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# REVERSE WIENER INDEX OF UNITARY ADDITION CAYLEY GRAPHS

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ABSTRACT. In this paper Reverse Wiener Index of Unitary Addition Cayley graph $G_n$is calculated. The vertices of Unitary Addition Cayley graph $G_n$ is $Z_n = 0, 1, ..., (n-1)$. If $gcd(x+y, n) = 1$,then the vertices x and y adjacent.

## 1. INTRODUCTION

Let G be a connected graph with vertex and edge sets V(G) and E(G) respectively. The distance between the vertices $v_i$ and $v_j$ of G is defined as the number of edges in a minimal path connecting the vertices $v_i$ and $v_j$ and is denoted by $d(v_(i), v_j)$.

"For a positive integer n > 1, the Unitary Addition Cayley graph Gn is the graph whose vertex set is $Z_n$, the integers modulo n and if $U_n$ be the set of all units of the ring $Z_n$, then two vertices x, y are adjacent if and only if $x + y \in U_n$" [3].

The topological index is a numerical parameters of a graph, and is invariant under graph automorphism. The Wiener Index was introduced by Harry Wiener in [2,4]. Let W(G) be the Wiener Index of G and is defined as the sum of the distances between all unordered pairs of vertices. i.e ., $W(G) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} d(v_i, v_j)$ [1] .

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The largest distance between any two vertices $v_i$ and $v_j$ of the graph G is the diameter d of the graph. In [1,5] the Reverse Wiener matrix

$$[RW]_{ij} = \begin{cases} d - d(v_i, v_j) & ifi \neq j \\ 0 & ifi = j \end{cases}.$$

The Reverse Wiener Index is $RW(G) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}[RW]_{ij}$.

**Theorem 1.1.** ([5]) The Wiener Index of Unitary Addition Cayley graph $G_n$ is as follows

$$W(G_n) = \begin{cases} \frac{n^2-1}{2} & n \text{ is a prime number} \\ \frac{3}{4}n^2 - 4 & n = 2^\alpha \ \alpha > 1 \\ \frac{5}{4}n^2 - n\phi(n) - n & n \text{ is even and prime divisor} \\ (n-1)(n - \frac{\phi(n)}{2}) & n \text{ odd but not prime} \end{cases},$$

where $\phi(n)$ denotes the Euler pi function.

### 1.1. **Reverse Wiener Index Of Unitary Addition Cayley Graph.**

**Theorem 1.2.** *If $G_n$ is the Unitary Addition Cayley graph on n vertices $(n \geq 2)$ then the Reverse Wiener Index of $G_n$ is as follows:*

$$RW(G_n) = \begin{cases} (\frac{n}{2})^2 & n = 2^\gamma, \gamma > 1 \\ \frac{(n-1)^2}{2} & n \textit{ is prime} \\ \frac{n^2-2n+4n\phi(n)}{4} & n \textit{ is even and ha prime divisor} \\ \frac{(n-1)\phi(n)}{2} & n \textit{ is odd not a prime number} \end{cases}.$$

*Proof.* Consider Unitary Addition Cayley graph Gn. To prove this theorem in four classifications of $n$.

**Case (i):** When $n = 2^\gamma, \gamma > 1$, $G_n$ is a complete bipartite graph with bipartition of set $V(G_n)$ into $X = \{0, 2, ..., (n-2)\}$ and $Y = \{1, 3, ..., (n-1)\}$. Hence d(u,v)=1 or 2 $u, v \in V(G_n)$. Therefore diameter of d is 2, d(u,v) =1 is $\frac{n^2}{4}$ times and $d(u,v) = 2$ is $\frac{n(n-2)}{4}$ times. Hence Reverse Wiener Index $RW(G_n) = \frac{n^2}{4}$.

**Case (ii):** Let n be prime. The distance between any two vertices are 1 or 2. Therefore, diameter is 2. Here $\frac{n-1}{2}$ vertices have distance 2 and $\frac{(n-1)^2}{2}$ vertices have distance 1. By the definition of Reverse Wiener Index consider the distance 1 as $\frac{(n-1)^2}{2}$. Therefore $RW(G_n) = \frac{(n-1)^2}{2}$.

**Case (iii):** Let n be an even number and has an odd prime divisor ,where $n \neq 2^\gamma, \gamma > 1$. $G_n$ is bipartite graph with vertex set $V(G_n)$ as $X \cup Y$. d(u,v) is either $u \in X$ or $u \in Y$.

Suppose $u \in X$ and $v \in X$ clearly u and v are not adjacent. Therefore d(u,v)=2 occurs in $\frac{(n^2-2n)}{4}$ times.

Suppose $u \in X$ and $v \in Y$. Now we take $Y = S \cup T$ where $S = \{v \in Y; uv \in E(G_n)\}$ and $T = \{v \in Y; uv \notin E(G_n)\}$. Clearly d(u,v)=1 is $\frac{n\phi(n)}{2}$ times. Let $v \in T$, u and v are not adjacent, hence d(u,v)=3 occurs in $\frac{n^2-2n\phi(n)}{4}$ times. Therefore, diameter is 3. Hence $RW(G_n) = \frac{n^2-2n+4n\phi(n)}{4}$.

**Case (iv):** Let n be odd but not prime. Let $p_1, p_2, ..., p_t$ be the different prime divisors of n, $n = p_1^{r_1} \times p_2^{r_2} \times .... \times p_t^{r_t}$ and $p_i \neq 2, 1 \leq i \leq t$. Hence d(u,v)=1or 2.Therefore, diameter is 2,here $\frac{(n-1)}{2}\phi(n)$ vertices have distance 1and $\frac{(n-1)}{2}(n - \phi(n))$ vertices have distance 2.

Therefore Reverse Wiener Index $RW(G_n) = \frac{(n-1)\phi(n)}{2}$. □

1.2. **Relation between Wiener Index and Reverse Wiener Index.** Let G be a graph with n vertices, diameter d ,Wiener Index W(G) and Reverse Wiener Index RW(G).Then $RW(G) = \frac{1}{2}n(n-1)d - W(G)$.

## REFERENCES


[1] A.T. BALABAN, D. MILLS, O. IVANCIUC, S.C. BASAK: *Reverse Wiener Indices*, Croat. Chem. Acta, **73**(4) (2000), 923-941.

[2] A. GRAOVAC, T. PISANAKI: *On the Wiener index of a graph*, Journal of Mathematical Chemistry, **8**(1) (1991), 53-62.

[3] D. SINHA, P. GARG, A. SINGH: *Some properties of Unitary addition Cayley Graphs*, Notes on Number Theory and Discrete Mathematics, **17**(3) (2011), 49-59.

[4] M. ELISI, R. RAEISU, B. TAERI: *Wiener index of some graph operations*, **160**(9) (2012), 1333-1344.

[5] C. THILAGA, P.B. SARASIJA: *Wiener and hyper Wiener indices of Unitary addition Cayley Graphs*, International journal of Recent Technology and Engineering, **8**(2S3) (2019), 131-132.

[6] X. CA, B. ZHOU: *Reverse Wiener indices of connected graphs*, Match Commun. Mat. Comput. Chem, **60** (2008), 95-105.

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