ψ-open in (X,τ,I). By hypothesis $cl^*(int(A)) \subseteq cl^*(int(cl^*(A)))$ and A is weakly ψ-$\mathcal{I}$-closed, $cl^*(int(A)) \subseteq cl^*(int(cl^*(A)))$ = A $\subseteq$ U. Therefore A is weakly ψ-$\mathcal{I}$-closed set in (X,τ,I). □

Example 8. *Let* $X = \{a, b, c, d\}$*,* $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}\}$*. Here* α*-*$\mathcal{I}$*-closed sets are* $\{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ *and weakly* ψ*-*$\mathcal{I}$*-closed sets are* $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$*. It is clear that* $\{b\}$ *is weakly* ψ*-*$\mathcal{I}$*-closed but it is not* α*-*$\mathcal{I}$*-closed.*

Theorem 4.4. *Every weakly* ψ*-*$\mathcal{I}$*-closed set is* $\mathcal{I}$*-closed set but not conversely*

Proof. Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $cl(A^*) \subseteq cl^*(int(A))$ and A is weakly ψ-$\mathcal{I}$-closed, $cl(A^* \subseteq cl^*(int(A))$ = A $\subseteq$ U. Therefore A is weakly ψ-closed set in (X, τ, I). □

Example 9. *Let* $X = \{a, b, c, d\}$*,* $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}\}$*. Here* $\mathcal{I}$*-closed sets are*
$\{X, \phi, \{a\}, \{d\}, \{a, d\}, \{b, c, d\}, \{b, c\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ *and weakly* ψ*-*$\mathcal{I}$*-closed sets are*
$\{X, \phi, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$*. It is clear that* $\{b, c\}$ *is weakly* ψ*-*$\mathcal{I}$*-closed but it is not* $\mathcal{I}$*-closed.*

Theorem 4.5. *Every* b*-closed set is weakly* ψ*-*$\mathcal{I}$*-closed set but not conversely*

Proof. Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $int(cl(int(A))) \subseteq cl^*(int(A))$ and A is weakly ψ- $\mathcal{I}$-closed, $int(cl(int(A))) \subseteq cl^*(int(A))$ = A $\subseteq$ U. Therefore A is b-closed set in X. □

Example 10. *Let* $X = \{a, b, c, d\}$*,* $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}\}$*. Here* b*-closed sets are* $\{X, \phi, \{a\}, \{b, c\}\}$ *and weakly* ψ $\mathcal{I}$*-closed sets are*
$\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$*. It is clear that* $\{b\}$ *is weakly* ψ $\mathcal{I}$*-closed but it is not* $\mathcal{I}$*-closed.*

Theorem 4.6. *Every* b*-*$\mathcal{I}$*-closed set is weakly* ψ*-*$\mathcal{I}$*-closed set but not conversely*

Proof. Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $int(cl^*(int(A))) \subseteq cl^*(int(A))$ and A is weakly ψ $\mathcal{I}$-closed, $int(cl^*(int(A))) \subseteq cl^*(int(A))$ = A $\subseteq$ U. Therefore A is b-$\mathcal{I}$-closed set in X. □

Example 11. *Let* $X = \{a, b, c, d\}$*,* $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{a\}\}$*. Here* b*-*$\mathcal{I}$*-closed sets are* $\{X, \phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ *and weakly* ψ*-*$\mathcal{I}$*-closed sets are* $\{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$*. It is clear that* $\{b\}$ *is weakly* ψ*-*$\mathcal{I}$*-closed but it is not* b*-*$\mathcal{I}$*-closed.*

Theorem 4.7. *Every α-closed set is weakly ψ-$\mathcal{I}$-closed set but not conversely*

Proof. Let $A \subseteq U$ and U is ψ-open in (X, τ, I). Since $cl^*(int(A)) \subseteq cl(int(cl(A)))$ and A is α-closed, $cl^*(int(A)) \subseteq cl(int(cl(A)))$ = A $\subseteq$ U. Therefore A is weakly ψ-closed set in (X, τ, I). $\square$

Example 12. *Let X =$\{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}\}$ and $\mathcal{I} = \{\phi\}$. Here α-closed sets are $\{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and weakly ψ-$\mathcal{I}$-closed sets are $\{X, \phi, \{a, b\}\}$. It is clear that $\{b\}$ is weakly ψ-$\mathcal{I}$-closed but it is not $\mathcal{I}$-closed.*

Remark 4.1. *In the following diagram we denote by arrows the implications between weakly ψ-$\mathcal{I}$-closed sets and other sets.*

$$\begin{array}{ccccc}
\textbf{pre}\mathcal{I}\textbf{CS} & \rightarrow & \alpha\mathcal{I}\textbf{CS} \rightarrow \alpha\textbf{CS} & & \\
\uparrow & \searrow & \downarrow \quad \nearrow & & \\
\mathcal{I}\textbf{CS} & \leftarrow & \textbf{weakly} \ \psi-\mathcal{I}\textbf{CS} \leftarrow \textbf{CS} & & \\
 & \nearrow & \uparrow & & \\
\textbf{b}\mathcal{I}\textbf{CS} & \rightarrow & \textbf{bCS} & &
\end{array}$$

5. CONTRA ψ-$\mathcal{I}$-CONTINUOUS MAPPING

Definition 5.1. *A function f : (X, τ,$\mathcal{I}$) $\rightarrow$ (Y, σ) is called contra ψ-$\mathcal{I}$-continuous if for every V$\in$ σ, $f^{-1}(V)$ is ψ-$\mathcal{I}$-open in (X, τ, I) for every closed set V of Y.*

Theorem 5.1. *Let (X, τ, $\mathcal{I}$)$\rightarrow$ (Y, σ) be any mapping. Then the following statements holds.*

(i) *Every contra α-$\mathcal{I}$-continuous function is contra ψ-$\mathcal{I}$-continuous.*
(ii) *Every contra α-continuous function is contra ψ-$\mathcal{I}$-continuous.*
(iii) *Every contra-continuous function is contra ψ-$\mathcal{I}$-continuous.*
(iv) *Every contra semi-$\mathcal{I}$-continuous function is contra ψ-$\mathcal{I}$-continuous*
(v) *Every contra semi-continuous function is contra ψ-$\mathcal{I}$-continuous.*
(vi) *Every contra strongly α-continuous function is contra ψ-$\mathcal{I}$-continuous.*
(vii) *Every contra β-$\mathcal{I}$-continuous function is contra ψ-$\mathcal{I}$-continuous.*
($viii$) *Every contra β-continuous function is contra ψ-$\mathcal{I}$-continuous.*

Proof.

(i) Suppose that $x \in$ X and V is any closed set of Y containing $f(x)$. Since f is α-$\mathcal{I}$ continuous and cl(V) is closed in Y, f^{-1}(V) is α-$\mathcal{I}$-open and f^{-1}(cl(V)) is α-$\mathcal{I}$ closed. Now put U = $f^{-1}(v)$. Then we have U $\in \psi\mathcal{I}O(x, X)$. This shows that f is contra ψ-$\mathcal{I}$-continuous functions.

(ii) Suppose that $x \in$ X and V is any closed set of Y containing $f(x)$. Since f is contra α-continuous and let A be a α open set. Then A = U $\cap$ V, where U $\in \tau$. Then $int(cl(int(V))) \supset A$. This shows that f is contra-ψ-$\mathcal{I}$ continuous function.

(iii) Suppose that $x \in$ X and V is any closed set of Y containing $f(x)$. Since f is contra-continuous. U $\in \psi\mathcal{I}O(x, X)$. Therefore f is contra-ψ-$\mathcal{I}$-continuous function.

(iv) Suppose that $x \in$ X and V is any closed set of Y containing $f(x)$. Since f is contra semi-$\mathcal{I}$ continuous and every ψ-$\mathcal{I}$ set is semi-$\mathcal{I}$-open. Therefore f is contra ψ-$\mathcal{I}$-continuous function.

(v) Suppose that $x \in$ X and V is any closed set of Y containing $f(x)$. Since f is contra semi-continuous and every ψ-$\mathcal{I}$ set is semi-$\mathcal{I}$-open. Therefore f is contra ψ-$\mathcal{I}$-continuous function.

(vi) Suppose that $x \in$ X and V is any closed set of Y containing $f(x)$. Since f is contra strongly α-$\mathcal{I}$-continuous and every ψ-$\mathcal{I}$ set is α-$\mathcal{I}$-open as well as B_I set for every closed set V of Y. Therefore f is contra ψ-$\mathcal{I}$-continuous function.

(vii) Suppose that $x \in$ X and V is any closed set of Y containing $f(x)$. Since f is contra β-$\mathcal{I}$ continuous and every β-$\mathcal{I}$ set is ψ-$\mathcal{I}$-open. Therefore f is contra ψ-$\mathcal{I}$-continuous function.

$(viii)$ Suppose that $x \in$ X and V is any closed set of Y containing $f(x)$. Since f is contra β-continuous and every ψ-$\mathcal{I}$ set is α-$\mathcal{I}$-open. Therefore f is contra ψ-$\mathcal{I}$-continuous function.

□

The converse of Theorem 5.2 are need not be true as shown in the following examples.

Example 13.

(i) *Let X* $= \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \{\phi\}, \{a\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I}= \{\phi\}, \{a\}$ *Define the identity mapping f:(X,* τ_1*,* $\mathcal{I}) \rightarrow (X, \tau_2$*) is contra* α-$\mathcal{I}$*-continuous but not contra* ψ-$\mathcal{I}$*-continuous.*

(ii) *Let X* $= \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \{\phi\}, \{a\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I}= \{\phi\}, \{a\}$ *Define the identity mapping* f:(X, τ_1, $\mathcal{I}$) $\rightarrow (X, \tau_2)$ *is contra* α*-continuous but not contra* ψ-$\mathcal{I}$*-continuous.*

(iii) *Let X* $= \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \{\phi\}, \{a\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I}= \{\phi\}, \{a\}$ *Define the identity mapping* f:(X, τ_1, $\mathcal{I}$) $\rightarrow (X, \tau_2)$ *is contra-continuous but not contra* ψ-$\mathcal{I}$*-continuous.*

(iv) *Let X* $= \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \{\phi\}, \{a\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I}= \{\phi\}, \{a\}$ *Define the identity mapping* f:(X, τ_1, $\mathcal{I}$) $\rightarrow (X, \tau_2)$ *is contra semi-*$\mathcal{I}$*-continuous but not contra* ψ-$\mathcal{I}$*-continuous.*

(v) *Let X* $= \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \{\phi\}, \{a\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I}= \{\phi\}, \{a\}$ *Define the identity mapping* f:(X, τ_1, $\mathcal{I}$) $\rightarrow (X, \tau_2)$ *is contra semi-continuous but not contra* ψ-$\mathcal{I}$*-continuous.*

(vi) *Let X* $= \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \{\phi\}, \{a\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I}= \{\phi\}, \{a\}$ *Define the identity mapping* f:(X, τ_1, $\mathcal{I}$) $\rightarrow (X, \tau_2)$ *is contra strongly* α-$\mathcal{I}$*-continuous but not contra* ψ-$\mathcal{I}$*-continuous.*

(vii) *Let X* $= \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \{\phi\}, \{a\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I}= \{\phi\}, \{a\}$. *Define the identity mapping* f:(X, τ_1, $\mathcal{I}$) $\rightarrow (X, \tau_2)$ *is contra* β-$\mathcal{I}$*-continuous but not contra* ψ-$\mathcal{I}$*-continuous.*

$(viii)$ *Let X* $= \{a, b, c\}$, $\tau_1 = \{X, \phi, \{a\}, \{a, b\}\}$, $\tau_2 = \{X, \{\phi\}, \{a\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I}= \{\phi\}, \{a\}$ *Define the identity mapping* f:(X, τ_1, $\mathcal{I}$) $\rightarrow (X, \tau_2)$ *is contra* β*-continuous but not contra* ψ-$\mathcal{I}$*-continuous.*

Remark 5.1. *In the following diagram we denote by arrows the implications between* ψ-$\mathcal{I}$*-closed sets and other sets.*

$$
\begin{array}{ccccccc}
 & \mathbf{cCS} & & \mathbf{cSCS} & & \mathbf{c\alpha\mathcal{I}CS} & \\
 & & \searrow & \downarrow & \swarrow & & \\
\mathbf{cS\mathcal{I}CS} & \rightarrow & & \mathbf{c\psi-\mathcal{I}CS} & & \leftarrow & \beta-\mathcal{I}\mathbf{CS} \\
 & & \nearrow & \uparrow & \nwarrow & & \\
 & \mathbf{c\beta\mathcal{I}CS} & & \mathbf{c\beta CS} & & \mathbf{cs\psi-CS} &
\end{array}
$$

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